Computer Algebra
Independent Integration Tests

Summer 2023 edition

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/202-7.6.1-u-a+b-arccsch-c-x-^n

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [178]. This is test number [202].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (178)	0.00 (0)
Mathematica	100.00 (178)	0.00 (0)
Fricas	69.10 (123)	30.90 (55)
Maple	57.87 (103)	42.13 (75)
Maxima	34.27 (61)	65.73 (117)
Mupad	27.53 (49)	72.47 (129)
Giac	25.84 (46)	74.16 (132)
Sympy	21.35 (38)	78.65 (140)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

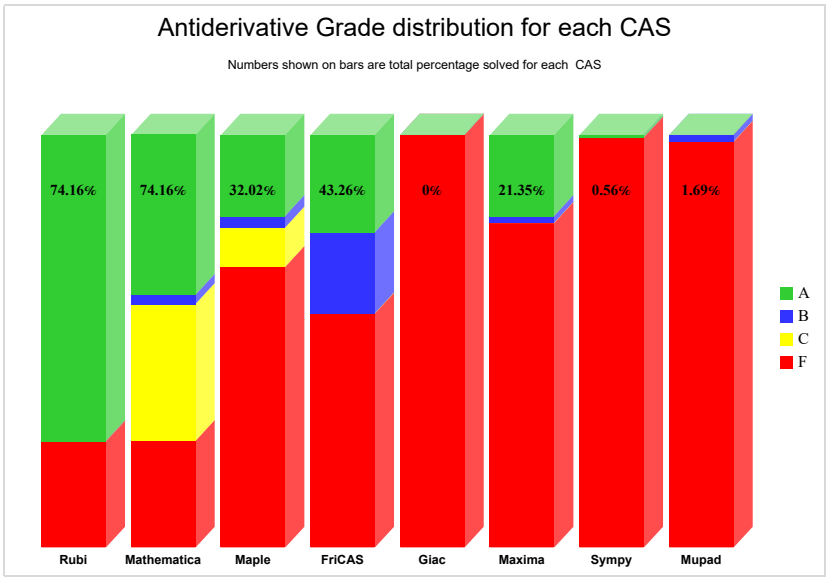
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

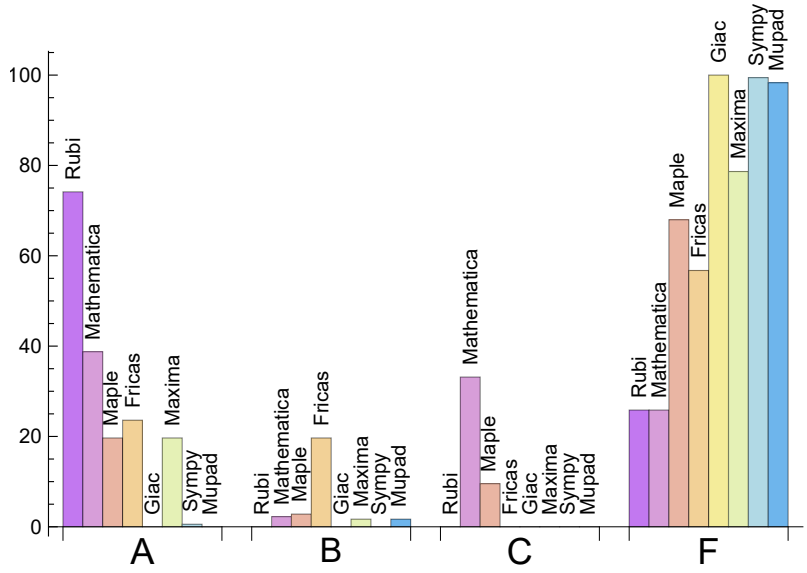
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.157	0.000	0.000	25.843
Mathematica	38.764	2.247	33.146	25.843
Fricas	23.596	19.663	0.000	56.742
Maple	19.663	2.809	9.551	67.978
Maxima	19.663	1.685	0.000	78.652
Sympy	0.562	0.000	0.000	99.438
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	1.685	0.000	98.315

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of

error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	55	89.09	10.91	0.00
Maple	75	100.00	0.00	0.00
Maxima	117	58.97	0.00	41.03
Mupad	129	0.00	100.00	0.00
Sympy	140	78.57	21.43	0.00
Giac	132	98.48	0.00	1.52

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Giac	0.28
Fricas	0.30
Rubi	0.38
Maxima	0.70
Maple	1.88
Mupad	5.44
Mathematica	6.03
Sympy	36.26

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	20.58	0.96	20.00	0.95
Giac	21.83	1.03	23.00	1.00
Mupad	26.80	1.21	27.00	1.17
Maxima	164.43	4.92	132.00	1.29
Rubi	223.66	1.00	132.50	1.00
Fricas	315.71	2.30	143.00	1.74
Mathematica	317.74	1.23	129.00	1.09
Maple	373.24	1.49	71.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

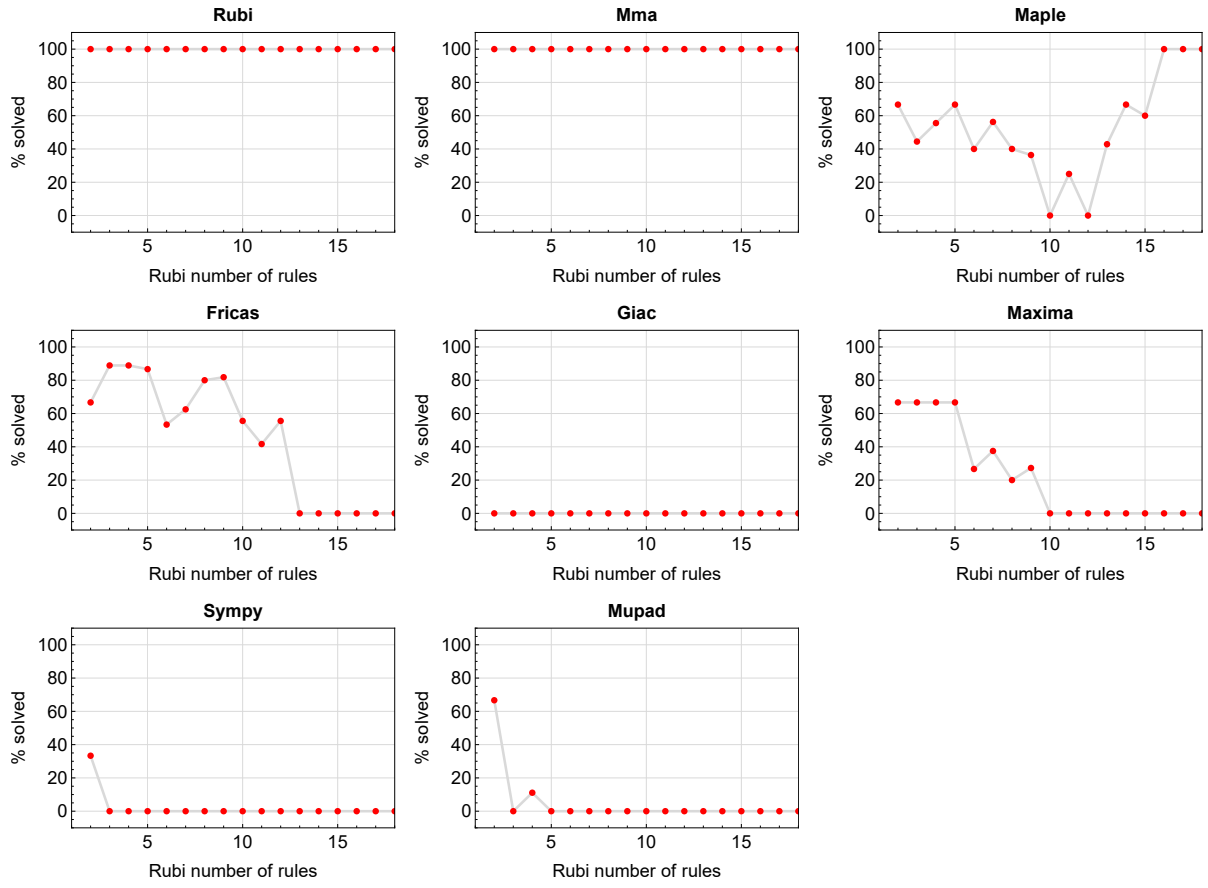


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

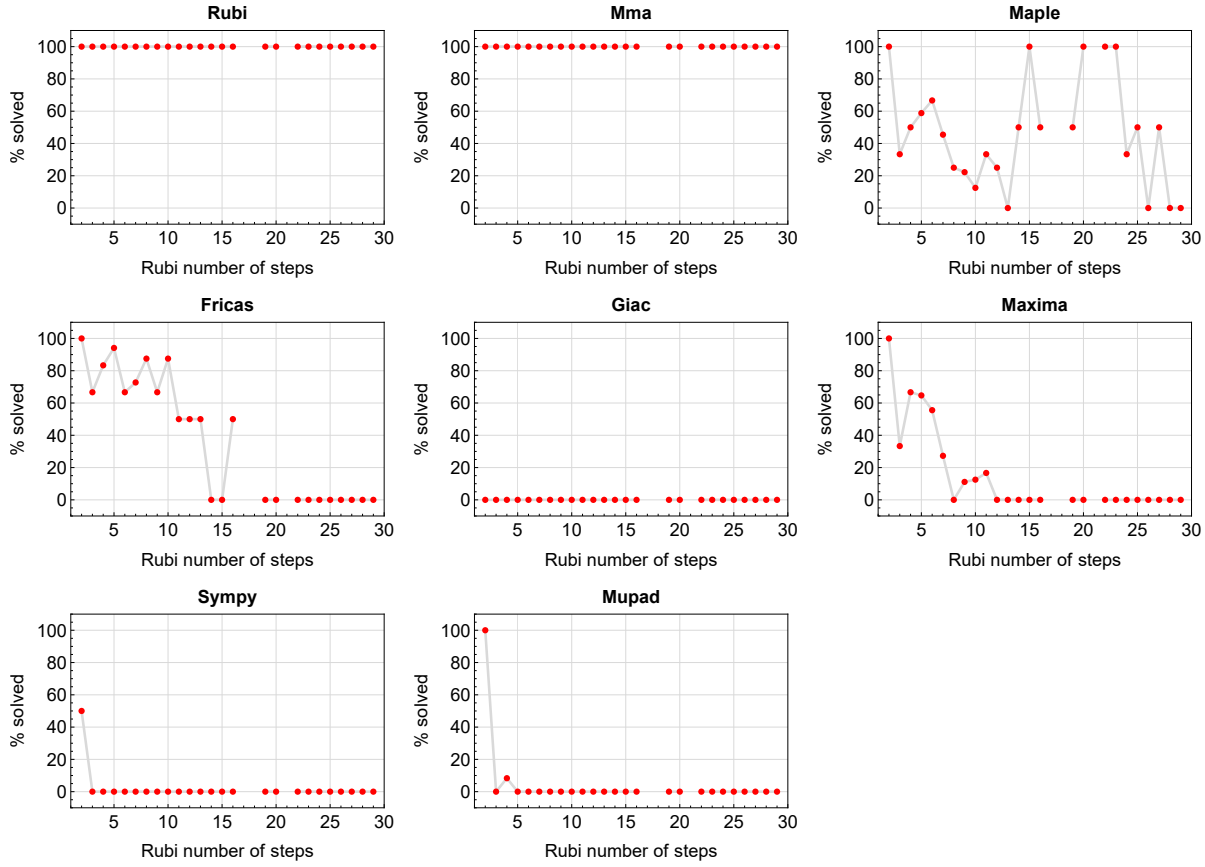


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

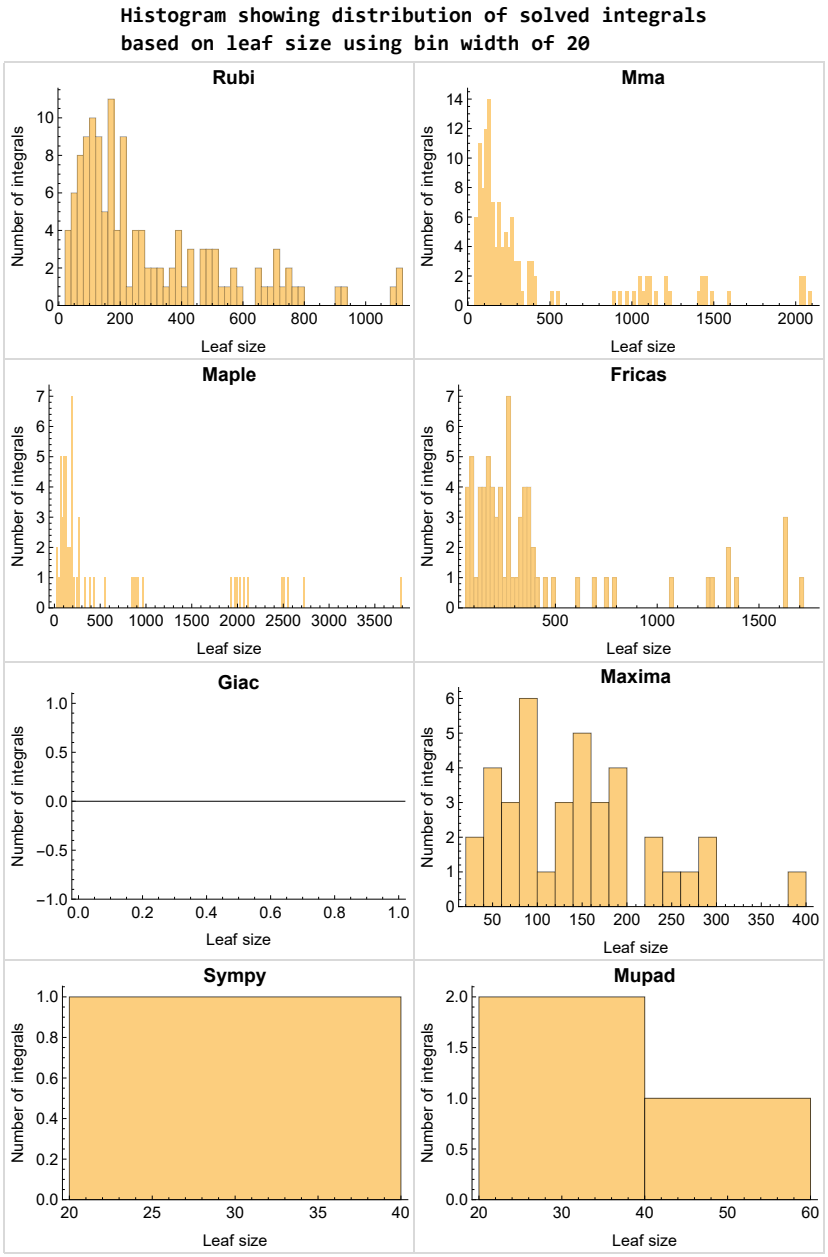


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

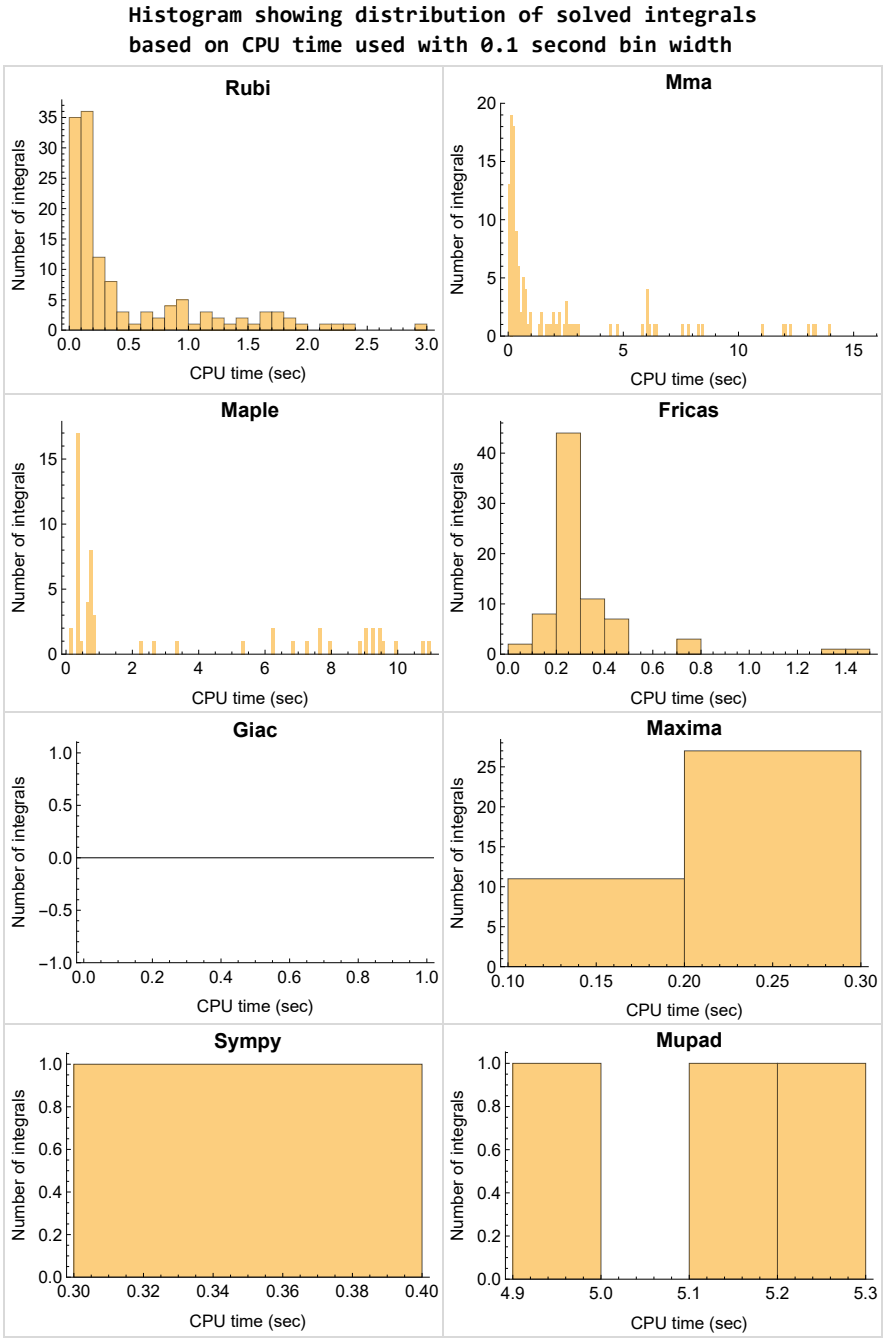


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

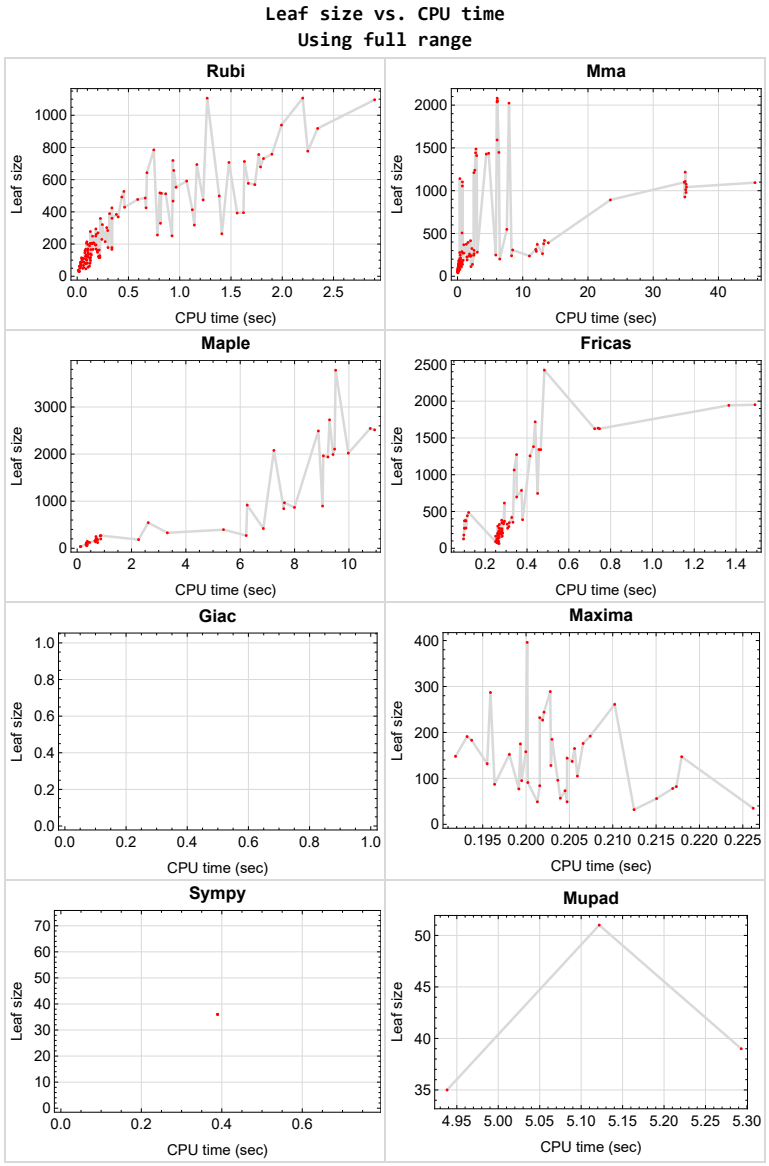


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {51, 53, 57, 58, 63, 69, 70, 75, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

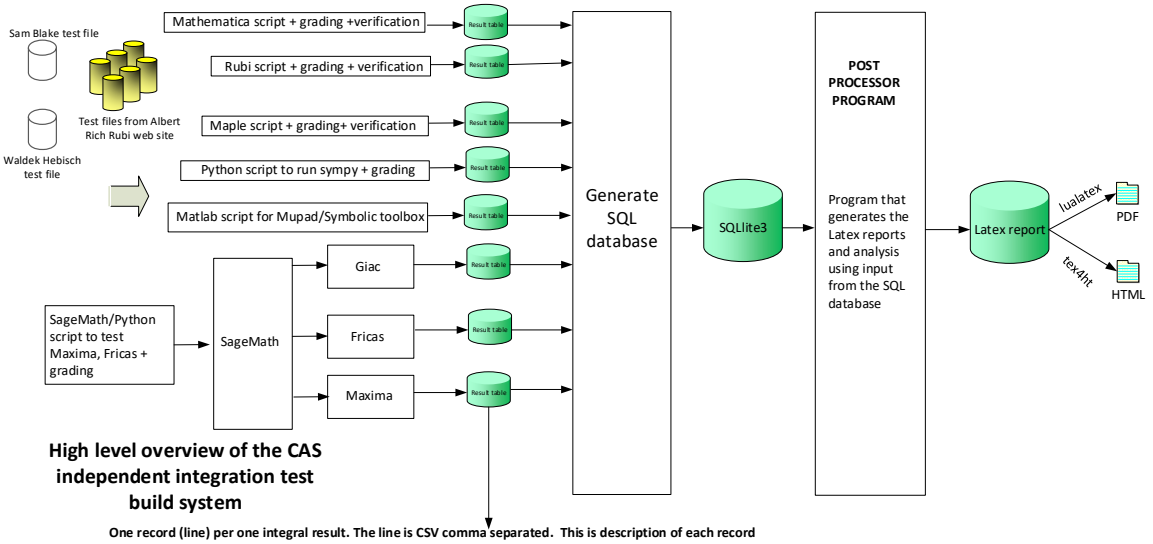
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
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2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 49, 50, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 145, 154, 163, 165, 166, 167, 174, 175, 176 }

B grade { 7, 25, 27, 47 }

C grade { 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 164 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 44, 45, 46, 47, 49, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { 10, 50, 105, 112, 113 }

C grade { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75 }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 4, 10, 11, 12, 13, 14, 23, 32, 76, 77, 80, 81, 82, 83, 84, 85, 88, 92, 93, 94, 95, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 145, 146, 147, 154, 155, 163, 164, 174, 175 }

B grade { 1, 3, 5, 6, 7, 9, 15, 17, 20, 21, 22, 29, 30, 31, 44, 45, 46, 47, 49, 50, 78, 79, 89, 90, 91, 105, 112, 113, 140, 148, 149, 156, 157, 158, 176 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 56, 57, 58, 59, 60, 63, 66, 70, 71, 72, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 165, 166, 167 }

F(-1) timedout fail { 52, 53, 64, 65, 69, 75 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 17, 20, 29, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { 10, 12, 14 }

C grade { }

F normal fail { 8, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 99, 101, 103, 104, 105, 106, 111, 112, 113, 114, 120, 129, 140, 149, 154, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 98, 100, 102, 107, 108, 109, 110, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 155, 156, 159, 160, 161, 162 }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 174, 175 }

Mupad

A grade { }

B grade { 6, 9, 10 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-2) exception fail { }

Sympy

A grade { 9 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 118, 119, 120, 126, 127, 129, 138, 139, 140, 145, 146, 148, 149, 154, 155, 166, 167, 176 }

F(-1) timedout fail { 74, 75, 110, 111, 112, 113, 114, 115, 116, 117, 128, 132, 136, 137, 147, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 169, 170, 174, 175 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	107	123	158	208	0	0	0
N.S.	1	1.00	0.97	1.12	1.44	1.89	0.00	0.00	0.00
time (sec)	N/A	0.046	0.163	0.458	0.200	0.283	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	72	79	77	97	0	0	0
N.S.	1	1.00	0.84	0.92	0.90	1.13	0.00	0.00	0.00
time (sec)	N/A	0.031	0.144	0.336	0.199	0.264	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	97	104	128	199	0	0	0
N.S.	1	1.00	1.13	1.21	1.49	2.31	0.00	0.00	0.00
time (sec)	N/A	0.037	0.062	0.347	0.203	0.278	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	70	57	87	0	0	0
N.S.	1	1.00	1.00	1.13	0.92	1.40	0.00	0.00	0.00
time (sec)	N/A	0.021	0.115	0.342	0.204	0.253	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	85	83	96	186	0	0	0
N.S.	1	1.00	1.37	1.34	1.55	3.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.069	0.326	0.204	0.265	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	61	35	70	0	0	39
N.S.	1	1.00	1.32	1.61	0.92	1.84	0.00	0.00	1.03
time (sec)	N/A	0.011	0.038	0.345	0.226	0.262	0.000	0.000	5.293

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	36	49	143	0	0	0
N.S.	1	1.00	2.13	1.20	1.63	4.77	0.00	0.00	0.00
time (sec)	N/A	0.015	0.128	0.135	0.201	0.263	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	51	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	40	58	32	64	36	0	35
N.S.	1	1.00	1.33	1.93	1.07	2.13	1.20	0.00	1.17
time (sec)	N/A	0.020	0.044	0.359	0.212	0.266	0.389	0.000	4.938

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	66	96	105	76	0	0	51
N.S.	1	1.00	1.32	1.92	2.10	1.52	0.00	0.00	1.02
time (sec)	N/A	0.027	0.052	0.357	0.206	0.261	0.000	0.000	5.122

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	59	71	56	77	0	0	0
N.S.	1	1.00	1.02	1.22	0.97	1.33	0.00	0.00	0.00
time (sec)	N/A	0.032	0.067	0.344	0.215	0.257	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	116	147	89	0	0	0
N.S.	1	1.00	1.05	1.57	1.99	1.20	0.00	0.00	0.00
time (sec)	N/A	0.035	0.065	0.358	0.218	0.250	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	69	79	73	87	0	0	0
N.S.	1	1.00	0.87	1.00	0.92	1.10	0.00	0.00	0.00
time (sec)	N/A	0.036	0.083	0.371	0.204	0.259	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	88	135	185	99	0	0	0
N.S.	1	1.00	0.90	1.38	1.89	1.01	0.00	0.00	0.00
time (sec)	N/A	0.047	0.092	0.386	0.203	0.252	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	272	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.088	0.223	0.000	0.000	0.266	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	225	0	0	0	0	0	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	1.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	87	0	82	234	0	0	0
N.S.	1	1.00	1.61	0.00	1.52	4.33	0.00	0.00	0.00
time (sec)	N/A	0.063	0.236	0.000	0.217	0.265	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	135	0	0	0	0	0	0
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	114	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	70	0	78	139	0	0	0
N.S.	1	1.00	1.43	0.00	1.59	2.84	0.00	0.00	0.00
time (sec)	N/A	0.055	0.163	0.000	0.217	0.261	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	163	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.066	0.145	0.000	0.000	0.280	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	106	0	0	178	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.093	0.203	0.000	0.000	0.260	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	147	0	0	202	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.087	0.178	0.000	0.000	0.259	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	271	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.984	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	548	0	0	0	0	0	0
N.S.	1	1.00	2.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	7.563	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	171	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	246	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	181	0	0	0	0	0	0
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.180	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	132	0	144	222	0	0	0
N.S.	1	1.00	1.69	0.00	1.85	2.85	0.00	0.00	0.00
time (sec)	N/A	0.085	0.234	0.000	0.205	0.265	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	182	0	0	267	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.081	0.315	0.000	0.000	0.272	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	200	0	0	301	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.134	0.320	0.000	0.000	0.260	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	277	0	0	346	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.144	0.342	0.000	0.000	0.280	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.012	2.904	0.026	0.255	0.251	0.461	0.264	5.263

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.005	3.334	0.026	0.256	0.244	0.403	0.249	4.748

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.018	0.320	0.028	0.261	0.241	0.910	0.256	5.207

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1351	44	15	18	22
N.S.	1	1.00	1.12	1.00	84.44	2.75	0.94	1.12	1.38
time (sec)	N/A	0.020	5.763	0.028	7.733	0.249	22.104	0.266	5.470

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	644	30	15	18	22
N.S.	1	1.00	1.12	1.00	40.25	1.88	0.94	1.12	1.38
time (sec)	N/A	0.019	3.864	0.023	3.606	0.267	10.082	0.263	5.080

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	81	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.022	0.976	0.027	0.291	0.253	1.129	0.264	4.722

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	566	32	15	18	22
N.S.	1	1.00	1.12	1.00	35.38	2.00	0.94	1.12	1.38
time (sec)	N/A	0.021	1.991	0.027	0.741	0.261	5.299	0.275	4.771

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	165	250	261	419	0	0	0
N.S.	1	1.00	0.99	1.50	1.56	2.51	0.00	0.00	0.00
time (sec)	N/A	0.339	0.260	0.692	0.210	0.327	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	186	192	328	0	0	0
N.S.	1	1.00	1.00	1.52	1.57	2.69	0.00	0.00	0.00
time (sec)	N/A	0.208	0.170	0.691	0.207	0.305	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	119	98	87	207	0	0	0
N.S.	1	1.00	1.47	1.21	1.07	2.56	0.00	0.00	0.00
time (sec)	N/A	0.126	0.222	0.335	0.196	0.277	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	36	49	143	0	0	0
N.S.	1	1.00	2.13	1.20	1.63	4.77	0.00	0.00	0.00
time (sec)	N/A	0.022	0.046	0.118	0.205	0.262	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	506	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	134	184	0	354	0	0	0
N.S.	1	1.00	1.37	1.88	0.00	3.61	0.00	0.00	0.00
time (sec)	N/A	0.130	0.219	2.260	0.000	0.287	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	204	544	0	745	0	0	0
N.S.	1	1.00	1.25	3.34	0.00	4.57	0.00	0.00	0.00
time (sec)	N/A	0.214	0.430	2.615	0.000	0.451	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	918	918	1094	2515	0	0	0	0	0
N.S.	1	1.00	1.19	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.345	45.597	10.945	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	679	418	1963	0	0	0	0	0
N.S.	1	1.00	0.62	2.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.788	13.327	9.058	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	926	840	0	0	0	0	0
N.S.	1	1.00	2.16	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	34.852	7.602	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	138	21	19	21	25
N.S.	1	1.00	1.10	0.90	6.57	1.00	0.90	1.00	1.19
time (sec)	N/A	0.058	43.245	0.161	2.170	0.266	13.244	0.265	5.008

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	149	21	20	21	25
N.S.	1	1.00	1.10	0.90	7.10	1.00	0.95	1.00	1.19
time (sec)	N/A	0.069	6.806	0.174	2.340	0.279	14.315	0.266	4.845

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	380	1939	0	0	0	0	0
N.S.	1	1.00	0.78	3.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.664	13.215	9.227	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	939	939	1098	2545	0	0	0	0	0
N.S.	1	1.00	1.17	2.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.992	34.763	10.786	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	707	707	1012	1991	0	0	0	0	0
N.S.	1	1.00	1.43	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.480	34.981	9.414	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	416	868	0	0	0	0	0
N.S.	1	1.00	0.88	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.229	1.999	7.996	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	307	395	0	0	0	0	0
N.S.	1	1.00	1.08	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	8.452	5.385	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	116	29	19	21	25
N.S.	1	1.00	1.10	0.90	5.52	1.38	0.90	1.00	1.19
time (sec)	N/A	0.057	4.861	0.172	1.111	0.251	8.543	0.288	4.867

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	175	31	20	21	25
N.S.	1	1.00	1.10	0.90	8.33	1.48	0.95	1.00	1.19
time (sec)	N/A	0.057	8.453	0.168	2.383	0.245	17.808	0.285	5.467

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	731	731	1042	2021	0	0	0	0	0
N.S.	1	1.00	1.43	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.817	35.093	9.978	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	979	896	0	0	0	0	0
N.S.	1	1.00	1.96	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.385	35.032	9.030	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	264	421	0	0	0	0	0
N.S.	1	1.00	0.83	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.143	13.028	6.850	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	166	328	0	0	0	0	0
N.S.	1	1.00	1.11	2.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.698	3.320	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	179	40	19	21	25
N.S.	1	1.00	1.10	0.90	8.52	1.90	0.90	1.00	1.19
time (sec)	N/A	0.066	14.894	0.179	2.184	0.269	32.577	0.265	5.037

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	252	42	20	21	25
N.S.	1	1.00	1.10	0.90	12.00	2.00	0.95	1.00	1.19
time (sec)	N/A	0.073	19.339	0.165	2.669	0.257	59.648	0.273	4.992

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	777	777	1108	2728	0	0	0	0	0
N.S.	1	1.00	1.43	3.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.249	34.883	9.286	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	1076	2492	0	0	0	0	0
N.S.	1	1.00	1.89	4.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.733	35.046	8.875	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	390	2106	0	0	0	0	0
N.S.	1	1.00	0.99	5.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.560	13.927	9.472	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	892	2079	0	0	0	0	0
N.S.	1	1.00	2.42	5.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	23.432	7.239	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	239	51	19	21	25
N.S.	1	1.00	1.10	0.90	11.38	2.43	0.90	1.00	1.19
time (sec)	N/A	0.065	38.786	0.164	2.651	0.253	162.422	0.256	4.995

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	316	53	0	21	25
N.S.	1	1.00	1.10	0.90	15.05	2.52	0.00	1.00	1.19
time (sec)	N/A	0.080	34.940	0.168	3.027	0.257	0.000	0.261	4.986

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	648	785	1217	3782	0	0	0	0	0
N.S.	1	1.21	1.88	5.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.747	34.906	9.511	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	138	198	289	295	0	0	0
N.S.	1	1.00	0.64	0.93	1.35	1.38	0.00	0.00	0.00
time (sec)	N/A	0.095	0.266	0.701	0.203	0.313	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	119	158	227	273	0	0	0
N.S.	1	1.00	0.71	0.95	1.36	1.63	0.00	0.00	0.00
time (sec)	N/A	0.083	0.204	0.652	0.202	0.307	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	155	109	148	245	0	0	0
N.S.	1	1.00	1.35	0.95	1.29	2.13	0.00	0.00	0.00
time (sec)	N/A	0.043	0.270	0.356	0.192	0.280	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	109	102	84	222	0	0	0
N.S.	1	1.00	1.20	1.12	0.92	2.44	0.00	0.00	0.00
time (sec)	N/A	0.049	0.159	0.360	0.202	0.279	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	109	91	105	0	0	0
N.S.	1	1.00	0.62	1.00	0.83	0.96	0.00	0.00	0.00
time (sec)	N/A	0.064	0.120	0.346	0.200	0.257	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	93	127	132	127	0	0	0
N.S.	1	1.00	0.59	0.80	0.84	0.80	0.00	0.00	0.00
time (sec)	N/A	0.077	0.151	0.355	0.196	0.256	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	109	145	165	146	0	0	0
N.S.	1	1.00	0.53	0.71	0.80	0.71	0.00	0.00	0.00
time (sec)	N/A	0.091	0.173	0.374	0.206	0.266	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	114	139	176	165	0	0	0
N.S.	1	1.00	0.56	0.68	0.86	0.81	0.00	0.00	0.00
time (sec)	N/A	0.127	0.273	0.674	0.207	0.281	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	97	121	137	144	0	0	0
N.S.	1	1.00	0.61	0.76	0.86	0.91	0.00	0.00	0.00
time (sec)	N/A	0.101	0.233	0.753	0.205	0.266	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	77	181	95	123	0	0	0
N.S.	1	1.00	0.53	1.24	0.65	0.84	0.00	0.00	0.00
time (sec)	N/A	0.076	0.132	0.749	0.199	0.258	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	107	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	124	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	182	270	396	388	0	0	0
N.S.	1	1.00	0.70	1.04	1.52	1.49	0.00	0.00	0.00
time (sec)	N/A	0.186	0.350	0.875	0.200	0.379	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	149	195	287	353	0	0	0
N.S.	1	1.00	0.76	0.99	1.46	1.79	0.00	0.00	0.00
time (sec)	N/A	0.104	0.219	0.714	0.196	0.333	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	134	173	191	347	0	0	0
N.S.	1	1.00	0.79	1.02	1.12	2.04	0.00	0.00	0.00
time (sec)	N/A	0.104	0.195	0.705	0.193	0.315	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	123	187	152	334	0	0	0
N.S.	1	1.00	0.75	1.14	0.93	2.04	0.00	0.00	0.00
time (sec)	N/A	0.106	0.244	0.717	0.198	0.288	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	126	175	175	165	0	0	0
N.S.	1	1.00	0.67	0.93	0.93	0.87	0.00	0.00	0.00
time (sec)	N/A	0.121	0.210	0.721	0.199	0.250	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	152	207	232	197	0	0	0
N.S.	1	1.00	0.61	0.83	0.93	0.79	0.00	0.00	0.00
time (sec)	N/A	0.150	0.265	0.714	0.202	0.272	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	159	198	244	224	0	0	0
N.S.	1	1.00	0.64	0.79	0.98	0.90	0.00	0.00	0.00
time (sec)	N/A	0.181	0.397	0.868	0.202	0.272	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	123	266	183	189	0	0	0
N.S.	1	1.00	0.61	1.31	0.90	0.93	0.00	0.00	0.00
time (sec)	N/A	0.136	0.293	0.845	0.194	0.267	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	150	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	186	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.866	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	512	512	1239	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.863	2.657	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	467	467	1103	0	0	0	0	0	0
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.935	0.755	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	477	477	1055	0	0	0	0	0	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.590	0.748	0.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	425	425	1141	0	0	0	0	0	0
N.S.	1	1.00	2.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	0.365	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	518	518	1211	0	0	0	0	0	0
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.807	2.517	0.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	591	591	1447	0	0	0	0	0	0
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.068	6.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	553	553	1410	0	0	0	0	0	0
N.S.	1	1.00	2.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.965	2.936	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	271	271	0	615	0	0	0
N.S.	1	1.00	1.95	1.95	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.130	0.792	6.219	0.000	0.292	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	515	515	1428	0	0	0	0	0	0
N.S.	1	1.00	2.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	4.411	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	756	756	1593	0	0	0	0	0	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.771	6.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	719	719	1442	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.934	2.774	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	713	713	1437	0	0	0	0	0	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.631	4.777	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	758	758	1487	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.899	2.833	0.000	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	694	694	2023	0	0	0	0	0	0
N.S.	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.167	7.894	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	375	967	0	1381	0	0	0
N.S.	1	1.00	2.25	5.79	0.00	8.27	0.00	0.00	0.00
time (sec)	N/A	0.155	1.356	7.626	0.000	0.431	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	368	916	0	1256	0	0	0
N.S.	1	1.00	1.80	4.47	0.00	6.13	0.00	0.00	0.00
time (sec)	N/A	0.165	0.963	6.258	0.000	0.414	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	657	657	2081	0	0	0	0	0	0
N.S.	1	1.00	3.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.942	6.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2045	0	0	0	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.268	6.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2053	0	0	0	0	0	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.199	6.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1096	1096	2038	0	0	0	0	0	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.900	6.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	413	413	324	0	0	1951	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	1.123	2.204	0.000	0.000	1.491	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	263	0	0	1625	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	5.38	0.00	0.00	0.00
time (sec)	N/A	0.289	2.541	0.000	0.000	0.747	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	233	0	0	1342	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	6.61	0.00	0.00	0.00
time (sec)	N/A	0.145	1.998	0.000	0.000	0.457	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.074	8.125	0.158	0.000	0.251	9.078	0.286	5.959

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.079	12.765	0.155	0.000	0.261	14.538	0.265	5.781

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.077	10.195	0.167	0.000	0.254	25.628	0.272	5.661

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.029	4.211	0.157	0.000	0.254	6.364	0.279	5.536

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.071	2.696	0.164	0.000	0.261	7.143	0.267	5.969

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	389	237	0	0	275	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.316	11.029	0.000	0.000	0.108	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	527	527	314	0	0	377	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.456	11.986	0.000	0.000	0.106	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	384	384	303	0	0	1943	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	5.06	0.00	0.00	0.00
time (sec)	N/A	0.382	2.535	0.000	0.000	1.366	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	270	246	0	0	1625	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	6.02	0.00	0.00	0.00
time (sec)	N/A	0.203	2.500	0.000	0.000	0.724	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.089	9.472	0.164	0.000	0.266	65.481	0.312	5.548

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.098	13.495	0.157	0.000	0.258	62.326	0.279	5.931

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	1.17
time (sec)	N/A	0.091	10.481	0.157	0.000	0.249	0.000	0.285	5.850

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.95	1.00	1.20
time (sec)	N/A	0.038	5.147	0.151	0.000	0.252	62.908	0.299	6.325

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.077	8.947	0.155	0.000	0.257	57.978	0.285	6.212

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.082	19.194	0.164	0.000	0.261	66.787	0.266	6.316

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	492	492	291	0	0	374	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.436	12.079	0.000	0.000	0.101	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	643	643	372	0	0	486	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.680	12.233	0.000	0.000	0.121	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	281	0	0	1633	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	4.96	0.00	0.00	0.00
time (sec)	N/A	0.812	3.034	0.000	0.000	0.740	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	236	0	0	1341	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	5.86	0.00	0.00	0.00
time (sec)	N/A	0.240	1.736	0.000	0.000	0.466	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	108	0	0	1064	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	7.88	0.00	0.00	0.00
time (sec)	N/A	0.110	0.611	0.000	0.000	0.339	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.069	1.720	0.202	0.000	0.255	7.469	0.280	5.767

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.078	7.413	0.169	0.000	0.266	28.520	0.292	5.722

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.070	10.027	0.139	0.000	0.260	13.925	0.277	5.792

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.025	1.113	0.159	0.000	0.264	3.109	0.269	5.594

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	0	179	0	0	0
N.S.	1	1.00	0.47	0.00	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.183	2.295	0.000	0.000	0.098	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	425	425	239	0	0	274	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.339	8.290	0.000	0.000	0.103	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	260	0	0	1719	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	6.71	0.00	0.00	0.00
time (sec)	N/A	0.783	1.841	0.000	0.000	0.439	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	191	0	0	1274	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	7.96	0.00	0.00	0.00
time (sec)	N/A	0.196	1.473	0.000	0.000	0.350	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	79	0	0	368	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	4.49	0.00	0.00	0.00
time (sec)	N/A	0.079	0.353	0.000	0.000	0.292	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.089	11.181	0.164	0.000	0.263	70.168	0.285	6.047

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	1.17
time (sec)	N/A	0.099	14.578	0.160	0.000	0.252	0.000	0.270	5.783

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.077	15.248	0.156	0.000	0.260	134.918	0.293	6.086

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.076	7.676	0.179	0.000	0.263	33.944	0.286	5.461

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	127	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.055	2.068	0.000	0.000	0.096	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	201	0	0	272	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.246	6.480	0.000	0.000	0.100	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	251	241	0	0	2421	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	9.65	0.00	0.00	0.00
time (sec)	N/A	0.925	2.123	0.000	0.000	0.483	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	139	0	0	786	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	4.65	0.00	0.00	0.00
time (sec)	N/A	0.184	0.770	0.000	0.000	0.373	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	0	698	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.106	0.514	0.000	0.000	0.351	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.094	19.102	0.156	0.000	0.257	0.000	0.274	5.697

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.106	23.849	0.156	0.000	0.258	0.000	0.283	5.532

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.091	17.345	0.183	0.000	0.257	0.000	0.299	5.386

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.083	13.460	0.154	0.000	0.252	0.000	0.287	5.266

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	359	189	0	0	371	0	0	0
N.S.	1	1.00	0.53	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.227	0.458	0.000	0.000	0.107	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	248	0	0	442	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.129	5.869	0.000	0.000	0.114	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	577	396	0	0	0	0	0	0
N.S.	1	0.97	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.669	1.623	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	379	360	288	0	0	0	0	0	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	0.679	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	208	167	0	0	0	0	0	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.060	3.187	0.165	0.309	0.256	41.891	0.261	4.772

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.056	6.777	0.167	0.303	0.252	0.000	0.271	4.913

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.112	1.531	0.160	0.317	0.260	0.000	0.304	5.038

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.105	0.196	0.172	0.314	0.264	53.733	0.296	5.506

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.078	1.548	0.159	0.347	0.260	23.026	0.276	5.542

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.082	1.863	0.156	0.328	0.263	146.974	0.282	5.119

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	395	395	214	0	0	382	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	1.622	0.461	0.000	0.000	0.279	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	180	0	0	324	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	1.411	0.522	0.000	0.000	0.268	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	133	141	0	0	265	0	0	0
N.S.	1	1.02	1.08	0.00	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	0.138	0.451	0.000	0.000	0.280	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	88	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.38	1.42	1.31	1.00	1.15
time (sec)	N/A	0.066	0.638	0.158	0.594	0.262	7.517	0.283	6.259

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	112	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.31	1.50	1.38	1.00	1.15
time (sec)	N/A	0.082	8.863	0.158	0.628	0.262	85.157	0.289	5.489

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [69] had the largest ratio of [.857099999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	11	8	1.00	14	0.571
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	N/A	0	0	1.00	12	0.000
34	N/A	0	0	1.00	10	0.000
35	N/A	0	0	1.00	14	0.000
36	A	4	4	1.00	14	0.286
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	N/A	0	0	1.00	16	0.000
40	N/A	0	0	1.00	16	0.000
41	A	3	3	1.00	14	0.214
42	N/A	0	0	1.00	16	0.000
43	N/A	0	0	1.00	16	0.000
44	A	11	9	1.00	16	0.562
45	A	10	9	1.00	16	0.562
46	A	9	9	1.00	14	0.643
47	A	5	4	1.00	8	0.500
48	A	4	2	1.00	16	0.125
49	A	7	7	1.00	16	0.438
50	A	8	8	1.00	16	0.500
51	A	31	16	1.00	21	0.762
52	A	24	15	1.00	19	0.790
53	A	15	11	1.00	18	0.611
54	N/A	0	0	1.00	21	0.000
55	N/A	0	0	1.00	21	0.000
56	A	22	13	1.00	18	0.722
57	A	27	17	1.00	21	0.810
58	A	20	15	1.00	21	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	14	13	1.00	19	0.684
60	A	9	9	1.00	18	0.500
61	N/A	0	0	1.00	21	0.000
62	N/A	0	0	1.00	21	0.000
63	A	23	15	1.00	21	0.714
64	A	16	13	1.00	21	0.619
65	A	11	11	1.00	19	0.579
66	A	6	6	1.00	18	0.333
67	N/A	0	0	1.00	21	0.000
68	N/A	0	0	1.00	21	0.000
69	A	31	18	1.00	21	0.857
70	A	25	17	1.00	21	0.810
71	A	19	14	1.00	19	0.737
72	A	12	11	1.00	18	0.611
73	N/A	0	0	1.00	21	0.000
74	N/A	0	0	1.00	21	0.000
75	A	19	14	1.21	18	0.778
76	A	7	7	1.00	19	0.368
77	A	6	7	1.00	19	0.368
78	A	5	5	1.00	16	0.312
79	A	4	5	1.00	19	0.263
80	A	4	5	1.00	19	0.263
81	A	5	6	1.00	19	0.316
82	A	6	6	1.00	19	0.316
83	A	5	5	1.00	19	0.263
84	A	5	5	1.00	19	0.263
85	A	6	5	1.00	17	0.294
86	A	11	11	1.00	19	0.579
87	A	13	13	1.00	19	0.684
88	A	7	8	1.00	21	0.381
89	A	6	7	1.00	18	0.389
90	A	6	7	1.00	21	0.333
91	A	6	7	1.00	21	0.333
92	A	5	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	6	7	1.00	21	0.333
94	A	5	6	1.00	21	0.286
95	A	6	5	1.00	19	0.263
96	A	12	13	1.00	21	0.619
97	A	14	15	1.00	21	0.714
98	A	25	12	1.00	21	0.571
99	A	26	9	1.00	19	0.474
100	A	19	7	1.00	18	0.389
101	A	19	7	1.00	21	0.333
102	A	24	10	1.00	21	0.476
103	A	31	14	1.00	21	0.667
104	A	29	12	1.00	21	0.571
105	A	7	6	1.00	19	0.316
106	A	24	10	1.00	21	0.476
107	A	51	15	1.00	21	0.714
108	A	27	10	1.00	21	0.476
109	A	47	11	1.00	18	0.611
110	A	50	13	1.00	21	0.619
111	A	33	13	1.00	21	0.619
112	A	6	7	1.00	21	0.333
113	A	8	7	1.00	19	0.368
114	A	28	11	1.00	21	0.524
115	A	35	11	1.00	21	0.524
116	A	63	12	1.00	21	0.571
117	A	81	12	1.00	18	0.667
118	A	12	12	1.00	23	0.522
119	A	11	12	1.00	23	0.522
120	A	9	9	1.00	21	0.429
121	N/A	0	0	1.00	23	0.000
122	N/A	0	0	1.00	23	0.000
123	N/A	0	0	1.00	23	0.000
124	N/A	0	0	1.00	20	0.000
125	N/A	0	0	1.00	23	0.000
126	A	8	9	1.00	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	9	10	1.00	23	0.435
128	A	12	12	1.00	23	0.522
129	A	10	10	1.00	21	0.476
130	N/A	0	0	1.00	23	0.000
131	N/A	0	0	1.00	23	0.000
132	N/A	0	0	1.00	23	0.000
133	N/A	0	0	1.00	20	0.000
134	N/A	0	0	1.00	23	0.000
135	N/A	0	0	1.00	23	0.000
136	A	9	10	1.00	23	0.435
137	A	10	10	1.00	23	0.435
138	A	11	12	1.00	23	0.522
139	A	10	12	1.00	23	0.522
140	A	9	9	1.00	21	0.429
141	N/A	0	0	1.00	23	0.000
142	N/A	0	0	1.00	23	0.000
143	N/A	0	0	1.00	23	0.000
144	N/A	0	0	1.00	20	0.000
145	A	8	9	1.00	23	0.391
146	A	8	10	1.00	23	0.435
147	A	10	11	1.00	23	0.478
148	A	9	11	1.00	23	0.478
149	A	4	4	1.00	21	0.190
150	N/A	0	0	1.00	23	0.000
151	N/A	0	0	1.00	23	0.000
152	N/A	0	0	1.00	23	0.000
153	N/A	0	0	1.00	23	0.000
154	A	3	4	1.00	20	0.200
155	A	7	9	1.00	23	0.391
156	A	10	11	1.00	23	0.478
157	A	7	8	1.00	23	0.348
158	A	5	5	1.00	21	0.238
159	N/A	0	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	N/A	0	0	1.00	23	0.000
161	N/A	0	0	1.00	23	0.000
162	N/A	0	0	1.00	23	0.000
163	A	7	8	1.00	23	0.348
164	A	5	7	1.00	20	0.350
165	A	6	7	0.97	23	0.304
166	A	6	7	0.95	23	0.304
167	A	5	6	0.95	21	0.286
168	N/A	0	0	1.00	23	0.000
169	N/A	0	0	1.00	23	0.000
170	N/A	0	0	1.00	25	0.000
171	N/A	0	0	1.00	25	0.000
172	N/A	0	0	1.00	25	0.000
173	N/A	0	0	1.00	25	0.000
174	A	16	11	1.00	26	0.423
175	A	13	11	1.00	26	0.423
176	A	8	9	1.02	26	0.346
177	N/A	0	0	1.00	26	0.000
178	N/A	0	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

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3.1 $\int x^6\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 110

$$\int x^6\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right) - \frac{5b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{112c^7}$$

```
[Out] 1/7*x^7*(a+b*arccsch(c*x))-5/112*b*arctanh((1+1/c^2/x^2)^(1/2))/c^7+5/112*b
*x^2*(1+1/c^2/x^2)^(1/2)/c^5-5/168*b*x^4*(1+1/c^2/x^2)^(1/2)/c^3+1/42*b*x^6
*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6419, 272, 44, 65, 214}

$$\int x^6\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right) - \frac{5b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{112c^7} + \frac{bx^6\sqrt{\frac{1}{c^2x^2}+1}}{42c} + \frac{5bx^2\sqrt{\frac{1}{c^2x^2}+1}}{112c^5} - \frac{5bx^4\sqrt{\frac{1}{c^2x^2}+1}}{168c^3}$$

```
[In] Int[x^6*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(112*c^5) - (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsch[c*x
]))/7 - (5*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(112*c^7)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^7\left(a + b\text{csch}^{-1}(cx)\right) + \frac{b\int \frac{x^5}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{7c}$$

$$= \frac{1}{7}x^7\left(a + b\text{csch}^{-1}(cx)\right) - \frac{b\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{14c}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a + b\text{csch}^{-1}(cx)\right) + \frac{(5b)\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{84c^3}$$

$$= -\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{112c^5}$$

$$= \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}$$
$$+ \frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{224c^7}$$

$$= \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}$$
$$+ \frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)}{112c^5}$$

$$= \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}$$
$$+ \frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{5b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{112c^7}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{ax^7}{7} + b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(\frac{5x^2}{112c^5} - \frac{5x^4}{168c^3} + \frac{x^6}{42c}\right)$$
$$+ \frac{1}{7}bx^7\operatorname{csch}^{-1}(cx) - \frac{5b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

```
[In] Integrate[x^6*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) - (5*x^4)/(1
68*c^3) + x^6/(42*c)) + (b*x^7*ArcCsch[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(1 +
c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

method	result	size
parts	$\frac{a\,x^7}{7}+\frac{b\left(\frac{c^7x^7\,\mathrm{arccsch}(cx)}{7}-\frac{\sqrt{c^2x^2+1}\left(-8c^5x^5\sqrt{c^2x^2+1}+10c^3x^3\sqrt{c^2x^2+1}-15cx\sqrt{c^2x^2+1}+15\,\mathrm{arcsinh}(cx)\right)}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^7}$	123
derivativedivides	$\frac{a\,c^7x^7}{7}+b\left(\frac{c^7x^7\,\mathrm{arccsch}(cx)}{7}-\frac{\sqrt{c^2x^2+1}\left(-8c^5x^5\sqrt{c^2x^2+1}+10c^3x^3\sqrt{c^2x^2+1}-15cx\sqrt{c^2x^2+1}+15\,\mathrm{arcsinh}(cx)\right)}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^7}$	127
default	$\frac{a\,c^7x^7}{7}+b\left(\frac{c^7x^7\,\mathrm{arccsch}(cx)}{7}-\frac{\sqrt{c^2x^2+1}\left(-8c^5x^5\sqrt{c^2x^2+1}+10c^3x^3\sqrt{c^2x^2+1}-15cx\sqrt{c^2x^2+1}+15\,\mathrm{arcsinh}(cx)\right)}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^7}$	127

```
[In] int(x^6*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*x^7+b/c^7*(1/7*c^7*x^7*arccsch(c*x)-1/336*(c^2*x^2+1)^(1/2)*(-8*c^5*x
^5*(c^2*x^2+1)^(1/2)+10*c^3*x^3*(c^2*x^2+1)^(1/2)-15*c*x*(c^2*x^2+1)^(1/2)+
15*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(92) = 184.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.89

$$\int x^6\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{48\,ac^7x^7+48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+15\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-\right.}{336\,c^7}$$

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 + 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
+ 1) - 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 15*b*log
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 48*(b*c^7*x^7 - b*c^7)*log((c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*b*c^6*x^6 - 10*b*c^4*x^4 +
15*b*c^2*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^6 \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \int x^6 (a + b\,\text{acsch}\,(cx))\, dx$$

```
[In] integrate(x**6*(a+b*acsch(c*x)),x)
[Out] Integral(x**6*(a + b*acsch(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.44

$$\int x^6 \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \frac{1}{7} a x^7 + \frac{1}{672}\left(96\, x^7\,\text{arcsch}\,(cx) + \frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3 - 3c^6\left(\frac{1}{c^2x^2}+1\right)^2 + 3c^6\left(\frac{1}{c^2x^2}+1\right) - c^6} - \frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6} + \frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right.$$

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/7*a*x^7 + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 4
0*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1
)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(s
qrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b
```

Giac [F]

$$\int x^6 \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \int (b\,\text{arcsch}\,(cx) + a) x^6\, dx$$

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + b\operatorname{csch}^{-1}(cx))\, dx = \int x^6 \left(a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right) dx$$

```
[In] int(x^6*(a + b*asinh(1/(c*x))),x)
[Out] int(x^6*(a + b*asinh(1/(c*x))), x)
```

3.2 $\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 82
Rubi [A] (verified) . . . 82
Mathematica [A] (verified) . . . 83
Maple [A] (verified) . . . 84
Fricas [A] (verification not implemented) . . . 84
Sympy [F] . . . 85
Maxima [A] (verification not implemented) . . . 85
Giac [F] . . . 85
Mupad [F(-1)] . . . 86

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{4b\sqrt{1+\frac{1}{c^2x^2}}x}{45c^5} - \frac{2b\sqrt{1+\frac{1}{c^2x^2}}x^3}{45c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c} + \frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/6*x^6*(a+b*arccsch(c*x))+4/45*b*x*(1+1/c^2/x^2)^(1/2)/c^5-2/45*b*x^3*(1+1
/c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6419, 277, 197}

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right) + \frac{bx^5\sqrt{\frac{1}{c^2x^2}+1}}{30c} + \frac{4bx\sqrt{\frac{1}{c^2x^2}+1}}{45c^5} - \frac{2bx^3\sqrt{\frac{1}{c^2x^2}+1}}{45c^3}$$

```
[In] Int[x^5*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*Sqrt[1 + 1/(c^2*x^2)]*x)/(45*c^5) - (2*b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsch[c*x]))/
6
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{b\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{6c} \\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c} + \frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(2b)\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{15c^3} \\
&= -\frac{2b\sqrt{1+\frac{1}{c^2x^2}}x^3}{45c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c} + \frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{(4b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{45c^5} \\
&= \frac{4b\sqrt{1+\frac{1}{c^2x^2}}x}{45c^5} - \frac{2b\sqrt{1+\frac{1}{c^2x^2}}x^3}{45c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c} + \frac{1}{6}x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{ax^6}{6} + b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(\frac{4x}{45c^5} - \frac{2x^3}{45c^3} + \frac{x^5}{30c}\right) + \frac{1}{6}bx^6\operatorname{csch}^{-1}(cx)$$

```
[In] Integrate[x^5*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) - (2*x^3)/(45*c
^3) + x^5/(30*c)) + (b*x^6*ArcCsch[c*x])/6
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{a\,x^6}{6}+\frac{b\left(\frac{c^6x^6\,\mathrm{arccsch}(cx)}{6}+\frac{\left(c^2x^2+1\right)\left(3c^4x^4-4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^6}$	79
derivativedivides	$\frac{\frac{a\,c^6x^6}{6}+b\left(\frac{c^6x^6\,\mathrm{arccsch}(cx)}{6}+\frac{\left(c^2x^2+1\right)\left(3c^4x^4-4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^6}$	83
default	$\frac{\frac{a\,c^6x^6}{6}+b\left(\frac{c^6x^6\,\mathrm{arccsch}(cx)}{6}+\frac{\left(c^2x^2+1\right)\left(3c^4x^4-4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^6}$	83

```
[In] int(x^5*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x^6+b/c^6*(1/6*c^6*x^6*arccsch(c*x)+1/90*(c^2*x^2+1)*(3*c^4*x^4-4*c^2
*x^2+8)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int x^5\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{15\,bc^5x^6\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+15\,ac^5x^6+\left(3\,bc^4x^5-4\,bc^2x^3+8\,bx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{90\,c^5}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^5*x^6*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 15*
a*c^5*x^6 + (3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2
)))/c^5
```

Sympy [F]

$$\int x^5(a+b\text{csch}^{-1}(cx))\,dx = \int x^5(a+b\,\text{acsch}\,(cx))\,dx$$

```
[In] integrate(x**5*(a+b*acsch(c*x)),x)
[Out] Integral(x**5*(a + b*acsch(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int x^5(a+b\text{csch}^{-1}(cx))\,dx\\&= \frac{1}{6}ax^6\\&\quad+\frac{1}{90}\left(15\,x^6\,\text{arcsch}\,(cx)+\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)b\end{aligned}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2)
- 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b
```

Giac [F]

$$\int x^5(a+b\text{csch}^{-1}(cx))\,dx = \int (b\,\text{arcsch}\,(cx)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \operatorname{csch}^{-1}(cx)\right)\, dx = \int x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int(x^5*(a + b*asinh(1/(c*x))),x)
[Out] int(x^5*(a + b*asinh(1/(c*x))), x)
```

3.3 $\int x^4\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 87
Rubi [A] (verified) . . . 87
Mathematica [A] (verified) . . . 89
Maple [A] (verified) . . . 89
Fricas [B] (verification not implemented) . . . 90
Sympy [F] . . . 90
Maxima [A] (verification not implemented) . . . 90
Giac [F] . . . 91
Mupad [F(-1)] . . . 91

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^4\left(a+b\text{csch}^{-1}(cx)\right)\,dx = -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{3b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{40c^5}$$

```
[Out] 1/5*x^5*(a+b*arccsch(c*x))+3/40*b*arctanh((1+1/c^2/x^2)^(1/2))/c^5-3/40*b*x
^2*(1+1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6419, 272, 44, 65, 214}

$$\int x^4\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{3b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{40c^5}+\frac{bx^4\sqrt{\frac{1}{c^2x^2}+1}}{20c}-\frac{3bx^2\sqrt{\frac{1}{c^2x^2}+1}}{40c^3}$$

```
[In] Int[x^4*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (-3*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^4)/(
20*c) + (x^5*(a + b*ArcCsch[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])
/(40*c^5)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{5c}\\
&= \frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{(3b)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{40c^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{80c^5} \\
&= -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)}{40c^3} \\
&= -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{3b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{40c^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$
\begin{aligned}
\int x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{ax^5}{5} &+ b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(-\frac{3x^2}{40c^3}+\frac{x^4}{20c}\right) \\
&+ \frac{1}{5}bx^5\operatorname{csch}^{-1}(cx) + \frac{3b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}
\end{aligned}
$$

```
[In] Integrate[x^4*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) + x^4/(20*c)
) + (b*x^5*ArcCsch[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])
])/(40*c^5)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

method	result	size
parts	$\frac{a\,x^5}{5} + \frac{b\left(\frac{c^5x^5\,\operatorname{arccsch}(cx)}{5} + \frac{\sqrt{c^2x^2+1}\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\,\operatorname{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	104
derivativedivides	$\frac{\frac{c^5x^5a}{5} + b\left(\frac{c^5x^5\,\operatorname{arccsch}(cx)}{5} + \frac{\sqrt{c^2x^2+1}\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\,\operatorname{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	108
default	$\frac{\frac{c^5x^5a}{5} + b\left(\frac{c^5x^5\,\operatorname{arccsch}(cx)}{5} + \frac{\sqrt{c^2x^2+1}\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\,\operatorname{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	108

```
[In] int(x^4*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arccsch(c*x)+1/40*(c^2*x^2+1)^(1/2)*(2*c^3*x^3
*(c^2*x^2+1)^(1/2)-3*c*x*(c^2*x^2+1)^(1/2)+3*arcsinh(c*x))/((c^2*x^2+1)/c^2
/x^2)^(1/2)/c/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(72) = 144$.

Time $= 0.28$ (sec) , antiderivative size $= 199$, normalized size of antiderivative $= 2.31$

$$\int x^4\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{8\,ac^5x^5 + 8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - 3\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right)}{40\,c^5}$$

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] 1/40*(8*a*c^5*x^5 + 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*b*log(c*x*
sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 8*(b*c^5*x^5 - b*c^5)*log((c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^4*x^4 - 3*b*c^2*x^2)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F]

$$\int x^4\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^4(a + b\operatorname{acsch}(cx))\,dx$$

```
[In] integrate(x**4*(a+b*acsch(c*x)),x)
[Out] Integral(x**4*(a + b*acsch(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.20$ (sec) , antiderivative size $= 128$, normalized size of antiderivative $= 1.49$

$$\int x^4\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}\,ax^5$$

$$+\frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx) - \frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2 - 2\,c^4\left(\frac{1}{c^2x^2}+1\right) + c^4} - \frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) +
c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) -
 1)/c^4)/c)*b
```

Giac [F]

$$\int x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int (b\operatorname{arcsch}(cx)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^4\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^4*(a + b*asinh(1/(c*x))), x)
```

3.4 $\int x^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 92
Rubi [A] (verified) . . . 92
Mathematica [A] (verified) . . . 93
Maple [A] (verified) . . . 93
Fricas [A] (verification not implemented) . . . 94
Sympy [F] . . . 94
Maxima [A] (verification not implemented) . . . 95
Giac [F] . . . 95
Mupad [F(-1)] . . . 95

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/4*x^4*(a+b*arccsch(c*x))-1/6*b*x*(1+1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1+1/
c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6419, 277, 197}

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{6c^3}$$

```
[In] Int[x^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/6*(b*Sqrt[1 + 1/(c^2*x^2)]*x)/c^3 + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c)
 + (x^4*(a + b*ArcCsch[c*x]))/4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{4c}\\ &= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{6c^3}\\ &= -\frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{ax^4}{4}+b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(-\frac{x}{6c^3}+\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\operatorname{csch}^{-1}(cx)$$

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^4)/4 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*(-1/6*x/c^3 + x^3/(12*c)) + (b*
x^4*ArcCsch[c*x])/4
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

method	result	size
parts	$\frac{a\,x^4}{4}+\frac{b\left(\frac{c^4x^4\,\mathrm{arccsch}(cx)}{4}+\frac{\left(c^2x^2+1\right)\left(c^2x^2-2\right)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^4}$	70
derivativedivides	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\,\mathrm{arccsch}(cx)}{4}+\frac{\left(c^2x^2+1\right)\left(c^2x^2-2\right)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^4}$	74
default	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\,\mathrm{arccsch}(cx)}{4}+\frac{\left(c^2x^2+1\right)\left(c^2x^2-2\right)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^4}$	74

```
[In] int(x^3*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arccsch(c*x)+1/12*(c^2*x^2+1)*(c^2*x^2-2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{3\,bc^3x^4\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+3\,ac^3x^4+\left(bc^2x^3-2\,bx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 3*a*c^3*x^4 + (b*c^2*x^3 - 2*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^3(a+b\,\mathrm{acsch}\,(cx))\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x^3(a+b\operatorname{csch}^{-1}(cx))\,dx$$
$$= \frac{1}{4}ax^4 + \frac{1}{12}\left(3\,x^4\operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b
```

Giac [F]

$$\int x^3(a+b\operatorname{csch}^{-1}(cx))\,dx = \int (b\operatorname{arcsch}(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\operatorname{csch}^{-1}(cx))\,dx = \int x^3\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(a + b*asinh(1/(c*x))), x)
```

3.5 $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 96
Rubi [A] (verified) . . . 96
Mathematica [A] (verified) . . . 98
Maple [A] (verified) . . . 98
Fricas [B] (verification not implemented) . . . 98
Sympy [F] . . . 99
Maxima [A] (verification not implemented) . . . 99
Giac [F] . . . 99
Mupad [F(-1)] . . . 100

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arccsch(c*x))-1/6*b*arctanh((1+1/c^2/x^2)^(1/2))/c^3+1/6*b*x^2
*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6419, 272, 44, 65, 214}

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}$$

```
[In] Int[x^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsch[c*x]))/3 - (b*Arc
Tanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{3c}\\
&= \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c}\\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\operatorname{csch}^{-1}(cx)-\frac{b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^3)/3 + (b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsch[c*
x])/3 - (b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

method	result	size
parts	$\frac{a\,x^3}{3}+\frac{b\left(\frac{c^3x^3\,\operatorname{arccsch}(cx)}{3}-\frac{\sqrt{c^2x^2+1}\left(-cx\sqrt{c^2x^2+1}+\operatorname{arcsinh}(cx)\right)}{6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xc}\right)}{c^3}$	83
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\operatorname{arccsch}(cx)}{3}+\frac{\sqrt{c^2x^2+1}\left(cx\sqrt{c^2x^2+1}-\operatorname{arcsinh}(cx)\right)}{6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^3}$	88
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\operatorname{arccsch}(cx)}{3}+\frac{\sqrt{c^2x^2+1}\left(cx\sqrt{c^2x^2+1}-\operatorname{arcsinh}(cx)\right)}{6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^3}$	88

```
[In] int(x^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arccsch(c*x)-1/6*(c^2*x^2+1)^(1/2)*(-c*x*(c^2*
x^2+1)^(1/2)+arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.00

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{2\,ac^3x^3+bc^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,bc^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-2\,bc^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+b\log\left(cx\right.}{6\,c^3}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 + b*c^2*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*b*c^3*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*b*c^3*log(c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) - c*x - 1) + b*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x)
 + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
)/c^3
```

Sympy [F]

$$\int x^2 (a + b\operatorname{csch}^{-1}(cx))\, dx = \int x^2 (a + b\operatorname{acsch}(cx))\, dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int x^2 (a + b\operatorname{csch}^{-1}(cx))\, dx$$
$$= \frac{1}{3} a x^3 + \frac{1}{12} \left(4 x^3 \operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c} \right) b$$

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^
2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x
^2) + 1) - 1)/c^2)/c)*b
```

Giac [F]

$$\int x^2 (a + b\operatorname{csch}^{-1}(cx))\, dx = \int (b\operatorname{arcsch}(cx) + a) x^2\, dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(a + b*asinh(1/(c*x))),x)
[Out] int(x^2*(a + b*asinh(1/(c*x))), x)
```

3.6 $\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 101
Rubi [A] (verified) . . . 101
Mathematica [A] (verified) . . . 102
Maple [A] (verified) . . . 102
Fricas [B] (verification not implemented) . . . 103
Sympy [F] . . . 103
Maxima [A] (verification not implemented) . . . 103
Giac [F] . . . 104
Mupad [B] (verification not implemented) . . . 104

Optimal result

Integrand size = 10, antiderivative size = 38

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/2*x^2*(a+b*arccsch(c*x))+1/2*b*x*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6419, 197}

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2c}$$

```
[In] Int[x*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsch[c*x]))/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
```

```
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{2c}\\ &= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{ax^2}{2}+\frac{bx\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\text{csch}^{-1}(cx)$$

```
[In] Integrate[x*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^2)/2 + (b*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsch[c*x]
)/2
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

method	result	size
parts	$\frac{ax^2}{2}+\frac{b\left(\frac{c^2x^2\,\text{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	61
derivativedivides	$\frac{\frac{ac^2x^2}{2}+b\left(\frac{c^2x^2\,\text{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	65
default	$\frac{\frac{ac^2x^2}{2}+b\left(\frac{c^2x^2\,\text{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	65

```
[In] int(x*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arccsch(c*x)+1/2/((c^2*x^2+1)/c^2/x^2)^(1/2)/c
/x*(c^2*x^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time $= 0.26$ (sec) , antiderivative size $= 70$, normalized size of antiderivative $= 1.84$

$$\int x\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{bcx^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + acx^2 + bx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,c}$$

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2 +
 b*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c
```

Sympy [F]

$$\int x\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x(a + b\operatorname{acsch}(cx))\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.23$ (sec) , antiderivative size $= 35$, normalized size of antiderivative $= 0.92$

$$\int x\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{2}\,ax^2 + \frac{1}{2}\left(x^2\operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)b$$

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b
```

Giac [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int (b\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x, x)
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{2}+\frac{b\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,c}$$

```
[In] int(x*(a + b*asinh(1/(c*x))),x)
[Out] (a*x^2)/2 + (b*x^2*asinh(1/(c*x)))/2 + (b*x*(1/(c^2*x^2) + 1)^(1/2))/(2*c)
```

3.7 $\int \left(a + b\mathbf{csch}^{-1}(cx)\right)\, dx$

Optimal result . . . 105
Rubi [A] (verified) . . . 105
Mathematica [B] (verified) . . . 106
Maple [A] (verified) . . . 107
Fricas [B] (verification not implemented) . . . 107
Sympy [F] . . . 107
Maxima [A] (verification not implemented) . . . 108
Giac [F] . . . 108
Mupad [F(-1)] . . . 108

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\, dx = ax + bx\text{csch}^{-1}(cx) + \frac{b\text{arctanh}\left(\sqrt{1 + \frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsch(c*x)+b*arctanh((1+1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6413, 272, 65, 214}

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\, dx = ax + \frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2} + 1}\right)}{c} + bx\text{csch}^{-1}(cx)$$

```
[In] Int[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6413

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \operatorname{csch}^{-1}(cx)\,dx \\
&= ax + bx\operatorname{csch}^{-1}(cx) + \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\,dx}{c} \\
&= ax + bx\operatorname{csch}^{-1}(cx) - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx\operatorname{csch}^{-1}(cx) - (bc)\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right) \\
&= ax + bx\operatorname{csch}^{-1}(cx) + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(30) = 60$.

Time $= 0.13$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 2.13$

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = ax + bx\operatorname{csch}^{-1}(cx) + \frac{2b\sqrt{1+\frac{1}{c^2x^2}}x\operatorname{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

```
[In] Integrate[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (2*b*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(-1 + Sqrt[1
+ c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

method	result	size
default	$ax + bx \operatorname{arccsch}(cx) + \frac{b \ln\left(cx + cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
parts	$ax + bx \operatorname{arccsch}(cx) + \frac{b \ln\left(cx + cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
derivativedivides	$\frac{acx + b\left(cx \operatorname{arccsch}(cx) + \ln\left(cx + cx\sqrt{1+\frac{1}{c^2x^2}}\right)\right)}{c}$	39

```
[In] int(a+b*arccsch(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.77

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx$$

$$= \frac{acx + bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (bcx -}{c}$$

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="fricas")
[Out] (a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)))/c
```

Sympy [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int \left(a + b\operatorname{acsch}(cx)\right)\, dx$$

```
[In] integrate(a+b*acsch(c*x),x)
[Out] Integral(a + b*acsch(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx$$
$$= ax + \frac{\left(2\,cx\,\operatorname{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b/c
```

Giac [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int b\,\operatorname{arcsch}\left(cx\right) + a\, dx$$

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arccsch(c*x) + a, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\, dx$$

```
[In] int(a + b*asinh(1/(c*x)),x)
```

```
[Out] int(a + b*asinh(1/(c*x)), x)
```

3.8 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx$

Optimal result . . . 109
Rubi [A] (verified) . . . 109
Mathematica [A] (verified) . . . 111
Maple [F] . . . 111
Fricas [F] . . . 112
Sympy [F] . . . 112
Maxima [F] . . . 112
Giac [F] . . . 112
Mupad [F(-1)] . . . 113

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx = -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right) + \frac{1}{2}b\,\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)$$

```
[Out] -1/2*(a+b*arccsch(c*x))^2/b-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^
(1/2))^2)+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6417, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx = -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2b} - \log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{2}b\,\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] -1/2*(a + b*ArcCsch[c*x])^2/b - (a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[
c*x])] + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6417

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{csch}^{-1}(cx)\right)}{b} \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)x}}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{csch}^{-1}(cx)\right)}{b}
\end{aligned}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right) + \text{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{csch}^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{csch}^{-1}(cx)}{b}\right)}\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right) + \frac{1}{2}b\,\text{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\text{csch}^{-1}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx = \frac{1}{2}b\text{csch}^{-1}(cx)^2 - b\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + a\log(x) - \frac{1}{2}b\,\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right)$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] (b*ArcCsch[c*x]^2)/2 - b*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + a*Log[x
] - (b*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Maple [F]

$$\int \frac{a+b\,\text{arccsch}\,(cx)}{x}dx$$

```
[In] int((a+b*arccsch(c*x))/x,x)
```

```
[Out] int((a+b*arccsch(c*x))/x,x)
```

Fricas [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx = \int \frac{a+b\operatorname{acsch}(cx)}{x}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x,x)
[Out] Integral((a + b*acsch(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="maxima")
[Out] -1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log(
x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x))*
log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) + 2*
integrate(log(x)/(c^2*x^3 + x), x))*b + a*log(x)
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/x,x)
[Out] int((a + b*asinh(1/(c*x)))/x, x)
```

3.9 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2}\,dx$

Optimal result . . . 114
Rubi [A] (verified) . . . 114
Mathematica [A] (verified) . . . 115
Maple [A] (verified) . . . 115
Fricas [B] (verification not implemented) . . . 116
Sympy [A] (verification not implemented) . . . 116
Maxima [A] (verification not implemented) . . . 116
Giac [F] . . . 117
Mupad [B] (verification not implemented) . . . 117

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = bc\sqrt{1+\frac{1}{c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{x}$$

```
[Out] (-a-b*arccsch(c*x))/x+b*c*(1+1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6419, 267}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = bc\sqrt{\frac{1}{c^2x^2}+1} - \frac{a+b\operatorname{csch}^{-1}(cx)}{x}$$

```
[In] Int[(a + b*ArcCsch[c*x])/x^2,x]
```

```
[Out] b*c*Sqrt[1 + 1/(c^2*x^2)] - (a + b*ArcCsch[c*x])/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
```

```
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\text{csch}^{-1}(cx)}{x} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^3}\,dx}{c} \\ &= bc\sqrt{1+\frac{1}{c^2x^2}} - \frac{a+b\text{csch}^{-1}(cx)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx = -\frac{a}{x} + bc\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{x}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^2,x]
[Out] -(a/x) + b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/x
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

method	result	size
parts	$-\frac{a}{x} + bc\left(-\frac{\text{arccsch}(cx)}{cx} + \frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)$	58
derivativedivides	$c\left(-\frac{a}{cx} + b\left(-\frac{\text{arccsch}(cx)}{cx} + \frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)\right)$	62
default	$c\left(-\frac{a}{cx} + b\left(-\frac{\text{arccsch}(cx)}{cx} + \frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)\right)$	62

```
[In] int((a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*c*(-1/c/x*arccsch(c*x)+1/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^
2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(28) = 56$.

Time $= 0.27$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 2.13$

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = \frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - a}{x}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) - a)/x
```

Sympy [A] (verification not implemented)

Time $= 0.39$ (sec) , antiderivative size $= 36$, normalized size of antiderivative $= 1.20$

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x} + bc\sqrt{1 + \frac{1}{c^2x^2}} - \frac{b\operatorname{acsch}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*acsch(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*sqrt(1 + 1/(c**2*x**2)) - b*acsch(c*x)/x, Ne(c, 0)),
(-(a + zoo*b)/x, True))
```

Maxima [A] (verification not implemented)

none

Time $= 0.21$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 1.07$

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = \left(c\sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x}\right)b - \frac{a}{x}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b - a/x
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx = b\,c\,\sqrt{\frac{1}{c^2\,x^2}+1}-\frac{a}{x}-\frac{b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x}$$

```
[In] int((a + b*asinh(1/(c*x)))/x^2,x)
[Out] b*c*(1/(c^2*x^2) + 1)^(1/2) - a/x - (b*asinh(1/(c*x)))/x
```

3.10 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx$

Optimal result . . . 118
Rubi [A] (verified) . . . 118
Mathematica [A] (verified) . . . 119
Maple [B] (verified) . . . 120
Fricas [A] (verification not implemented) . . . 120
Sympy [F] . . . 121
Maxima [B] (verification not implemented) . . . 121
Giac [F] . . . 121
Mupad [B] (verification not implemented) . . . 122

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2\operatorname{csch}^{-1}(cx) - \frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2}$$

```
[Out] -1/4*b*c^2*arccsch(c*x)+1/2*(-a-b*arccsch(c*x))/x^2+1/4*b*c*(1+1/c^2/x^2)^(
1/2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6419, 342, 327, 221}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2\operatorname{csch}^{-1}(cx)$$

```
[In] Int[(a + b*ArcCsch[c*x])/x^3,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*ArcCsch[c*x])/4 - (a + b*ArcCsch
[c*x])/(2*x^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}\,dx}{2c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2} + \frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2} - \frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2\operatorname{csch}^{-1}(cx) - \frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2} + \frac{bc\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{4x} - \frac{b\operatorname{csch}^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\operatorname{arcsinh}\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsch[c*x])/(
2*x^2) - (b*c^2*ArcSinh[1/(c*x)])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

method	result	size
parts	$-\frac{a}{2x^2} + b\,c^2\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2} - \frac{\sqrt{c^2x^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2-\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x^3c^3}\right)$	96
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2} + b\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2} + \frac{\sqrt{c^2x^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2+\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3x^3}\right)\right)$	99
default	$c^2\left(-\frac{a}{2c^2x^2} + b\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2} + \frac{\sqrt{c^2x^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2+\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3x^3}\right)\right)$	99

```
[In] int((a+b*arccsch(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arccsch(c*x)-1/4*(c^2*x^2+1)^(1/2)*(arctanh(
1/(c^2*x^2+1)^(1/2))*c^2*x^2-(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)
/x^3/c^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = \frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - (bc^2x^2 + 2\,b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - 2\,a}{4\,x^2}$$

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b*c^2*x^2 + 2*b)*log((c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x^3}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**3,x)
[Out] Integral((a + b*acsch(c*x))/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

Time $= 0.21$ (sec) , antiderivative size $= 105$, normalized size of antiderivative $= 2.10$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3}\,dx$$
$$= \frac{1}{8}\,b\left(\frac{\frac{2\,c^4 x\sqrt{\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}+1\right)-1} - c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right) + c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c} - \frac{4\,\text{arcsch}\,(cx)}{x^2}\right)$$
$$- \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
[Out] 1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3
*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1
))/c - 4*arccsch(c*x)/x^2) - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = \frac{b\,c\,\sqrt{\frac{1}{c^2\,x^2}+1}}{4\,x} - \frac{b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\,\left(\frac{c^2\,x}{4}+\frac{1}{2\,x}\right)}{x} - \frac{a}{2\,x^2}$$

```
[In] int((a + b*asinh(1/(c*x)))/x^3,x)
```

```
[Out] (b*c*(1/(c^2*x^2) + 1)^(1/2))/(4*x) - (b*asinh(1/(c*x))*((c^2*x)/4 + 1/(2*x
)))/x - a/(2*x^2)
```

3.11 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx$

Optimal result . . . 123
Rubi [A] (verified) . . . 123
Mathematica [A] (verified) . . . 124
Maple [A] (verified) . . . 125
Fricas [A] (verification not implemented) . . . 125
Sympy [F] . . . 125
Maxima [A] (verification not implemented) . . . 126
Giac [F] . . . 126
Mupad [F(-1)] . . . 126

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = -\frac{1}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{1}{9}bc^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}$$

```
[Out] 1/9*b*c^3*(1+1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsch(c*x))/x^3-1/3*b*c^3*(1+1/c^
2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6419, 272, 45}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} + \frac{1}{9}bc^3\left(\frac{1}{c^2x^2}+1\right)^{3/2} - \frac{1}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}$$

```
[In] Int[(a + b*ArcCsch[c*x])/x^4,x]
```

```
[Out] -1/3*(b*c^3*Sqrt[1 + 1/(c^2*x^2)]) + (b*c^3*(1 + 1/(c^2*x^2))^(3/2))/9 - (a
 + b*ArcCsch[c*x])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}\,dx}{3c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} + \frac{b\text{Subst}\left(\int \left(-\frac{c^2}{\sqrt{1+\frac{x}{c^2}}} + c^2\sqrt{1+\frac{x}{c^2}}\right)\,dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{1}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{1}{9}bc^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = -\frac{a}{3x^3} + b\left(-\frac{2c^3}{9} + \frac{c}{9x^2}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\operatorname{csch}^{-1}(cx)}{3x^3}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*((-2*c^3)/9 + c/(9*x^2))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*
ArcCsch[c*x])/(3*x^3)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3}-\frac{(c^2x^2+1)\,(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)$	71
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3}-\frac{(c^2x^2+1)\,(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)\right)$	75
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3}-\frac{(c^2x^2+1)\,(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)\right)$	75

```
[In] int((a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arccsch(c*x)-1/9*(c^2*x^2+1)*(2*c^2*x^2-1)/(
(c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx=-\frac{3\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(2\,bc^3x^3-bcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+3\,a}{9\,x^3}$$

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^3*x^3
 - b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 3*a)/x^3
```

Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx=\int \frac{a+b\operatorname{acsch}(cx)}{x^4}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\,b\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\operatorname{arcsch}(cx)}{x^3}\right) - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*ar
ccsch(c*x)/x^3) - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = \int \frac{a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^4}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/x^4,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx$

Optimal result . . . 127
Rubi [A] (verified) . . . 127
Mathematica [A] (verified) . . . 129
Maple [A] (verified) . . . 129
Fricas [A] (verification not implemented) . . . 130
Sympy [F] . . . 130
Maxima [B] (verification not implemented) . . . 130
Giac [F] . . . 131
Mupad [F(-1)] . . . 131

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\operatorname{csch}^{-1}(cx) - \frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}$$

```
[Out] 3/32*b*c^4*arccsch(c*x)+1/4*(-a-b*arccsch(c*x))/x^4+1/16*b*c*(1+1/c^2/x^2)^
(1/2)/x^3-3/32*b*c^3*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6419, 342, 327, 221}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\operatorname{csch}^{-1}(cx) + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{16x^3} - \frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}}{32x}$$

```
[In] Int[(a + b*ArcCsch[c*x])/x^5,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(32*
x) + (3*b*c^4*ArcCsch[c*x])/32 - (a + b*ArcCsch[c*x])/(4*x^4)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^6}\,dx}{4c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} + \frac{b\text{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c} \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} - \frac{1}{16}(3bc)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} + \frac{1}{32}\left(3bc^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\operatorname{csch}^{-1}(cx) - \frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx = -\frac{a}{4x^4} + b\left(\frac{c}{16x^3} - \frac{3c^3}{32x}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\text{arcsinh}\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(c/(16*x^3) - (3*c^3)/(32*x))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]
- (b*ArcCsch[c*x])/(4*x^4) + (3*b*c^4*ArcSinh[1/(c*x)])/32
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{a}{4x^4} + b\,c^4\left(-\frac{\text{arccsch}(cx)}{4c^4x^4} - \frac{\sqrt{c^2x^2+1}\left(-3\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-2\sqrt{c^2x^2+1}\right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right)$
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4} + b\left(-\frac{\text{arccsch}(cx)}{4c^4x^4} - \frac{\sqrt{c^2x^2+1}\left(-3\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-2\sqrt{c^2x^2+1}\right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right)\right)$
default	$c^4\left(-\frac{a}{4c^4x^4} + b\left(-\frac{\text{arccsch}(cx)}{4c^4x^4} - \frac{\sqrt{c^2x^2+1}\left(-3\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-2\sqrt{c^2x^2+1}\right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right)\right)$

```
[In] int((a+b*arccsch(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4+b*c^4*(-1/4/c^4/x^4*arccsch(c*x)-1/32*(c^2*x^2+1)^(1/2)*(-3*arct
anh(1/(c^2*x^2+1)^(1/2))*c^4*x^4+3*(c^2*x^2+1)^(1/2)*c^2*x^2-2*(c^2*x^2+1)^
(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^5/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx = \frac{(3\,bc^4x^4-8\,b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(3\,bc^3x^3-2\,bcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-8\,a}{32\,x^4}$$

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) - (3*b*c^3*x^3 - 2*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 8*a)/x^4
```

Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx = \int \frac{a+b\operatorname{acsch}(cx)}{x^5}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(62) = 124.

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx$$

$$= \frac{1}{64}\,b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}+1\right)+1}}{c} - \frac{16\ \text{arc}}{} - \frac{a}{4\,x^4}\right)$$

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c
^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1/(
c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1) +
 1))/c - 16*arccsch(c*x)/x^4) - 1/4*a/x^4
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5}\,dx = \int \frac{a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/x^5,x)
[Out] int((a + b*asinh(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx$

Optimal result . . . 132
Rubi [A] (verified) . . . 132
Mathematica [A] (verified) . . . 133
Maple [A] (verified) . . . 134
Fricas [A] (verification not implemented) . . . 134
Sympy [F] . . . 134
Maxima [A] (verification not implemented) . . . 135
Giac [F] . . . 135
Mupad [F(-1)] . . . 135

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = \frac{1}{5}bc^5\sqrt{1+\frac{1}{c^2x^2}} - \frac{2}{15}bc^5\left(1+\frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{25}bc^5\left(1+\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5}$$

```
[Out] -2/15*b*c^5*(1+1/c^2/x^2)^(3/2)+1/25*b*c^5*(1+1/c^2/x^2)^(5/2)+1/5*(-a-b*ar
ccsch(c*x))/x^5+1/5*b*c^5*(1+1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6419, 272, 45}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5} + \frac{1}{25}bc^5\left(\frac{1}{c^2x^2}+1\right)^{5/2} - \frac{2}{15}bc^5\left(\frac{1}{c^2x^2}+1\right)^{3/2} + \frac{1}{5}bc^5\sqrt{\frac{1}{c^2x^2}+1}$$

```
[In] Int[(a + b*ArcCsch[c*x])/x^6,x]
```

```
[Out] (b*c^5*Sqrt[1 + 1/(c^2*x^2)])/5 - (2*b*c^5*(1 + 1/(c^2*x^2))^(3/2))/15 + (b
*c^5*(1 + 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsch[c*x])/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^7}\,dx}{5c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{10c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \left(\frac{c^4}{\sqrt{1+\frac{x}{c^2}}} - 2c^4\sqrt{1+\frac{x}{c^2}} + c^4\left(1+\frac{x}{c^2}\right)^{3/2}\right)\,dx, x, \frac{1}{x^2}\right)}{10c} \\
&= \frac{1}{5}bc^5\sqrt{1+\frac{1}{c^2x^2}} - \frac{2}{15}bc^5\left(1+\frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{25}bc^5\left(1+\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5} + b\left(\frac{8c^5}{75} + \frac{c}{25x^4} - \frac{4c^3}{75x^2}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\operatorname{csch}^{-1}(cx)}{5x^5}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 + b*((8*c^5)/75 + c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(1 + c^2*x
^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(5*x^5)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arccsch}(cx)}{5c^5x^5}+\frac{(c^2x^2+1)\left(8c^4x^4-4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsch}(cx)}{5c^5x^5}+\frac{(c^2x^2+1)\left(8c^4x^4-4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsch}(cx)}{5c^5x^5}+\frac{(c^2x^2+1)\left(8c^4x^4-4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

```
[In] int((a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arccsch(c*x)+1/75*(c^2*x^2+1)*(8*c^4*x^4-4*c
^2*x^2+3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx$$
$$=-\frac{15\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(8\,bc^5x^5-4\,bc^3x^3+3\,bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+15\,a}{75\,x^5}$$

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(15*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*x
^5 - 4*b*c^3*x^3 + 3*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 15*a)/x^5
```

Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx=\int \frac{a+b\operatorname{acsch}(cx)}{x^6}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx$$
$$= \frac{1}{75}b\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{15\,\operatorname{arcsch}(cx)}{x^5}\right) - \frac{a}{5\,x^5}$$

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 1
5*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) - 1/5*a/x^5
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{x^6}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = \int \frac{a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^6}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/x^6,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx$

Optimal result . . . 136
Rubi [A] (verified) . . . 136
Mathematica [A] (verified) . . . 138
Maple [A] (verified) . . . 138
Fricas [A] (verification not implemented) . . . 139
Sympy [F] . . . 139
Maxima [B] (verification not implemented) . . . 139
Giac [F] . . . 140
Mupad [F(-1)] . . . 140

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x} - \frac{5}{96}bc^6\operatorname{csch}^{-1}(cx) - \frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}$$

```
[Out] -5/96*b*c^6*arccsch(c*x)+1/6*(-a-b*arccsch(c*x))/x^6+1/36*b*c*(1+1/c^2/x^2)
^(1/2)/x^5-5/144*b*c^3*(1+1/c^2/x^2)^(1/2)/x^3+5/96*b*c^5*(1+1/c^2/x^2)^(1/
2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6419, 342, 327, 221}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\operatorname{csch}^{-1}(cx) + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{36x^5} + \frac{5bc^5\sqrt{\frac{1}{c^2x^2}+1}}{96x} - \frac{5bc^3\sqrt{\frac{1}{c^2x^2}+1}}{144x^3}$$

```
[In] Int[(a + b*ArcCsch[c*x])/x^7,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(144
*x^3) + (5*b*c^5*Sqrt[1 + 1/(c^2*x^2)])/(96*x) - (5*b*c^6*ArcCsch[c*x])/96
- (a + b*ArcCsch[c*x])/(6*x^6)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^8}\,dx}{6c} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} + \frac{b\text{Subst}\left(\int \frac{x^6}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c} \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} - \frac{1}{36}(5bc)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} - \frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} + \frac{1}{48}(5bc^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x} - \frac{a+b\text{csch}^{-1}(cx)}{6x^6} - \frac{1}{96}(5bc^5)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x} - \frac{5}{96}bc^6\text{csch}^{-1}(cx) - \frac{a+b\text{csch}^{-1}(cx)}{6x^6}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} + b\left(\frac{c}{36x^5} - \frac{5c^3}{144x^3} + \frac{5c^5}{96x}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\text{arcsinh}\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(c/(36*x^5) - (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(1 +
c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(6*x^6) - (5*b*c^6*ArcSinh[1/(c*x)])
/96
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

method	result
parts	$-\frac{a}{6x^6} + b\,c^6\left(-\frac{\text{arccsch}(cx)}{6c^6x^6} - \frac{\sqrt{c^2x^2+1}\left(15\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6 - 15\sqrt{c^2x^2+1}\,c^4x^4 + 10\sqrt{c^2x^2+1}\,c^2x^2 - 8\sqrt{c}\right.}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^7x^7}$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6} + b\left(-\frac{\text{arccsch}(cx)}{6c^6x^6} - \frac{\sqrt{c^2x^2+1}\left(15\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6 - 15\sqrt{c^2x^2+1}\,c^4x^4 + 10\sqrt{c^2x^2+1}\,c^2x^2 -\right.}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^7x^7}\right.$
default	$c^6\left(-\frac{a}{6c^6x^6} + b\left(-\frac{\text{arccsch}(cx)}{6c^6x^6} - \frac{\sqrt{c^2x^2+1}\left(15\,\text{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6 - 15\sqrt{c^2x^2+1}\,c^4x^4 + 10\sqrt{c^2x^2+1}\,c^2x^2 -\right.}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^7x^7}\right.$

```
[In] int((a+b*arccsch(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6+b*c^6*(-1/6/c^6/x^6*arccsch(c*x)-1/288*(c^2*x^2+1)^(1/2)*(15*arc
tanh(1/(c^2*x^2+1)^(1/2))*c^6*x^6-15*(c^2*x^2+1)^(1/2)*c^4*x^4+10*(c^2*x^2+
1)^(1/2)*c^2*x^2-8*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^7/x^7)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx$$
$$= -\frac{3\left(5\,bc^6x^6+16\,b\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(15\,bc^5x^5-10\,bc^3x^3+8\,bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+48\,a}{288\,x^6}$$

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/288*(3*(5*b*c^6*x^6 + 16*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)) - (15*b*c^5*x^5 - 10*b*c^3*x^3 + 8*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 48*a)/x^6
```

Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx = \int \frac{a+b\,\operatorname{acsch}(cx)}{x^7}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**7,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(82) = 164.

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.89

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx =$$
$$-\frac{1}{576}\,b\left(\frac{15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(15\,c^{12}x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\,c^{10}x^3\left(\frac{1}{c^2x^2}+1\right)\right.}{c^6x^6\left(\frac{1}{c^2x^2}+1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2+3}}{c}\right.$$
$$-\frac{a}{6\,x^6}$$

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 15*c^7*log(c*x*sqrt(
1/(c^2*x^2) + 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) + 1)^(5/2) - 40*c^10*x^
3*(1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^
2*x^2) + 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) + 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) + 1)
 - 1))/c + 96*arccsch(c*x)/x^6) - 1/6*a/x^6
```

Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^7}\,dx = \int \frac{b\operatorname{arcsch}(cx) + a}{x^7}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^7}\,dx = \int \frac{a + b\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^7}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/x^7,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^7, x)
```

3.15 $\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 105

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx = \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{3c^4}$$

```
[Out] 1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsch(c*x))^2-1/3*b^2*ln(x)/c^4-1/3*b*x*(a+
b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsch(c*x))*(1+1/c
^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6421, 5560, 4270, 4269, 3556}

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx = \frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{6c} - \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{3c^4} + \frac{b^2x^2}{12c^2}$$

```
[In] Int[x^3*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*x^2)/(12*c^2) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/(3*c^
3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*x]))/(6*c) + (x^4*(a + b
*ArcCsch[c*x])^2)/4 - (b^2*Log[x])/(3*c^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2 \coth(x)\text{csch}^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\text{csch}^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{2c^4} \\
&= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2 \\
&\quad + \frac{b\text{Subst}\left(\int (a+bx)\text{csch}^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{3c^4}
\end{aligned}$$

$$= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{b^2\operatorname{Subst}\left(\int \coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{3c^4}$$

$$= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{3c^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 dx$$

$$= \frac{cx\left(b^2cx + 3a^2c^3x^3 + 2ab\sqrt{1+\frac{1}{c^2x^2}}\left(-2+c^2x^2\right)\right) + 2bcx\left(3ac^3x^3 + b\sqrt{1+\frac{1}{c^2x^2}}\left(-2+c^2x^2\right)\right)\operatorname{csch}^{-1}(cx) +}{12c^4}$$

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))
 + 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))*ArcCsch[c
*x] + 3*b^2*c^4*x^4*ArcCsch[c*x]^2 - 4*b^2*Log[x])/(12*c^4)
```

Maple [F]

$$\int x^3(a+b\ \operatorname{arccsch}(cx))^2\,dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(91) = 182.

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.59

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 dx$$

$$= \frac{3\,b^2c^4x^4\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 + 3\,a^2c^4x^4 + 6\,abc^4\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 6\,abc^4\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right.}{}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 +
3*a^2*c^4*x^4 + 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1)
- 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + b^2*c^2*x^2
- 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 + (b^2*c^3*x^3 - 2*b^2*c*x)*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 2*(a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^4
```

Sympy [F]

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int x^3 (a + b \operatorname{acsch}(cx))^2 \, dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))**2, x)
```

Maxima [F]

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a*b + 1/288*(72*x^4*log(sqrt(c^2*x^2 + 1) +
 1)^2 + 1152*c^2*integrate(1/2*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 1152*c^2*integrate(1/2*x^5*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x)*log(c) + 576*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(
sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*c^2
*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqr
t(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 576*c^2*int
egrate(1/2*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2
 + 1) + 1), x) - 1152*c^2*integrate(1/2*x^5*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 1152
*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) - 1152*integrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
 24*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^
2 + 1) + 6)*log(c)^2/c^4 - 48*(3*c^2*x^2 - 2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c
^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*log(c)^2/c^4 + 144*(c^2*x^2 - 2*sqrt(
```

```
c^2*x^2 + 1) + 1)*log(c)^2/c^4 + 144*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1
))*log(c)^2/c^4 - 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2)
 - 12*sqrt(c^2*x^2 + 1) + 6)*log(c)*log(x)/c^4 + 288*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)*log(x)/c^4 + 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^
2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*log(c)*log(sqrt(c^2*x^2 + 1) +
 1)/c^4 - 288*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)*log(sqrt(c^2*x^2 +
 1) + 1)/c^4 + 4*(18*c^2*x^2 - 9*(c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) -
 96*sqrt(c^2*x^2 + 1) + 66*log(sqrt(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2
 + 1) - 1) + 18)*log(c)/c^4 + 4*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^
2 + 1)^(3/2) + 132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*
log(c)/c^4 - 144*(c^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) +
 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)*log(c)/c^4 + 144*(c^2*x^2 - 6*sqrt(c^
2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*log(c)/c^4 + 12*(6*c^2*x^2 -
 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*log(
sqrt(c^2*x^2 + 1) + 1)/c^4 + (6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 +
 1)^(3/2) + 132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)/c^4
 + 576*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*integrate(1/2*sqrt(c^2*x^
2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 576*integrate(1/2*x^3*log(x)^2/(sqrt(
c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*integrat
e(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^
2*x^2 + sqrt(c^2*x^2 + 1) + 1), x))*b^2
```

Giac [F]

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int x^3 \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

```
[In] int(x^3*(a + b*asinh(1/(c*x)))^2,x)
[Out] int(x^3*(a + b*asinh(1/(c*x)))^2, x)
```

3.16 $\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 122

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx = \frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} - \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsch(c*x))^2-2/3*b*(a+b*arccsch(c*x))*arctanh
(1/c/x+(1+1/c^2/x^2)^(1/2))/c^3-1/3*b^2*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2
))/c^3+1/3*b^2*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b*arcc
sch(c*x))*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5560, 4270, 4267, 2317, 2438}

$$\begin{aligned}\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx = &-\frac{2b\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^3}\\ &+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c}\\ &+\frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2-\frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}\\ &+\frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2x}{3c^2}\end{aligned}$$

```
[In] Int[x^2*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcCsch[c*x]))/(3*c)
+ (x^3*(a + b*ArcCsch[c*x])^2)/3 - (2*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcC
sch[c*x]])/(3*c^3) - (b^2*PolyLog[2, -E^ArcCsch[c*x]])/(3*c^3) + (b^2*PolyL
og[2, E^ArcCsch[c*x]])/(3*c^3)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
```

```
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2 \coth(x)\text{csch}^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^3} \\
&= \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\text{csch}^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{3c^3} \\
&= \frac{b^2 x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 \\
&\quad + \frac{b\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{3c^3} \\
&= \frac{b^2 x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2 \\
&\quad - \frac{2b\left(a+b\text{csch}^{-1}(cx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{3c^3} \\
&\quad - \frac{b^2\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{3c^3} \\
&\quad + \frac{b^2\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{3c^3}
\end{aligned}$$

$$= \frac{b^2 x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} - \frac{b^2\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} + \frac{b^2\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}$$

$$= \frac{b^2 x}{3c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} - \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} + \frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.84

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx$$

$$= \frac{b^2cx + abc^2\sqrt{1+\frac{1}{c^2x^2}}x^2 + a^2c^3x^3 + b^2c^2\sqrt{1+\frac{1}{c^2x^2}}x^2\operatorname{csch}^{-1}(cx) + 2abc^3x^3\operatorname{csch}^{-1}(cx) + b^2c^3x^3\operatorname{csch}^{-1}(cx)}{}$$

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*c*x + a*b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2 + a^2*c^3*x^3 + b^2*c^2*Sqrt[1
 + 1/(c^2*x^2)]*x^2*ArcCsch[c*x] + 2*a*b*c^3*x^3*ArcCsch[c*x] + b^2*c^3*x^3
*ArcCsch[c*x]^2 + b^2*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])] - b^2*ArcCsch
[c*x]*Log[1 + E^(-ArcCsch[c*x])] + (a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Log[-(c*x
) + Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] + b^2*PolyLog[2, -E^(-ArcCsch[c*x
])] - b^2*PolyLog[2, E^(-ArcCsch[c*x])])/(3*c^3)
```

Maple [F]

$$\int x^2\left(a+b\operatorname{arccsch}(cx)\right)^2\,dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))^2,x)
```

Fricas [F]

$$\int x^2 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arccsch(c*x)^2 + 2*a*b*x^2*arccsch(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int x^2 (a + b \operatorname{acsch}(cx))^2 \, dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))**2,x)
[Out] Integral(x**2*(a + b*acsch(c*x))**2, x)
```

Maxima [F]

$$\int x^2 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*
x^2) + 1) - 1)/c^2)/c)*a*b + 1/3*(x^3*log(sqrt(c^2*x^2 + 1) + 1)^2 - 3*inte
grate(-1/3*(3*c^2*x^4*log(c)^2 + 3*x^2*log(c)^2 + 3*(c^2*x^4 + x^2)*log(x)^
2 + 6*(c^2*x^4*log(c) + x^2*log(c))*log(x) - 2*(3*c^2*x^4*log(c) + 3*x^2*lo
g(c) + 3*(c^2*x^4 + x^2)*log(x) + (c^2*x^4*(3*log(c) + 1) + 3*x^2*log(c) +
3*(c^2*x^4 + x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 3
*(c^2*x^4*log(c)^2 + x^2*log(c)^2 + (c^2*x^4 + x^2)*log(x)^2 + 2*(c^2*x^4*l
og(c) + x^2*log(c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/
2) + 1), x))*b^2
```

Giac [F]

$$\int x^2 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int x^2 \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

```
[In] int(x^2*(a + b*asinh(1/(c*x)))^2,x)
[Out] int(x^2*(a + b*asinh(1/(c*x)))^2, x)
```

3.17 $\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 54

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx = \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arccsch(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsch(c*x))*(1+1/c^2/
x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6421, 5560, 4269, 3556}

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx = \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{c^2}$$

```
[In] Int[x*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/c + (x^2*(a + b*ArcCsch[c*
x])^2)/2 + (b^2*Log[x])/c^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2 \coth(x)\text{csch}^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\text{csch}^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2 \\
&\quad - \frac{b^2\text{Subst}\left(\int \coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx$$
$$=\frac{acx\left(2b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)+2bcx\left(b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)\operatorname{csch}^{-1}(cx)+b^2c^2x^2\operatorname{csch}^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

```
[In] Integrate[x*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 + 1/(c^2*x^2
)] + a*c*x)*ArcCsch[c*x] + b^2*c^2*x^2*ArcCsch[c*x]^2 + 2*b^2*Log[c*x])/(2*
c^2)
```

Maple [F]

$$\int x(a+b\ \operatorname{arccsch}(cx))^2\,dx$$

```
[In] int(x*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x*(a+b*arccsch(c*x))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.33

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx$$
$$=\frac{b^2c^2x^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2+a^2c^2x^2+2\,abc^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-2\,abc^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right.}{2\,c^2}$$

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^2
*c^2*x^2 + 2*a*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*a
*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*a*b*c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 + b^2*c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)))/c^2
```

Sympy [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=\int x(a+b\operatorname{acsch}(cx))^2\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))**2,x)
[Out] Integral(x*(a + b*acsch(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\begin{aligned}\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=&\frac{1}{2}\,b^2x^2\operatorname{arcsch}(cx)^2+\frac{1}{2}\,a^2x^2\\&+\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)ab\\&+\left(\frac{x\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2\end{aligned}$$

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
[Out] 1/2*b^2*x^2*arccsch(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c^
2*x^2) + 1)/c)*a*b + (x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + log(x)/c^2)*
b^2
```

Giac [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=\int (b\operatorname{arcsch}(cx)+a)^2x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=\int x\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int(x*(a + b*asinh(1/(c*x)))^2,x)
[Out] int(x*(a + b*asinh(1/(c*x)))^2, x)
```

3.18 $\int \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$

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Rubi [A] (verified) . . . 157
Mathematica [A] (verified) . . . 159
Maple [F] . . . 159
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Sympy [F] . . . 160
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Mupad [F(-1)] . . . 161

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = x\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{2b^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c} - \frac{2b^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arccsch(c*x))^2+4*b*(a+b*arccsch(c*x))*arctanh(1/c/x+(1+1/c^2/x^2)^(
1/2))/c+2*b^2*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c-2*b^2*polylog(2,1/c/x
+(1+1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6415, 5560, 4267, 2317, 2438}

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = \frac{4b\operatorname{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c} + x\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{2b^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c} - \frac{2b^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] x*(a + b*ArcCsch[c*x])^2 + (4*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcCsch[c*x]
])/c + (2*b^2*PolyLog[2, -E^ArcCsch[c*x]])/c - (2*b^2*PolyLog[2, E^ArcCsch[
c*x]])/c
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6415

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{Subst}\left(\int (a+bx)^2 \coth(x)\operatorname{csch}(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\
&= x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{(2b)\operatorname{Subst}\left(\int (a+bx)\operatorname{csch}(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\
&= x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{(2b^2)\operatorname{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\
&\quad - \frac{(2b^2)\operatorname{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c}
\end{aligned}$$

$$= x\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\text{csch}^{-1}(cx)}\right)}{c} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\text{csch}^{-1}(cx)}\right)}{c}$$

$$= x\left(a+b\text{csch}^{-1}(cx)\right)^2 + \frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\operatorname{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{2b^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c} - \frac{2b^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.99

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$$

$$= \frac{a^2cx + 2abcx\text{csch}^{-1}(cx) + b^2cx\text{csch}^{-1}(cx)^2 - 2b^2\text{csch}^{-1}(cx)\log\left(1-e^{-\text{csch}^{-1}(cx)}\right) + 2b^2\text{csch}^{-1}(cx)\log\left(1\right.}{}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcCsch[c*x] + b^2*c*x*ArcCsch[c*x]^2 - 2*b^2*ArcCsch[
c*x]*Log[1 - E^(-ArcCsch[c*x])] + 2*b^2*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*
x])] + 2*a*b*Log[Cosh[ArcCsch[c*x]/2]] - 2*a*b*Log[Sinh[ArcCsch[c*x]/2]] -
2*b^2*PolyLog[2, -E^(-ArcCsch[c*x])] + 2*b^2*PolyLog[2, E^(-ArcCsch[c*x])])
/c
```

Maple [F]

$$\int \left(a+b\,\text{arccsch}\left(cx\right)\right)^2\,dx$$

```
[In] int((a+b*arccsch(c*x))^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^2,x)
```

Fricas [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^2 \, dx$$

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2, x)
```

Sympy [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int \left(a + b \operatorname{acsch}(cx)\right)^2 \, dx$$

```
[In] integrate((a+b*acsch(c*x))**2,x)
[Out] Integral((a + b*acsch(c*x))**2, x)
```

Maxima [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^2 \, dx$$

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="maxima")
[Out] (x*log(sqrt(c^2*x^2 + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 +
 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x) - 2*(c^2*x^2*l
og(c) + (c^2*x^2 + 1)*log(x) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 + 1)*log(x)
 + log(c))*sqrt(c^2*x^2 + 1) + log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^
2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c)
)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x))*b^2 +
 a^2*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/
(c^2*x^2) + 1) - 1))*a*b/c
```

Giac [F]

$$\int \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^2 dx$$

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

```
[In] int((a + b*asinh(1/(c*x)))^2,x)
[Out] int((a + b*asinh(1/(c*x)))^2, x)
```

3.19 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx$

Optimal result . . . 162
Rubi [A] (verified) . . . 162
Mathematica [A] (verified) . . . 164
Maple [F] . . . 165
Fricas [F] . . . 165
Sympy [F] . . . 165
Maxima [F] . . . 165
Giac [F] . . . 166
Mupad [F(-1)] . . . 166

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right) + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right)$$

```
[Out] 1/3*(a+b*arccsch(c*x))^3/b-(a+b*arccsch(c*x))^2*ln(1-(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)-b*(a+b*arccsch(c*x))*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/2*
b^2*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 3797, 2221, 2611, 2320, 6724}

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = -b\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcCsch[c*x])^2/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^3/(3*b) - (a + b*ArcCsch[c*x])^2*Log[1 - E^(2*ArcCsch[
c*x])] - b*(a + b*ArcCsch[c*x])*PolyLog[2, E^(2*ArcCsch[c*x])] + (b^2*PolyL
og[3, E^(2*ArcCsch[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^2 \coth(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} + 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2 \log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad + (2b)\text{Subst}\left(\int (a+bx)\log\left(1-e^{2x}\right)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2 \log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad - b\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad + b^2\text{Subst}\left(\int \text{PolyLog}\left(2, e^{2x}\right)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2 \log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad - b\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad + \frac{1}{2}b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2\text{csch}^{-1}(cx)}\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2 \log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad - b\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right) + \frac{1}{2}b^2\,\text{PolyLog}\left(3, e^{2\text{csch}^{-1}(cx)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\begin{aligned}
\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}\,dx &= ab\text{csch}^{-1}(cx)^2 + \frac{1}{3}b^2\text{csch}^{-1}(cx)^3 \\
&\quad - 2ab\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad - b^2\text{csch}^{-1}(cx)^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + a^2\log(cx) \\
&\quad - b\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad + \frac{1}{2}b^2\,\text{PolyLog}\left(3, e^{2\text{csch}^{-1}(cx)}\right)
\end{aligned}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x,x]
```

```
[Out] a*b*ArcCsch[c*x]^2 + (b^2*ArcCsch[c*x]^3)/3 - 2*a*b*ArcCsch[c*x]*Log[1 - E^
(2*ArcCsch[c*x])] - b^2*ArcCsch[c*x]^2*Log[1 - E^(2*ArcCsch[c*x])] + a^2*Lo
g[c*x] - b*(a + b*ArcCsch[c*x])*PolyLog[2, E^(2*ArcCsch[c*x])] + (b^2*PolyL
og[3, E^(2*ArcCsch[c*x])])/2
```

Maple [F]

$$\int \frac{(a+b\,\operatorname{arccsch}(cx))^2}{x}dx$$

```
[In] int((a+b*arccsch(c*x))^2/x,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))^2}{x}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**2/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2 + a^2*log(x) - integrate(-(b^2*log(
c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2 - 2*a*b*log(c) + (b^2*c^2*
```

```
x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)
*log(x) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x
^2 + b^2)*log(x) + sqrt(c^2*x^2 + 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*
log(c) - a*b + (2*b^2*c^2*x^2 + b^2)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1) +
sqrt(c^2*x^2 + 1)*(b^2*log(c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2
 - 2*a*b*log(c) + (b^2*c^2*x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c
^2)*x^2 + b^2*log(c) - a*b)*log(x)))/(c^2*x^3 + (c^2*x^3 + x)*sqrt(c^2*x^2
+ 1) + x), x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^2/x,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal result . . . 167
Rubi [A] (verified) . . . 167
Mathematica [A] (verified) . . . 168
Maple [F] . . . 169
Fricas [B] (verification not implemented) . . . 169
Sympy [F] . . . 169
Maxima [A] (verification not implemented) . . . 170
Giac [F] . . . 170
Mupad [F(-1)] . . . 170

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{2b^2}{x} + 2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}$$

```
[Out] -2*b^2/x-(a+b*arccsch(c*x))^2/x+2*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6421, 3377, 2717}

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx = 2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x} - \frac{2b^2}{x}$$

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^2,x]
```

```
[Out] (-2*b^2)/x + 2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]) - (a + b*ArcC
sch[c*x])^2/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int (a+bx)^2\cosh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= 2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right) - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x} \\
&\quad - \left(2b^2c\right)\text{Subst}\left(\int \cosh(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{2b^2}{x} + 2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right) - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\begin{aligned}
&\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^2}\,dx \\
&= -\frac{a^2+2b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x + 2b\left(a-bc\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{csch}^{-1}(cx) + b^2\text{csch}^{-1}(cx)^2}{x}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^2,x]
```

```
[Out] -((a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 2*b*(a - b*c*Sqrt[1 + 1/
(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*ArcCsch[c*x]^2)/x)
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arccsch}\,(cx))^2}{x^2}dx$$

```
[In] int((a+b*arccsch(c*x))^2/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.

Time $= 0.26$ (sec) , antiderivative size $= 139$, normalized size of antiderivative $= 2.84$

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx$$
$$= \frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - a^2 - 2\,b^2 + 2\left(b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - ab\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{x}$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b^2*log((c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - a*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F]

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))^2}{x^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx = 2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)ab + 2\left(c\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx)-\frac{1}{x}\right)b^2 - \frac{b^2\operatorname{arcsch}(cx)^2}{x} - \frac{a^2}{x}$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x) - 1/x)*b^2 - b^2*arccsch(c*x)^2/x - a^2/x
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^2/x^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\mathbf{csch}^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal result . . . 171
Rubi [A] (verified) . . . 171
Mathematica [A] (verified) . . . 173
Maple [F] . . . 173
Fricas [B] (verification not implemented) . . . 173
Sympy [F] . . . 174
Maxima [F] . . . 174
Giac [F] . . . 174
Mupad [F(-1)] . . . 175

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{b^2}{4x^2} - \frac{1}{2}abc^2\text{csch}^{-1}(cx) - \frac{1}{4}b^2c^2\text{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{2x} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2}$$

```
[Out] -1/4*b^2/x^2-1/2*a*b*c^2*arccsch(c*x)-1/4*b^2*c^2*arccsch(c*x)^2-1/2*(a+b*a
rccsch(c*x))^2/x^2+1/2*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6421, 5554, 3391}

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx = \frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{2x} - \frac{1}{2}abc^2\text{csch}^{-1}(cx) - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2} - \frac{1}{4}b^2c^2\text{csch}^{-1}(cx)^2 - \frac{b^2}{4x^2}$$

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^3,x]
```

```
[Out] -1/4*b^2/x^2 - (a*b*c^2*ArcCsch[c*x])/2 - (b^2*c^2*ArcCsch[c*x]^2)/4 + (b*c
*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*x) - (a + b*ArcCsch[c*x])^2
/(2*x^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^2 \text{Subst}\left(\int (a+bx)^2 \cosh(x)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2} + \left(bc^2\right)\text{Subst}\left(\int (a+bx)\sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{4x^2} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{2x} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2} \\
&\quad - \frac{1}{2}\left(bc^2\right)\text{Subst}\left(\int (a+bx)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{4x^2} - \frac{1}{2}abc^2\text{csch}^{-1}(cx) - \frac{1}{4}b^2c^2\text{csch}^{-1}(cx)^2 \\
&\quad + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{2x} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx =$$
$$-\frac{2a^2+b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x-2b\left(-2a+bc\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{csch}^{-1}(cx)+b^2(2+c^2x^2)\,\text{csch}^{-1}(cx)^2+2abc^2$$
$$4x^2$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^3,x]
```

```
[Out] -1/4*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 2*b*(-2*a + b*c*Sqrt[
1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*(2 + c^2*x^2)*ArcCsch[c*x]^2 + 2*a*b
*c^2*x^2*ArcSinh[1/(c*x)])/x^2
```

Maple [F]

$$\int \frac{\left(a+b\,\text{arccsch}\left(cx\right)\right)^2}{x^3}dx$$

```
[In] int((a+b*arccsch(c*x))^2/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(75) = 150.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx$$
$$=\frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-(b^2c^2x^2+2\,b^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2-2\,a^2-b^2-2\left(abc^2x^2-b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,ab\right)\log}{4\,x^2}$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b^2*c^2*x^2 + 2*b^2)*log((c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 - 2*(a*b*c^2*x
^2 - b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx = \int \frac{\left(a+b\,\operatorname{acsch}\left(cx\right)\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**2/x**3,x)
[Out] Integral((a + b*acsch(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx = \int \frac{\left(b\,\operatorname{arcsch}\left(cx\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="maxima")
[Out] 1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c
^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) -
 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^2 +
2*integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2
*x^2*log(c) + log(c))*log(x) - (2*c^2*x^2*log(c) + 2*(c^2*x^2 + 1)*log(x) +
 (c^2*x^2*(2*log(c) - 1) + 2*(c^2*x^2 + 1)*log(x) + 2*log(c))*sqrt(c^2*x^2
+ 1) + 2*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^2*log(c)^2 + (c^2*x^2
+ 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*x^2
 + 1))/(c^2*x^5 + x^3 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a^2/x
^2
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx = \int \frac{\left(b\,\operatorname{arcsch}\left(cx\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^2/x^3,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal result . . . 176
Rubi [A] (verified) . . . 176
Mathematica [A] (verified) . . . 178
Maple [F] . . . 178
Fricas [B] (verification not implemented) . . . 179
Sympy [F] . . . 179
Maxima [F] . . . 179
Giac [F] . . . 180
Mupad [F(-1)] . . . 180

Optimal result

Integrand size = 14, antiderivative size = 100

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}$$

```
[Out] -2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsch(c*x))^2/x^3-4/9*b*c^3*(a+b*arc
csch(c*x))*(1+1/c^2/x^2)^(1/2)+2/9*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/
2)/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6421, 5554, 3391, 3377, 2717}

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = \frac{2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2} - \frac{4}{9}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}+\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^4,x]
```

```
[Out] (-2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a +
 b*ArcCsch[c*x]))/9 + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(9
*x^2) - (a + b*ArcCsch[c*x])^2/(3*x^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(c^3 \text{Subst}\left(\int (a+bx)^2 \cosh(x) \sinh^2(x)\, dx, x, \text{csch}^{-1}(cx)\right)\right) \\ &= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}(2bc^3)\,\text{Subst}\left(\int (a+bx)\sinh^3(x)\, dx, x, \text{csch}^{-1}(cx)\right)\end{aligned}$$

$$= -\frac{2b^2}{27x^3} + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3} - \frac{1}{9}(4bc^3)\operatorname{Subst}\left(\int (a+bx)\sinh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$

$$= -\frac{2b^2}{27x^3} - \frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3} + \frac{1}{9}(4b^2c^3)\operatorname{Subst}\left(\int \cosh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$

$$= -\frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} - \frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = \frac{-9a^2 + 6abc\sqrt{1+\frac{1}{c^2x^2}}x(1-2c^2x^2) + 2b^2(-1+6c^2x^2) - 6b\left(3a + bc\sqrt{1+\frac{1}{c^2x^2}}x(-1+2c^2x^2)\right)\operatorname{csch}^{-1}(cx)}{27x^3}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^4,x]
```

```
[Out] (-9*a^2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c
^2*x^2) - 6*b*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2))*ArcCsch[
c*x] - 9*b^2*ArcCsch[c*x]^2)/(27*x^3)
```

Maple [F]

$$\int \frac{\left(a+b\ \operatorname{arccsch}(cx)\right)^2}{x^4}dx$$

```
[In] int((a+b*arccsch(c*x))^2/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(86) = 172$.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx$$

$$= \frac{12\,b^2c^2x^2 - 9\,b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 9\,a^2 - 2\,b^2 - 6\left(3\,ab + (2\,b^2c^3x^3 - b^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+}{cx}\right.}{27\,x^3}$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x))^2 - 9*a^2 - 2*b^2 - 6*(3*a*b + (2*b^2*c^3*x^3 - b^2*c*x)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 6*(2
*a*b*c^3*x^3 - a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^3
```

Sympy [F]

$$\int \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a + b\operatorname{acsch}(cx)\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx) + a\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*
arccsch(c*x)/x^3) - 1/3*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^3 + 3*integrate
(-1/3*(3*c^2*x^2*log(c)^2 + 3*(c^2*x^2 + 1)*log(x)^2 + 3*log(c)^2 + 6*(c^2*
x^2*log(c) + log(c))*log(x) - 2*(3*c^2*x^2*log(c) + 3*(c^2*x^2 + 1)*log(x)
 + (c^2*x^2*(3*log(c) - 1) + 3*(c^2*x^2 + 1)*log(x) + 3*log(c))*sqrt(c^2*x^2
 + 1) + 3*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 3*(c^2*x^2*log(c)^2 + (c^2*x
^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*
x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1)), x)) - 1/3*a^
2/x^3
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^4}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^2/x^4,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 132

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\operatorname{csch}^{-1}(cx) + \frac{3}{32}b^2c^4\operatorname{csch}^{-1}(cx)^2+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16x}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x^4}$$

```
[Out] -1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsch(c*x)+3/32*b^2*c^4*arccs
ch(c*x)^2-1/4*(a+b*arccsch(c*x))^2/x^4+1/8*b*c*(a+b*arccsch(c*x))*(1+1/c^2/
x^2)^(1/2)/x^3-3/16*b*c^3*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6421, 5554, 3391}

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \frac{3}{16}abc^4\operatorname{csch}^{-1}(cx)+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16x}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x^4} + \frac{3}{32}b^2c^4\operatorname{csch}^{-1}(cx)^2+\frac{3b^2c^2}{32x^2}-\frac{b^2}{32x^4}$$

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^5,x]
```

```
[Out] -1/32*b^2/x^4 + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsch[c*x])/16 + (3*b^2
*c^4*ArcCsch[c*x]^2)/32 + (b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/
(8*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(16*x) - (a
+ b*ArcCsch[c*x])^2/(4*x^4)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^4 \text{Subst}\left(\int (a+bx)^2 \cosh(x) \sinh^3(x)\, dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right) \text{Subst}\left(\int (a+bx) \sinh^4(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{32x^4} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4} \\
&\quad - \frac{1}{8}\left(3bc^4\right) \text{Subst}\left(\int (a+bx) \sinh^2(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{16x} \\
&\quad - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{16}\left(3bc^4\right) \text{Subst}\left(\int (a+bx)\, dx, x, \text{csch}^{-1}(cx)\right)
\end{aligned}$$

$$= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\text{csch}^{-1}(cx) + \frac{3}{32}b^2c^4\text{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{16x} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx$$

$$= \frac{-8a^2 - b^2 + 4abc\sqrt{1+\frac{1}{c^2x^2}}x + 3b^2c^2x^2 - 6abc^3\sqrt{1+\frac{1}{c^2x^2}}x^3 - 2b\left(8a + bc\sqrt{1+\frac{1}{c^2x^2}}x(-2+3c^2x^2)\right)\text{csch}}{32x^4}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 - b^2 + 4*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^3
*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3
*c^2*x^2))*ArcCsch[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^2 + 6*a*b*c^4*x
^4*ArcSinh[1/(c*x)])/(32*x^4)
```

Maple [F]

$$\int \frac{\left(a+b\,\text{arccsch}\left(cx\right)\right)^2}{x^5}dx$$

```
[In] int((a+b*arccsch(c*x))^2/x^5,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx$$

$$= \frac{3\,b^2c^2x^2 + \left(3\,b^2c^4x^4 - 8\,b^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 8\,a^2 - b^2 + 2\left(3\,abc^4x^4 - 8\,ab - \left(3\,b^2c^3x^3 - 2\,b^2cx\right)\sqrt{\frac{c^2}{}}\right.}{32\,x^4}$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x))^2 - 8*a^2 - b^2 + 2*(3*a*b*c^4*x^4 - 8*a*b - (3*b^2*c
^3*x^3 - 2*b^2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 +
1)/(c^2*x^2)))/x^4
```

Sympy [F]

$$\int \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(a + b\operatorname{acsch}(cx)\right)^2}{x^5}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**5, x)
```

Maxima [F]

$$\int \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx) + a\right)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/
(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1)
 + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^4
 + 4*integrate(-1/2*(2*c^2*x^2*log(c)^2 + 2*(c^2*x^2 + 1)*log(x)^2 + 2*log(
c)^2 + 4*(c^2*x^2*log(c) + log(c))*log(x) - (4*c^2*x^2*log(c) + 4*(c^2*x^2
+ 1)*log(x) + (c^2*x^2*(4*log(c) - 1) + 4*(c^2*x^2 + 1)*log(x) + 4*log(c))*
sqrt(c^2*x^2 + 1) + 4*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(c^2*x^2*log(c
)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x
))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1)),
x)) - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^5}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^2/x^5,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 195

$$\begin{aligned}\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx = {}& \frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}\\ & -\frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^3}\\ & +\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\\ & +\frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{c^4}\\ & +\frac{b^3\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^4}\end{aligned}$$

```
[Out] 1/4*b^2*x^2*(a+b*arccsch(c*x))/c^2-1/2*b*(a+b*arccsch(c*x))^2/c^4+1/4*x^4*(
a+b*arccsch(c*x))^3+b^2*(a+b*arccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))
^2)/c^4+1/2*b^3*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(1+1
/c^2/x^2)^(1/2)/c^3-1/2*b*x*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c^3+1/
4*b*x^3*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6421, 5560, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\begin{aligned}\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx = &\frac{b^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^4}\\ &+\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4}\\ &+\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4c}\\ &-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^3}+\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\\ &+\frac{b^3\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^4}+\frac{b^3x\sqrt{\frac{1}{c^2x^2}+1}}{4c^3}\end{aligned}$$

```
[In] Int[x^3*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[1 + 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsch[c*x]))/(4*c
^2) - (b*(a + b*ArcCsch[c*x])^2)/(2*c^4) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a +
b*ArcCsch[c*x])^2)/(2*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*
x])^2)/(4*c) + (x^4*(a + b*ArcCsch[c*x])^3)/4 + (b^2*(a + b*ArcCsch[c*x])*L
og[1 - E^(2*ArcCsch[c*x])])/c^4 + (b^3*PolyLog[2, E^(2*ArcCsch[c*x])])/(2*c
^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
```

```
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^3 \coth(x)\text{csch}^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{csch}^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{4c^4} \\
&= \frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 \\
&\quad + \frac{b\text{Subst}\left(\int (a+bx)^2\text{csch}^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{2c^4} - \frac{b^3\text{Subst}\left(\int \text{csch}^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{4c^4} \\
&= \frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3} \\
&\quad + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 \\
&\quad + \frac{b^2\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^4} \\
&\quad + \frac{(ib^3)\,\text{Subst}\left(\int 1\,dx, x, -ic\sqrt{1+\frac{1}{c^2x^2}}x\right)}{4c^4} \\
&= \frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4} \\
&\quad - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c} \\
&\quad + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{(2b^2)\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)}{c^4} \\
&= \frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4} \\
&\quad - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c} \\
&\quad + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{b^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{c^4} \\
&\quad - \frac{b^3\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4} \\
&\quad - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4c} \\
&\quad + \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 + \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{c^4} \\
&\quad - \frac{b^3\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^4} \\
&= \frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2} \\
&\quad - \frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^3} \\
&\quad + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \\
&\quad + \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{c^4} + \frac{b^3\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.39

$$
\begin{aligned}
&\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx \\
&= \frac{-2a^2bc\sqrt{1+\frac{1}{c^2x^2}}x + b^3c\sqrt{1+\frac{1}{c^2x^2}}x + ab^2c^2x^2 + a^2bc^3\sqrt{1+\frac{1}{c^2x^2}}x^3 + a^3c^4x^4 + b^2\left(3ac^4x^4 + b\left(2 - 2c\sqrt{1-\right.\right.}\right.}{}
\end{aligned}
$$

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x])^3,x]

[Out] (-2*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x + a*b^2
*c^2*x^2 + a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^4
*x^4 + b*(2 - 2*c*Sqrt[1 + 1/(c^2*x^2)]*x + c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3))
*ArcCsch[c*x]^2 + b^3*c^4*x^4*ArcCsch[c*x]^3 + b*ArcCsch[c*x]*(c*x*(b^2*c*x
 + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2)) + 4*b^2*Log[
1 - E^(-2*ArcCsch[c*x])]) + 4*a*b^2*Log[1/(c*x)] - 2*b^3*PolyLog[2, E^(-2*A
rcCsch[c*x])])/(4*c^4)
```

Maple [F]

$$\int x^3(a+b\,\mathrm{arccsch}\,(cx))^3\,dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))^3,x)
[Out] int(x^3*(a+b*arccsch(c*x))^3,x)
```

Fricas [F]

$$\int x^3\left(a+b\mathrm{csch}^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arcsch}\,(cx)+a)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^3*arccsch(c*x)^3 + 3*a*b^2*x^3*arccsch(c*x)^2 + 3*a^2*b*x^3*
arccsch(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3\left(a+b\mathrm{csch}^{-1}(cx)\right)^3\,dx=\int x^3(a+b\,\mathrm{acsch}\,(cx))^3\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))**3,x)
[Out] Integral(x**3*(a + b*acsch(c*x))**3, x)
```

Maxima [F]

$$\int x^3\left(a+b\mathrm{csch}^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arcsch}\,(cx)+a)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4*log(sqrt(c^2*x^2 + 1) + 1)^3 - 12*b^3*c^2*integrate(1/4*x^5*log
(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c
)^2 + 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 1/4*a^3*x^4
 - 12*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x^2 +
1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integ
rate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^2
```

```
*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2
*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^
2*integrate(1/4*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^
2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c
^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
 1), x)*log(c) - 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) +
24*a*b^2*c^2*integrate(1/4*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
+ sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^5*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x)*log(c) - 4*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^3
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3
*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1
)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 12*b^
3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)
^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 4*b^
3*c^2*integrate(1/4*x^5*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqr
t(c^2*x^2 + 1) + 1), x) + 12*b^3*c^2*integrate(1/4*x^5*log(x)^2*log(sqrt(c^
2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x) - 12*b^3*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(
sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2
*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*c^2*integrate(1/4*sqrt(
c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*sqrt
(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
 c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 3*b^3*c^2*integrate(1/4*sqrt(c^2*x^
2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^
2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*x^5*log(x)^2/(s
qrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*
c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*
c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4
*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x) - 12*b^3*integrate(1/4*x^3*log(x)/(sqrt(c^2*x^2 +
1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 12*b^3*integra
te(1/4*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
+ sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 12*b^3*integrate(1/4*sqrt(c^2*x^2 +
 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
 1), x)*log(c) + 24*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt
(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
 + 1), x)*log(c) - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*
x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x)*log(c) - 12*b^3*integrate(1/4*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*integrate(1/4*x^3*
log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
```

```
rt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*integrate(1/4*x^3*log(sqrt(c^2*x^2
 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1),
 x)*log(c) + 24*a*b^2*integrate(1/4*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
 c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*integrate(1/4*x^3*l
og(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) + 1/4*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) +
 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a^2*b - 4*b^3*integrate(1/4*sqr
t(c^2*x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x) + 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)^2*lo
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x) - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqr
t(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
 1) + 1), x) - 4*b^3*integrate(1/4*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2
 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3*integrate(1/4*x^3*log(x)^2*
log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*
x^2 + 1) + 1), x) - 12*b^3*integrate(1/4*x^3*log(x)*log(sqrt(c^2*x^2 + 1) +
 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 1
2*a*b^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2
*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*integrate(1/4*sqrt(c
^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*integrate(1/4*sqrt(c^2*
x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*integrate(1/4*x^3*log(x)^2/(sqr
t(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*in
tegrate(1/4*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*integrate(1/4*x^3*log(s
qrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2
 + 1) + 1), x) + 1/12*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2
) - 12*sqrt(c^2*x^2 + 1) + 6)*b^3*log(c)^3/c^4 + 1/6*(3*c^2*x^2 - 2*(c^2*x^
2 + 1)^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*b^3*log(c)^3/c
^4 - 1/2*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*b^3*log(c)^3/c^4 - 1/2*b^3*(2*
sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*log(c)^3/c^4 + 1/4*(6*c^2*x^2 - 3*(c^
2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*b^3*log(c)
^2*log(x)/c^4 - 3/2*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*b^3*log(c)^2*log(x)
/c^4 - 1/4*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt
(c^2*x^2 + 1) + 6)*b^3*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^4 + 3/2*(c^2*x
^2 - 2*sqrt(c^2*x^2 + 1) + 1)*b^3*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^4 -
 1/4*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x
^2 + 1) + 6)*a*b^2*log(c)^2/c^4 - 1/2*(3*c^2*x^2 - 2*(c^2*x^2 + 1)^(3/2) +
6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*a*b^2*log(c)^2/c^4 + 3/2*(c^2
*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*a*b^2*log(c)^2/c^4 - 1/48*(18*c^2*x^2 - 9*(
c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) - 96*sqrt(c^2*x^2 + 1) + 66*log(sqr
t(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2 + 1) - 1) + 18)*b^3*log(c)^2/c^4
 - 1/48*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 + 1)^(3/2) + 132*sqrt(c
^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*b^3*log(c)^2/c^4 + 3/4*(c
```

```
^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log(sqrt(c^2*
x^2 + 1) - 1) + 1)*b^3*log(c)^2/c^4 - 3/4*(c^2*x^2 - 6*sqrt(c^2*x^2 + 1) +
6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*b^3*log(c)^2/c^4 + 3/2*a*b^2*(2*sqrt(c^2*
x^2 + 1) - log(c^2*x^2 + 1))*log(c)^2/c^4 - 1/2*(6*c^2*x^2 - 3*(c^2*x^2 + 1
)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*a*b^2*log(c)*log(x)
/c^4 + 3*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*a*b^2*log(c)*log(x)/c^4 + 1/2*
(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 +
1) + 6)*a*b^2*log(c)*log(sqrt(c^2*x^2 + 1) + 1)/c^4 - 3*(c^2*x^2 - 2*sqrt(c
^2*x^2 + 1) + 1)*a*b^2*log(c)*log(sqrt(c^2*x^2 + 1) + 1)/c^4 + 1/24*(18*c^2
*x^2 - 9*(c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) - 96*sqrt(c^2*x^2 + 1) +
66*log(sqrt(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2 + 1) - 1) + 18)*a*b^2*l
og(c)/c^4 + 1/24*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 + 1)^(3/2) +
132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*a*b^2*log(c)/c^
4 - 3/2*(c^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log
(sqrt(c^2*x^2 + 1) - 1) + 1)*a*b^2*log(c)/c^4 + 3/2*(c^2*x^2 - 6*sqrt(c^2*x
^2 + 1) + 6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*a*b^2*log(c)/c^4
```

Giac [F]

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int x^3 \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int(x^3*(a + b*asinh(1/(c*x)))^3,x)
[Out] int(x^3*(a + b*asinh(1/(c*x)))^3, x)
```

3.25 $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

Optimal result . . . 195
Rubi [A] (verified) . . . 196
Mathematica [B] (verified) . . . 199
Maple [F] . . . 200
Fricas [F] . . . 200
Sympy [F] . . . 201
Maxima [F] . . . 201
Giac [F] . . . 202
Mupad [F(-1)] . . . 202

Optimal result

Integrand size = 14, antiderivative size = 194

$$\begin{aligned}\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx = &\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}\\&+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^3\\&-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c^3}\\&+\frac{b^3\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^3}\\&-\frac{b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c^3}\\&+\frac{b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c^3}\\&+\frac{b^3\,\text{PolyLog}\left(3,-e^{\text{csch}^{-1}(cx)}\right)}{c^3}-\frac{b^3\,\text{PolyLog}\left(3,e^{\text{csch}^{-1}(cx)}\right)}{c^3}\end{aligned}$$

```
[Out] b^2*x*(a+b*arccsch(c*x))/c^2+1/3*x^3*(a+b*arccsch(c*x))^3-b*(a+b*arccsch(c*
x))^2*arctanh(1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1+1/c^2/x^2)^(1/2
))/c^3-b^2*(a+b*arccsch(c*x))*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c^3+b^2
*(a+b*arccsch(c*x))*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+b^3*polylog(3,
-1/c/x-(1+1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,1/c/x+(1+1/c^2/x^2)^(1/2))/c^
3+1/2*b*x^2*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6421, 5560, 4271, 3855, 4267, 2611, 2320, 6724}

$$\begin{aligned}\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx &= -\frac{b\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{c^3}\\ &- \frac{b^2\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^3}\\ &+ \frac{b^2\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^3}\\ &+ \frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}\\ &+ \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 + \frac{b^3\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^3}\\ &+ \frac{b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} - \frac{b^3\operatorname{PolyLog}\left(3,e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3}\end{aligned}$$

```
[In] Int[x^2*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcCsch[c*x]))/c^2 + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcC
sch[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsch[c*x])^3)/3 - (b*(a + b*ArcCsch[c*x
])^2*ArcTanh[E^ArcCsch[c*x]])/c^3 + (b^3*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c^
3 - (b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c^3 + (b^2*(a +
b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c^3 + (b^3*PolyLog[3, -E^ArcCsc
h[c*x]])/c^3 - (b^3*PolyLog[3, E^ArcCsch[c*x]])/c^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (a+bx)^3 \coth(x)\text{csch}^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{b\operatorname{Subst}\left(\int (a+bx)^2\operatorname{csch}^3(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} + \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3$$
$$+ \frac{b\operatorname{Subst}\left(\int (a+bx)^2\operatorname{csch}(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2c^3} - \frac{b^3\operatorname{Subst}\left(\int \operatorname{csch}(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}$$
$$+ \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^3} - \frac{b^2\operatorname{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3}$$
$$+ \frac{b^2\operatorname{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}$$
$$+ \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^3} - \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2, -e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2, e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -e^x\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3}$$
$$- \frac{b^3\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, e^x\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3}$$

$$
\begin{aligned}
&= \frac{b^2 x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} \\
&\quad + \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^3 \operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^3} - \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^3 \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad - \frac{b^3 \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&= \frac{b^2 x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} \\
&\quad + \frac{1}{3}x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^3 \operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^3} - \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^3 \operatorname{PolyLog}\left(3,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3} - \frac{b^3 \operatorname{PolyLog}\left(3,e^{\operatorname{csch}^{-1}(cx)}\right)}{c^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 548 vs. $2(194) = 388$.

Time = 7.56 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.82

$$
\begin{aligned}
&\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx \\
&= \frac{a^3x^3}{3} + \frac{a^2bx^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c} + a^2bx^3\operatorname{csch}^{-1}(cx) - \frac{a^2b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{2c^3} \\
&\quad + \frac{ab^2\left(8\operatorname{PolyLog}\left(2,-e^{-\operatorname{csch}^{-1}(cx)}\right) + 2c^3x^3\left(-2+4\operatorname{csch}^{-1}(cx)^2+2\cosh\left(2\operatorname{csch}^{-1}(cx)\right) - \frac{3\operatorname{csch}^{-1}(cx)\log\left(1\right.}{cx}\right.\right.}{} \\
&\quad + \frac{b^3\left(24\operatorname{csch}^{-1}(cx)\coth\left(\frac{1}{2}\operatorname{csch}^{-1}(cx)\right) - 4\operatorname{csch}^{-1}(cx)^3\coth\left(\frac{1}{2}\operatorname{csch}^{-1}(cx)\right) + 6\operatorname{csch}^{-1}(cx)^2\operatorname{csch}^2\left(\frac{1}{2}\operatorname{csch}^{-1}(\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3*A
rcCsch[c*x] - (a^2*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3) +
(a*b^2*(8*PolyLog[2, -E^(-ArcCsch[c*x])] + 2*c^3*x^3*(-2 + 4*ArcCsch[c*x]^2
 + 2*Cosh[2*ArcCsch[c*x]] - (3*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])])/(c*
x) + (3*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*x])])/(c*x) - (4*PolyLog[2, E^(-
ArcCsch[c*x])])/(c^3*x^3) + 2*ArcCsch[c*x]*Sinh[2*ArcCsch[c*x]] + ArcCsch[c
*x]*Log[1 - E^(-ArcCsch[c*x])]*Sinh[3*ArcCsch[c*x]] - ArcCsch[c*x]*Log[1 +
E^(-ArcCsch[c*x])]*Sinh[3*ArcCsch[c*x]])))/(8*c^3) + (b^3*(24*ArcCsch[c*x]*
Coth[ArcCsch[c*x]/2] - 4*ArcCsch[c*x]^3*Coth[ArcCsch[c*x]/2] + 6*ArcCsch[c*
x]^2*Csch[ArcCsch[c*x]/2]^2 + (ArcCsch[c*x]^3*Csch[ArcCsch[c*x]/2]^4)/(c*x)
 + 24*ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] - 24*ArcCsch[c*x]^2*Log[1 +
 E^(-ArcCsch[c*x])] - 48*Log[Tanh[ArcCsch[c*x]/2]] + 48*ArcCsch[c*x]*PolyLo
g[2, -E^(-ArcCsch[c*x])] - 48*ArcCsch[c*x]*PolyLog[2, E^(-ArcCsch[c*x])] +
48*PolyLog[3, -E^(-ArcCsch[c*x])] - 48*PolyLog[3, E^(-ArcCsch[c*x])] + 6*Ar
cCsch[c*x]^2*Sech[ArcCsch[c*x]/2]^2 + 16*c^3*x^3*ArcCsch[c*x]^3*Sinh[ArcCsc
h[c*x]/2]^4 - 24*ArcCsch[c*x]*Tanh[ArcCsch[c*x]/2] + 4*ArcCsch[c*x]^3*Tanh[
ArcCsch[c*x]/2]))/(48*c^3)
```

Maple [F]

$$\int x^2(a + b \operatorname{arccsch}(cx))^3 \, dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))^3,x)
```

Fricas [F]

$$\int x^2\left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arccsch(c*x)^3 + 3*a*b^2*x^2*arccsch(c*x)^2 + 3*a^2*b*x^2*
arccsch(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b \text{csch}^{-1}(cx)\right)^3 \, dx = \int x^2 (a + b \, \text{acsch}\,(cx))^3 \, dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))**3, x)
```

Maxima [F]

$$\int x^2 \left(a + b \text{csch}^{-1}(cx)\right)^3 \, dx = \int (b \, \text{arcsch}\,(cx) + a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log(sqrt(c^2*x^2 + 1) + 1)^3 + 1/3*a^3*x^3 + 1/4*(4*x^3*arccsch
(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1
/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b - i
ntegrate(((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b^3*c^2*x^4 + b^
3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*((b^3*c^2*log(c
) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + (3*(b^3*c^2*log(c
) - a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)
*log(x) + ((b^3*c^2*(3*log(c) + 1) - 3*a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b
^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2
*x^2 + 1) + 1)^2 + 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*lo
g(c)^2 - 2*a*b^2*log(c))*x^2)*log(x) - 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*l
og(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*
log(x)^2 + 2*((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*
log(x) + ((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a
*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*log(x)^2 + 2*((b^3*c^2*log(c) -
a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(s
qrt(c^2*x^2 + 1) + 1) + ((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b
^3*c^2*x^4 + b^3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*
((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + 3*
((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(
c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

Giac [F]

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int \left(b\operatorname{arcsch}(cx)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int x^2\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int(x^2*(a + b*asinh(1/(c*x)))^3,x)
[Out] int(x^2*(a + b*asinh(1/(c*x)))^3, x)
```

3.26 $\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 117

$$\begin{aligned}\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx = {}& \frac{3b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c}\\ &+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\\ &-\frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{c^2}\\ &-\frac{3b^3\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^2}\end{aligned}$$

```
[Out] 3/2*b*(a+b*arccsch(c*x))^2/c^2+1/2*x^2*(a+b*arccsch(c*x))^3-3*b^2*(a+b*arcc
sch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/c^2-3/2*b^3*polylog(2,(1/c/x+
(1+1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2
)/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used

$= \{6421, 5560, 4269, 3797, 2221, 2317, 2438\}$

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx = -\frac{3b^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{3bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} + \frac{3b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{3b^3\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^2}$$

```
[In] Int[x*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcCsch[c*x])^2)/(2*c^2) + (3*b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*
ArcCsch[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsch[c*x])^3)/2 - (3*b^2*(a + b*Arc
Csch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/c^2 - (3*b^3*PolyLog[2, E^(2*ArcCsc
h[c*x])])/(2*c^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{Subst}\left(\int (a+bx)^3 \coth(x)\operatorname{csch}^2(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{(3b)\operatorname{Subst}\left(\int (a+bx)^2\operatorname{csch}^2(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2c^2} \\
&= \frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \\
&\quad - \frac{(3b^2)\operatorname{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{3b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} \\
&\quad + \frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 + \frac{(6b^2)\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{3b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c} \\
&\quad + \frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{c^2} \\
&\quad + \frac{(3b^3)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2}
\end{aligned}$$

$$= \frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{c^2} + \frac{(3b^3)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\text{csch}^{-1}(cx)}\right)}{2c^2}$$

$$= \frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{c^2} - \frac{3b^3\,\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$$

$$= \frac{3b^2\left(ac^2x^2+b\left(-1+c\sqrt{1+\frac{1}{c^2x^2}}x\right)\right)\text{csch}^{-1}(cx)^2 + b^3c^2x^2\text{csch}^{-1}(cx)^3 + 3b\text{csch}^{-1}(cx)\left(acx\left(2b\sqrt{1+\frac{1}{c^2x^2}}-\right.\right.}{}$$

```
[In] Integrate[x*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (3*b^2*(a*c^2*x^2 + b*(-1 + c*Sqrt[1 + 1/(c^2*x^2)]*x))*ArcCsch[c*x]^2 + b^
3*c^2*x^2*ArcCsch[c*x]^3 + 3*b*ArcCsch[c*x]*(a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2
)] + a*c*x) - 2*b^2*Log[1 - E^(-2*ArcCsch[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 +
1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + 3*b^3*PolyLog[2, E^(-2*ArcCsc
h[c*x])])/(2*c^2)
```

Maple [F]

$$\int x(a+b\,\text{arccsch}\,(cx))^3\,dx$$

```
[In] int(x*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x*(a+b*arccsch(c*x))^3,x)
```

Fricas [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int (b\operatorname{arcsch}(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arccsch(c*x)^3 + 3*a*b^2*x*arccsch(c*x)^2 + 3*a^2*b*x*arccsc
h(c*x) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int x(a+b\operatorname{acsch}(cx))^3\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))**3, x)
```

Maxima [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int (b\operatorname{arcsch}(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arccsch(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsch(c*x) + x*sqrt
(1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + lo
g(x)/c^2)*a*b^2 - 1/4*(24*c^2*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*c^2*integrate(1
/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 2*x^2*log(sqrt(c^2*x^2 + 1) + 1)^3 + 24
*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 48*c^2*integrate(1/2*sqrt
(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integrate(1/2*sq
rt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2
 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integrate(1/2*x^3*l
og(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*l
og(c) - 48*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^
2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*i
ntegrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
```

```
c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 8*c^2*integrate(1/2*sqrt(c^2*
x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
 1) + 1), x) - 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2*log(sqrt
(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
 + 1), x) + 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*
x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x) + 8*c^2*integrate(1/2*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*c^2*integrate(1/2*x^3*log(x)^2*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x) + 24*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*c^2*
integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*
sqrt(c^2*x^2 + 1)*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*
x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*integrate(1/2*x
*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
 48*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*
integrate(1/2*x*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x)*log(c) - 48*integrate(1/2*x*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)
 + 24*integrate(1/2*x*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 2*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)^3/c^2 + 2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*log
(c)^3/c^2 + 2*(log(c^2*x^2 + 1) - 2*log(sqrt(c^2*x^2 + 1) + 1))*log(c)^3/c^
2 + 6*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(x)/c^2 - 6*(c^2*x^2
 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^2 + 4*log(
c)^3*log(sqrt(c^2*x^2 + 1) + 1)/c^2 - 6*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)
^2/c^2 + 4*log(c)*log(sqrt(c^2*x^2 + 1) + 1)^3/c^2 - 3*(c^2*x^2 - 4*sqrt(c^
2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)
*log(c)^2/c^2 + 3*(c^2*x^2 - 6*sqrt(c^2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1)
+ 1) + 1)*log(c)^2/c^2 + 8*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^3/(sqrt
(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integrate
(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
+ 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*sqrt
(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 8*integrate(1/2*x*log(x)^3/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integra
te(1/2*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*x*log(x)*log(sqrt(c^
2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
+ 1), x))*b^3
```

Giac [F]

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int (b\operatorname{arcsch}(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int x\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int(x*(a + b*asinh(1/(c*x)))^3,x)
[Out] int(x*(a + b*asinh(1/(c*x)))^3, x)
```

3.27 $\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx$

Optimal result . . . 210
Rubi [A] (verified) . . . 210
Mathematica [B] (verified) . . . 213
Maple [F] . . . 214
Fricas [F] . . . 214
Sympy [F] . . . 214
Maxima [F] . . . 214
Giac [F] . . . 215
Mupad [F(-1)] . . . 215

Optimal result

Integrand size = 10, antiderivative size = 120

$$\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx = x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3+\frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c}+\frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c}-\frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{c}-\frac{6b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c}+\frac{6b^3\operatorname{PolyLog}\left(3,e^{\operatorname{csch}^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arccsch(c*x))^3+6*b*(a+b*arccsch(c*x))^2*arctanh(1/c/x+(1+1/c^2/x^2)
^(1/2))/c+6*b^2*(a+b*arccsch(c*x))*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c-
6*b^2*(a+b*arccsch(c*x))*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c-6*b^3*polyl
og(3,-1/c/x-(1+1/c^2/x^2)^(1/2))/c+6*b^3*polylog(3,1/c/x+(1+1/c^2/x^2)^(1/2
))/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

$= \{6415, 5560, 4267, 2611, 2320, 6724\}$

$$\int \left(a + b\text{csch}^{-1}(cx)\right)^3 dx = \frac{6b\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a + b\text{csch}^{-1}(cx)\right)^2}{c} + \frac{6b^2\,\text{PolyLog}\left(2, -e^{\text{csch}^{-1}(cx)}\right)\left(a + b\text{csch}^{-1}(cx)\right)}{c} - \frac{6b^2\,\text{PolyLog}\left(2, e^{\text{csch}^{-1}(cx)}\right)\left(a + b\text{csch}^{-1}(cx)\right)}{c} + x\left(a + b\text{csch}^{-1}(cx)\right)^3 - \frac{6b^3\,\text{PolyLog}\left(3, -e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{6b^3\,\text{PolyLog}\left(3, e^{\text{csch}^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] x*(a + b*ArcCsch[c*x])^3 + (6*b*(a + b*ArcCsch[c*x])^2*ArcTanh[E^ArcCsch[c*
x]])/c + (6*b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c - (6*b^
2*(a + b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c - (6*b^3*PolyLog[3, -E
^ArcCsch[c*x]])/c + (6*b^3*PolyLog[3, E^ArcCsch[c*x]])/c
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6415

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^3 \coth(x)\text{csch}(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c} \\
&= x\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{c} \\
&= x\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2 \text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{(6b^2)\,\text{Subst}\left(\int (a+bx)\log\left(1-e^x\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{c} \\
&\quad - \frac{(6b^2)\,\text{Subst}\left(\int (a+bx)\log\left(1+e^x\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{c} \\
&= x\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2 \text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2, -e^{\text{csch}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2, e^{\text{csch}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{(6b^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2, -e^x\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{c} \\
&\quad + \frac{(6b^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2, e^x\right)\,dx, x, \text{csch}^{-1}(cx)\right)}{c}
\end{aligned}$$

$$
\begin{aligned}
&= x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{(6b^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{(6b^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&= x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 + \frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{6b^3\operatorname{PolyLog}\left(3,-e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{6b^3\operatorname{PolyLog}\left(3,e^{\operatorname{csch}^{-1}(cx)}\right)}{c}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 246 vs. $2(120) = 240$.

Time $= 0.42$ (sec) , antiderivative size $= 246$, normalized size of antiderivative $= 2.05$

$$
\begin{aligned}
\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^3 dx &= a^3x + 3a^2bx\operatorname{csch}^{-1}(cx) + \frac{3a^2b\log\left(cx\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{c} \\
&\quad + \frac{3ab^2\left(\operatorname{csch}^{-1}(cx)\left(cx\operatorname{csch}^{-1}(cx) - 2\log\left(1-e^{-\operatorname{csch}^{-1}(cx)}\right) + 2\log\left(1+e^{-\operatorname{csch}^{-1}(cx)}\right)\right) - 2\operatorname{PolyLog}\left(2,-\right.\right.}{c} \\
&\quad + \frac{b^3\left(cx\operatorname{csch}^{-1}(cx)^3 - 3\operatorname{csch}^{-1}(cx)^2\log\left(1-e^{-\operatorname{csch}^{-1}(cx)}\right) + 3\operatorname{csch}^{-1}(cx)^2\log\left(1+e^{-\operatorname{csch}^{-1}(cx)}\right) - 6\operatorname{csch}^{-1}\right.}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcCsch[c*x] + (3*a^2*b*Log[c*x*(1 + Sqrt[(1 + c^2*x^2)/(
c^2*x^2)])])/c + (3*a*b^2*(ArcCsch[c*x]*(c*x*ArcCsch[c*x] - 2*Log[1 - E^(-A
rcCsch[c*x])] + 2*Log[1 + E^(-ArcCsch[c*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c
*x])] + 2*PolyLog[2, E^(-ArcCsch[c*x])]))/c + (b^3*(c*x*ArcCsch[c*x]^3 - 3*
```

```
ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] + 3*ArcCsch[c*x]^2*Log[1 + E^(-Ar
cCsch[c*x])] - 6*ArcCsch[c*x]*PolyLog[2, -E^(-ArcCsch[c*x])] + 6*ArcCsch[c*
x]*PolyLog[2, E^(-ArcCsch[c*x])] - 6*PolyLog[3, -E^(-ArcCsch[c*x])] + 6*Pol
yLog[3, E^(-ArcCsch[c*x])]))/c
```

Maple [F]

$$\int (a + b \operatorname{arccsch}(cx))^3 \, dx$$

```
[In] int((a+b*arccsch(c*x))^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^3,x)
```

Fricas [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 \, dx$$

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x)
 + a^3, x)
```

Sympy [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (a + b \operatorname{acsch}(cx))^3 \, dx$$

```
[In] integrate((a+b*acsch(c*x))**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3, x)
```

Maxima [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 \, dx$$

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*x + 3/2*(2*c*x*arccsch(c*x) + log(
sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*a^2*b/c - inte
grate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^
```

```
3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c
^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log
(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(b^3*
log(c) - a*b^2 + (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^
3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c)
 + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x) - 3*(b^3*log(c)^2 -
2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2
 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2
)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2
*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*
c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 +
1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 +
 (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c)
 + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/
(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

Giac [F]

$$\int \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int \left(b \operatorname{arcsch}(cx) + a\right)^3 \, dx$$

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((a + b*asinh(1/(c*x)))^3,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^3, x)
```

3.28 $\int \frac{\left(a+b\mathbf{csch}^{-1}(cx)\right)^3}{x}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 110

$$\begin{aligned}\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x}\,dx = {}& \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\\ & - \frac{3}{2}b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)\\ & + \frac{3}{2}b^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right)\\ & - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4, e^{2\operatorname{csch}^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/4*(a+b*arccsch(c*x))^4/b-(a+b*arccsch(c*x))^3*ln(1-(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)-3/2*b*(a+b*arccsch(c*x))^2*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2
)+3/2*b^2*(a+b*arccsch(c*x))*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)-3/4*b
^3*polylog(4,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{6421, 3797, 2221, 2611, 6744, 2320, 6724\}$

$$\int \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^3}{x}\,dx = \frac{3}{2}b^2 \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a + b\operatorname{csch}^{-1}(cx)\right) - \frac{3}{2}b \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a + b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{\left(a + b\operatorname{csch}^{-1}(cx)\right)^4}{4b} - \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a + b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4, e^{2\operatorname{csch}^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcCsch[c*x])^3/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^4/(4*b) - (a + b*ArcCsch[c*x])^3*Log[1 - E^(2*ArcCsch[
c*x])] - (3*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2 + (3
*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])])/2 - (3*b^3*PolyLo
g[4, E^(2*ArcCsch[c*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
```

```
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^3 \coth(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} + 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3 \log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad + (3b)\text{Subst}\left(\int (a+bx)^2 \log\left(1-e^{2x}\right)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3 \log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2 \text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right) \\
&\quad + \left(3b^2\right)\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2, e^{2x}\right)\,dx, x, \text{csch}^{-1}(cx)\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}b^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad - \frac{1}{2}\left(3b^3\right) \operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, e^{2x}\right)\, dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}b^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad - \frac{1}{4}\left(3b^3\right) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\, dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}b^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right) - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.65

$$
\begin{aligned}
\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x}\, dx = \frac{1}{4}\Big(&6a^2b\operatorname{csch}^{-1}(cx)^2 + 4ab^2\operatorname{csch}^{-1}(cx)^3 + b^3\operatorname{csch}^{-1}(cx)^4 \\
&- 12a^2b\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&- 12ab^2\operatorname{csch}^{-1}(cx)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&- 4b^3\operatorname{csch}^{-1}(cx)^3\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + 4a^3\log(cx) \\
&- 6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&+ 6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&- 3b^3 \operatorname{PolyLog}\left(4, e^{2\operatorname{csch}^{-1}(cx)}\right)\Big)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x,x]
```

```
[Out] (6*a^2*b*ArcCsch[c*x]^2 + 4*a*b^2*ArcCsch[c*x]^3 + b^3*ArcCsch[c*x]^4 - 12*
a^2*b*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] - 12*a*b^2*ArcCsch[c*x]^2*Lo
```

```
g[1 - E^(2*ArcCsch[c*x])] - 4*b^3*ArcCsch[c*x]^3*Log[1 - E^(2*ArcCsch[c*x])
] + 4*a^3*Log[c*x] - 6*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch[c*x
])] + 6*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])] - 3*b^3*Pol
yLog[4, E^(2*ArcCsch[c*x])])/4
```

Maple [F]

$$\int \frac{(a+b\,\operatorname{arccsch}(cx))^3}{x}dx$$

```
[In] int((a+b*arccsch(c*x))^3/x,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x,x)
```

Fricas [F]

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^3}{x}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^3}{x}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))^3}{x}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^3}{x}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*log(x) - integrate((b^3*log(c
)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b
```

```
^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2*log(c))*x^2 + 3*(b^3*l
og(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)
- a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + s
qrt(c^2*x^2 + 1)*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (
2*b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^
2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b
*c^2)*x^2)*log(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log
(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2
+ 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*l
og(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) +
 a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2
 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x
^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2
*log(c))*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*lo
g(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a
*b^2*c^2*log(c) + a^2*b*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^3 + (c^
2*x^3 + x)*sqrt(c^2*x^2 + 1) + x), x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^3/x,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal result . . . 222
Rubi [A] (verified) . . . 222
Mathematica [A] (verified) . . . 224
Maple [F] . . . 224
Fricas [B] (verification not implemented) . . . 224
Sympy [F] . . . 225
Maxima [A] (verification not implemented) . . . 225
Giac [F] . . . 225
Mupad [F(-1)] . . . 226

Optimal result

Integrand size = 14, antiderivative size = 78

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx = 6b^3c\sqrt{1+\frac{1}{c^2x^2}} - \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x}$$

```
[Out] -6*b^2*(a+b*arccsch(c*x))/x-(a+b*arccsch(c*x))^3/x+6*b^3*c*(1+1/c^2/x^2)^(1
/2)+3*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6421, 3377, 2718}

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} + 6b^3c\sqrt{\frac{1}{c^2x^2}+1}$$

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^2,x]
```

```
[Out] 6*b^3*c*Sqrt[1 + 1/(c^2*x^2)] - (6*b^2*(a + b*ArcCsch[c*x]))/x + 3*b*c*Sqrt
[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2 - (a + b*ArcCsch[c*x])^3/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int (a+bx)^3\cosh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} + (3bc)\text{Subst}\left(\int (a+bx)^2\sinh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} \\
&\quad - \left(6b^2c\right)\text{Subst}\left(\int (a+bx)\cosh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x} + \left(6b^3c\right)\text{Subst}\left(\int \sinh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= 6b^3c\sqrt{1+\frac{1}{c^2x^2}} - \frac{6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} \\
&\quad + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx =$$
$$-\frac{a^3+6ab^2-3a^2bc\sqrt{1+\frac{1}{c^2x^2}}x-6b^3c\sqrt{1+\frac{1}{c^2x^2}}x+3b\left(a^2+2b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{csch}^{-1}(cx)+3b^2\left(a\right.}{x}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^2,x]
```

```
[Out] -((a^3 + 6*a*b^2 - 3*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 + 1/(
c^2*x^2)]*x + 3*b*(a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c
*x] + 3*b^2*(a - b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x]^2 + b^3*ArcCsch[
c*x]^3)/x)
```

Maple [F]

$$\int \frac{\left(a+b\,\text{arccsch}\,(cx)\right)^3}{x^2}dx$$

```
[In] int((a+b*arccsch(c*x))^3/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.85

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx =$$
$$-\frac{b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-3\left(a^2b+2\,b^3\right)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+a^3+6\,ab^2-3\left(b^3cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-ab^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+}{cx}\right.}{x}$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2*b
^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^2 - 3*(2*a*b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - a^2*b - 2*b^3)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\operatorname{acsch}(cx)\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**3/x**2,x)
[Out] Integral((a + b*acsch(c*x))**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.85

$$\begin{aligned}
&\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx \\
&= -\frac{b^3\operatorname{arcsch}(cx)^3}{x} + 3\left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsch}(cx)}{x}\right)a^2b \\
&\quad + 6\left(c\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx) - \frac{1}{x}\right)ab^2 \\
&\quad + 3\left(c\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx)^2 + 2c\sqrt{\frac{1}{c^2x^2}+1} - \frac{2\operatorname{arcsch}(cx)}{x}\right)b^3 \\
&\quad - \frac{3ab^2\operatorname{arcsch}(cx)^2}{x} - \frac{a^3}{x}
\end{aligned}$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="maxima")
[Out] -b^3*arccsch(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a^2*b
+ 6*(c*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2*x
^2) + 1)*arccsch(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) + 1) - 2*arccsch(c*x)/x)*b^3
 - 3*a*b^2*arccsch(c*x)^2/x - a^3/x
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^3/x^2,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^2, x)
```

3.30 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal result . . . 227
Rubi [A] (verified) . . . 227
Mathematica [A] (verified) . . . 229
Maple [F] . . . 230
Fricas [B] (verification not implemented) . . . 230
Sympy [F] . . . 230
Maxima [F] . . . 231
Giac [F] . . . 231
Mupad [F(-1)] . . . 232

Optimal result

Integrand size = 14, antiderivative size = 123

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx = \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\operatorname{csch}^{-1}(cx) - \frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{2x^2}$$

```
[Out] -3/8*b^3*c^2*arccsch(c*x)-3/4*b^2*(a+b*arccsch(c*x))/x^2-1/4*c^2*(a+b*arccs
ch(c*x))^3-1/2*(a+b*arccsch(c*x))^3/x^2+3/8*b^3*c*(1+1/c^2/x^2)^(1/2)/x+3/4
*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5554, 3392, 32, 2715, 8}

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{2x^2} + \frac{3b^3c\sqrt{\frac{1}{c^2x^2}+1}}{8x} - \frac{3}{8}b^3c^2\operatorname{csch}^{-1}(cx)$$

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^3,x]
```

```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsch[c*x])/8 - (3*b^2
*(a + b*ArcCsch[c*x]))/(4*x^2) + (3*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsc
h[c*x])^2)/(4*x) - (c^2*(a + b*ArcCsch[c*x])^3)/4 - (a + b*ArcCsch[c*x])^3/
(2*x^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2} + \frac{1}{2}(3bc^2)\,\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x} \\
&\quad - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2} - \frac{1}{4}(3bc^2)\,\text{Subst}\left(\int (a+bx)^2\,dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{4}(3b^3c^2)\,\text{Subst}\left(\int \sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\big(a \\
&\quad + b\text{csch}^{-1}(cx)\big)^3 - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2} - \frac{1}{8}(3b^3c^2)\,\text{Subst}\left(\int 1\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\text{csch}^{-1}(cx) - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} \\
&\quad + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\left(a+b\text{csch}^{-1}(cx)\right)^3 - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx = \\
-\frac{4a^3+6ab^2-6a^2bc\sqrt{1+\frac{1}{c^2x^2}}x-3b^3c\sqrt{1+\frac{1}{c^2x^2}}x+6b\left(2a^2+b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{csch}^{-1}(cx)+6b^2(}{8}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^3,x]
```

```
[Out] -1/8*(4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1
+ 1/(c^2*x^2)]*x + 6*b*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcC
sch[c*x] + 6*b^2*(-(b*c*Sqrt[1 + 1/(c^2*x^2)]*x) + a*(2 + c^2*x^2))*ArcCsch
[c*x]^2 + 2*b^3*(2 + c^2*x^2)*ArcCsch[c*x]^3 + 3*b*(2*a^2 + b^2)*c^2*x^2*Ar
cSinh[1/(c*x)])/x^2
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arccsch}\,(cx))^3}{x^3}dx$$

```
[In] int((a+b*arccsch(c*x))^3/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(107) = 214$.

Time $= 0.27$ (sec) , antiderivative size $= 267$, normalized size of antiderivative $= 2.17$

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{x^3}\,dx =$$

$$-\frac{2\left(b^3c^2x^2+2\,b^3\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-3\left(2\,a^2b+b^3\right)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+4\,a^3+6\,ab^2+6\left(ab^2c^2x^2-b^3cx\sqrt{\frac{c^2x^2}{c^2}}\right.}{}$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(b^3*c^2*x^2 + 2*b^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x))^3 - 3*(2*a^2*b + b^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 4*a^3 + 6*a
*b^2 + 6*(a*b^2*c^2*x^2 - b^3*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b^2)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(4*a*b^2*c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F]

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))^3}{x^3}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) -
 c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
 - 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^2 -
 1/2*a^3/x^2 - integrate(1/2*(2*b^3*log(c)^3 - 6*a*b^2*log(c)^2 + 2*(b^3*c^
2*x^2 + b^3)*log(x)^3 + 2*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 6
*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(2*b^
3*log(c) - 2*a*b^2 + 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 +
b^3)*log(x) + sqrt(c^2*x^2 + 1)*(2*b^3*log(c) - 2*a*b^2 + (b^3*c^2*(2*log(c
) - 1) - 2*a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2
 + 1) + 1)^2 + 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b
^2*c^2*log(c))*x^2)*log(x) - 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*lo
g(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*lo
g(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2
*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2
+ b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)
*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(b^3*log(c)^3 -
3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b
^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)
*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a
*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^5 + x^3 + (c^2*x^5
+ x^3)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^3/x^3,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 166

$$\begin{aligned}\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = &-\frac{14}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}}+\frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2}\\ &-\frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3}+\frac{4b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x}\\ &-\frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\\ &+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3}\end{aligned}$$

```
[Out] 2/27*b^3*c^3*(1+1/c^2/x^2)^(3/2)-2/9*b^2*(a+b*arccsch(c*x))/x^3+4/3*b^2*c^2
*(a+b*arccsch(c*x))/x-1/3*(a+b*arccsch(c*x))^3/x^3-14/9*b^3*c^3*(1+1/c^2/x^
2)^(1/2)-2/3*b*c^3*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)+1/3*b*c*(a+b*ar
ccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{6421, 5554, 3392, 3377, 2718, 2713\}$

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \frac{4b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x} - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^2} - \frac{2}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3} + \frac{2}{27}b^3c^3\left(\frac{1}{c^2x^2}+1\right)^{3/2} - \frac{14}{9}b^3c^3\sqrt{\frac{1}{c^2x^2}+1}$$

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^4,x]
```

```
[Out] (-14*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)])/9 + (2*b^3*c^3*(1 + 1/(c^2*x^2))^(3/2))
/27 - (2*b^2*(a + b*ArcCsch[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsch[c*x]
))/(3*x) - (2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/3 + (b*c*
Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(3*x^2) - (a + b*ArcCsch[c*x]
)^3/(3*x^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3 \text{Subst}\left(\int (a+bx)^3 \cosh(x) \sinh^2(x)\, dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right) \text{Subst}\left(\int (a+bx)^2 \sinh^3(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{2b^2\left(a+b\text{csch}^{-1}(cx)\right)}{9x^3} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^2} \\
&\quad - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3x^3} - \frac{1}{3}\left(2bc^3\right) \text{Subst}\left(\int (a+bx)^2 \sinh(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{9}\left(2b^3c^3\right) \text{Subst}\left(\int \sinh^3(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{2b^2\left(a+b\text{csch}^{-1}(cx)\right)}{9x^3} - \frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2 \\
&\quad + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3x^3} \\
&\quad + \frac{1}{3}\left(4b^2c^3\right) \text{Subst}\left(\int (a+bx)\cosh(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad - \frac{1}{9}\left(2b^3c^3\right) \text{Subst}\left(\int \left(1-x^2\right)\, dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} \\
&\quad - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x} \\
&\quad - \frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3} - \frac{1}{3}\left(4b^3c^3\right)\operatorname{Subst}\left(\int \sinh(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{14}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} \\
&\quad + \frac{4b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x} - \frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 \\
&\quad + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.20

$$
\begin{aligned}
&\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx \\
&= \frac{-9a^3 + 2b^3c\sqrt{1+\frac{1}{c^2x^2}}x(1-20c^2x^2) + 9a^2bc\sqrt{1+\frac{1}{c^2x^2}}x(1-2c^2x^2) + 6ab^2(-1+6c^2x^2) + 3b\Big(-9a^2+6ab
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^4,x]

[Out] (-9*a^3 + 2*b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 20*c^2*x^2) + 9*a^2*b*c*Sqrt
[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 6*a*b^2*(-1 + 6*c^2*x^2) + 3*b*(-9*a^
2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c^2*x^2
))*ArcCsch[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2)
)*ArcCsch[c*x]^2 - 9*b^3*ArcCsch[c*x]^3)/(27*x^3)
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsch}\,(cx))^3}{x^4}dx$$

```
[In] int((a+b*arccsch(c*x))^3/x^4,x)
[Out] int((a+b*arccsch(c*x))^3/x^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(144) = 288$.

Time $= 0.26$ (sec) , antiderivative size $= 301$, normalized size of antiderivative $= 1.81$

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{x^4}\,dx$$
$$= \frac{36\,ab^2c^2x^2 - 9\,b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 9\,a^3 - 6\,ab^2 - 9\left(3\,ab^2 + (2\,b^3c^3x^3 - b^3cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2}{c^2x}}\right.}{}$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="fricas")
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^3 - 9*a^3 - 6*a*b^2 - 9*(3*a*b^2 + (2*b^3*c^3*x^3 - b^3*c*x)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^
2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b - 2*b^3 - 6*(2*a*b^2*c^3*x^3 - a*b^2*c*x)*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 - (9*a^2*b + 2*b^3)*c*x)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))^3}{x^4}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**3/x**4,x)
[Out] Integral((a + b*acsch(c*x))**3/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c -
3*arccsch(c*x)/x^3) - 1/3*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^3 - 1/3*a^3/x^
3 - integrate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)
^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2
+ (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (3*b^3*log(c) - 3*a*b^2 + 3*
(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 3*(b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*
x^2 + 1)*(3*b^3*log(c) - 3*a*b^2 + (b^3*c^2*(3*log(c) - 1) - 3*a*b^2*c^2)*x
^2 + 3*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*l
og(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log
(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*lo
g(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2
*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^
2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^
3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 +
 1))*log(sqrt(c^2*x^2 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c
^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*
(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*l
og(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log
(x))*sqrt(c^2*x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))
, x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^4}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^3/x^4,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 204

$$\begin{aligned}\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx = {}& \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} + \frac{45}{256}b^3c^4\operatorname{csch}^{-1}(cx) \\ & - \frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^2} \\ & + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{16x^3} \\ & - \frac{9bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{32x} \\ & + \frac{3}{32}c^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{4x^4}\end{aligned}$$

```
[Out] 45/256*b^3*c^4*arccsch(c*x)-3/32*b^2*(a+b*arccsch(c*x))/x^4+9/32*b^2*c^2*(a
+b*arccsch(c*x))/x^2+3/32*c^4*(a+b*arccsch(c*x))^3-1/4*(a+b*arccsch(c*x))^3
/x^4+3/128*b^3*c*(1+1/c^2/x^2)^(1/2)/x^3-45/256*b^3*c^3*(1+1/c^2/x^2)^(1/2)
/x+3/16*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+b*ar
ccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5554, 3392, 32, 2715, 8}

$$\begin{aligned}\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx = {} & \frac{9b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^2} - \frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^4} \\ & + \frac{3}{32}c^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 + \frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{16x^3} \\ & - \frac{9bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{32x} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{4x^4} \\ & + \frac{45}{256}b^3c^4\operatorname{csch}^{-1}(cx) + \frac{3b^3c\sqrt{\frac{1}{c^2x^2}+1}}{128x^3} - \frac{45b^3c^3\sqrt{\frac{1}{c^2x^2}+1}}{256x}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(128*x^3) - (45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2
)])/(256*x) + (45*b^3*c^4*ArcCsch[c*x])/256 - (3*b^2*(a + b*ArcCsch[c*x]))/
(32*x^4) + (9*b^2*c^2*(a + b*ArcCsch[c*x]))/(32*x^2) + (3*b*c*Sqrt[1 + 1/(c
^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*
(a + b*ArcCsch[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsch[c*x])^3)/32 - (a + b
*ArcCsch[c*x])^3/(4*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^4 \text{Subst}\left(\int (a+bx)^3 \cosh(x) \sinh^3(x)\, dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{4}\left(3bc^4\right) \text{Subst}\left(\int (a+bx)^2 \sinh^4(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&= -\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{16x^3} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4} \\
&\quad - \frac{1}{16}\left(9bc^4\right) \text{Subst}\left(\int (a+bx)^2 \sinh^2(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{32}\left(3b^3c^4\right) \text{Subst}\left(\int \sinh^4(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^2} \\
&\quad + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{16x^3} - \frac{9bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{32x} \\
&\quad - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{32}\left(9bc^4\right) \text{Subst}\left(\int (a+bx)^2\, dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad - \frac{1}{128}\left(9b^3c^4\right) \text{Subst}\left(\int \sinh^2(x)\, dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad - \frac{1}{32}\left(9b^3c^4\right) \text{Subst}\left(\int \sinh^2(x)\, dx, x, \text{csch}^{-1}(cx)\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} - \frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^4} \\
&\quad + \frac{9b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{16x^3} \\
&\quad - \frac{9bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{32x} + \frac{3}{32}c^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{256}\left(9b^3c^4\right)\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{64}\left(9b^3c^4\right)\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} + \frac{45}{256}b^3c^4\operatorname{csch}^{-1}(cx) \\
&\quad - \frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{32x^2} \\
&\quad + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{16x^3} - \frac{9bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{32x} \\
&\quad + \frac{3}{32}c^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3 - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{4x^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.36

$$
\begin{aligned}
&\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx \\
&= \frac{-64a^3 - 24ab^2 + 48a^2bc\sqrt{1+\frac{1}{c^2x^2}}x + 6b^3c\sqrt{1+\frac{1}{c^2x^2}}x + 72ab^2c^2x^2 - 72a^2bc^3\sqrt{1+\frac{1}{c^2x^2}}x^3 - 45b^3c^3\sqrt{1+}}{}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 - 24*a*b^2 + 48*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 +
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3
 - 45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 24*b*(8*a^2 + b^2*(1 - 3*c^2*x^2)
 + 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3*c^2*x^2))*ArcCsch[c*x] + 24*b^2*
(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(2 - 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcCsch[
c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^3 + 9*b*(8*a^2 + 5*b^2)*c^4*x^
4*ArcSinh[1/(c*x)])/(256*x^4)
```

Maple [F]

$$\int \frac{(a+b\ \mathrm{arccsch}\,(cx))^3}{x^5}dx$$

```
[In] int((a+b*arccsch(c*x))^3/x^5,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{x^5}\,dx$$

$$= \frac{72\,ab^2c^2x^2+8\left(3\,b^3c^4x^4-8\,b^3\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-64\,a^3-24\,ab^2+24\left(3\,ab^2c^4x^4-8\,ab^2-(3\,b^3c^3x^3-\right.}{}$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x))^3 - 64*a^3 - 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8
*a*b^2 - (3*b^3*c^3*x^3 - 2*b^3*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(3*(8*a^2*b + 5*b^3)*c^4*
x^4 + 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 - 16*(3*a*b^2*c^3*x^3 - 2*a*b^2*c*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 - 2*(8*a^2*b + b^3)*c*x)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/x^4
```

Sympy [F]

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{x^5}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))^3}{x^5}\,dx$$

```
[In] integrate((a+b*acsch(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**5, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx = \int \frac{\left(b\,\operatorname{arcsch}\left(cx\right)+a\right)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(
1/(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt
(1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) +
1) + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^
4 - 1/4*a^3/x^4 - integrate(1/4*(4*b^3*log(c)^3 - 12*a*b^2*log(c)^2 + 4*(b^
3*c^2*x^2 + b^3)*log(x)^3 + 4*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2
 + 12*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*
(4*b^3*log(c) - 4*a*b^2 + 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 4*(b^3*c^2*x
^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(4*b^3*log(c) - 4*a*b^2 + (b^3*c^2*(4*
log(c) - 1) - 4*a*b^2*c^2)*x^2 + 4*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^
2*x^2 + 1) + 1)^2 + 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 -
 2*a*b^2*c^2*log(c))*x^2)*log(x) - 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3
*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*
(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c
)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c
^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^
2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 4*(b^3*log(
c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3
 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b
^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^
2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c
^2*x^7 + x^5)*sqrt(c^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx = \int \frac{\left(b\,\operatorname{arcsch}\left(cx\right)+a\right)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^5}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))^3/x^5,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\mathbf{csch}^{-1}(cx)}\,dx$

Optimal result . . . 247
Rubi [N/A] . . . 247
Mathematica [N/A] . . . 248
Maple [N/A] (verified) . . . 248
Fricas [N/A] . . . 248
Sympy [N/A] . . . 248
Maxima [N/A] . . . 249
Giac [N/A] . . . 249
Mupad [N/A] . . . 249

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\operatorname{csch}^{-1}(cx)}\,dx = \operatorname{Int}\left(\frac{x}{a+b\operatorname{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{a+b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{x}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

```
[In] Int[x/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][x/(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$$

```
[In] Integrate[x/(a + b*ArcCsch[c*x]),x]
[Out] Integrate[x/(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\,\text{arccsch}\,(cx)}dx$$

```
[In] int(x/(a+b*arccsch(c*x)),x)
[Out] int(x/(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{x}{b\,\text{arcsch}\,(cx)+a}\,dx$$

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arccsch(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{x}{a+b\,\text{acsch}\,(cx)}\,dx$$

```
[In] integrate(x/(a+b*acsch(c*x)),x)
[Out] Integral(x/(a + b*acsch(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{x}{b\operatorname{arcsch}(cx) + a}\,dx$$

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arccsch(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{x}{b\operatorname{arcsch}(cx) + a}\,dx$$

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(x/(b*arccsch(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{x}{a + b\operatorname{asinh}\left(\frac{1}{cx}\right)}\,dx$$

```
[In] int(x/(a + b*asinh(1/(c*x))),x)
[Out] int(x/(a + b*asinh(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\mathbf{csch}^{-1}(cx)}\,dx$

Optimal result . . . 250
Rubi [N/A] . . . 250
Mathematica [N/A] . . . 251
Maple [N/A] (verified) . . . 251
Fricas [N/A] . . . 251
Sympy [N/A] . . . 251
Maxima [N/A] . . . 252
Giac [N/A] . . . 252
Mupad [N/A] . . . 252

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx = \text{Int}\left(\frac{1}{a+b\text{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcCsch[c*x])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\,\text{arccsch}\,(cx)}dx$$

```
[In] int(1/(a+b*arccsch(c*x)),x)
[Out] int(1/(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{1}{b\,\text{arcsch}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arccsch(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{1}{a+b\,\text{acsch}\,(cx)}\,dx$$

```
[In] integrate(1/(a+b*acsch(c*x)),x)
[Out] Integral(1/(a + b*acsch(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{1}{b\operatorname{arcsch}(cx) + a}\,dx$$

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arccsch(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{1}{b\operatorname{arcsch}(cx) + a}\,dx$$

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arccsch(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{1}{a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int(1/(a + b*asinh(1/(c*x))),x)
[Out] int(1/(a + b*asinh(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}\,dx$

Optimal result . . . 253
Rubi [N/A] . . . 253
Mathematica [N/A] . . . 254
Maple [N/A] (verified) . . . 254
Fricas [N/A] . . . 254
Sympy [N/A] . . . 254
Maxima [N/A] . . . 255
Giac [N/A] . . . 255
Mupad [N/A] . . . 255

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCsch[c*x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCsch[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCsch[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcCsch[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\,\text{arccsch}\left(cx\right)\right)}dx$$

```
[In] int(1/x/(a+b*arccsch(c*x)),x)
[Out] int(1/x/(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\text{arcsch}\left(cx\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arccsch(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\,\text{acsch}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*acsch(c*x)),x)
[Out] Integral(1/(x*(a + b*acsch(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsch}(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsch(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsch}(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsch(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x\,\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x*(a + b*asinh(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx$

Optimal result . . . 256
Rubi [A] (verified) . . . 256
Mathematica [A] (verified) . . . 257
Maple [F] . . . 258
Fricas [F] . . . 258
Sympy [F] . . . 258
Maxima [F] . . . 258
Giac [F] . . . 259
Mupad [F(-1)] . . . 259

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = -\frac{c\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b} + \frac{c\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}$$

```
[Out] -c*Chi(a/b+arccsch(c*x))*cosh(a/b)/b+c*Shi(a/b+arccsch(c*x))*sinh(a/b)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6421, 3384, 3379, 3382}

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \frac{c\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b} - \frac{c\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{b}$$

```
[In] Int[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -((c*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/b) + (c*Sinh[a/b]*SinhInte
gral[a/b + ArcCsch[c*x]])/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\ &= -\left(\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\ &\quad + \left(c\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) \\ &= -\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\begin{aligned}&\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx \\ &= -\frac{c\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right) - \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)}{b}\end{aligned}$$

```
[In] Integrate[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -((c*(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcCsch[c*x]]))/b)
```

Maple [F]

$$\int \frac{1}{x^2 \left(a + b \operatorname{arccsch}\left(cx\right)\right)} dx$$

```
[In] int(1/x^2/(a+b*arccsch(c*x)),x)
[Out] int(1/x^2/(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a + b\mathrm{csch}^{-1}(cx)\right)} dx = \int \frac{1}{(b \operatorname{arcsch}\left(cx\right) + a)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*arccsch(c*x) + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a + b\mathrm{csch}^{-1}(cx)\right)} dx = \int \frac{1}{x^2 \left(a + b \operatorname{acsch}\left(cx\right)\right)} dx$$

```
[In] integrate(1/x**2/(a+b*acsch(c*x)),x)
[Out] Integral(1/(x**2*(a + b*acsch(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a + b\mathrm{csch}^{-1}(cx)\right)} dx = \int \frac{1}{(b \operatorname{arcsch}\left(cx\right) + a)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsch}(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}\,dx$$

```
[In] int(1/(x^2*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x^2*(a + b*asinh(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx$

Optimal result . . . 260
Rubi [A] (verified) . . . 260
Mathematica [A] (verified) . . . 262
Maple [F] . . . 262
Fricas [F] . . . 262
Sympy [F] . . . 263
Maxima [F] . . . 263
Giac [F] . . . 263
Mupad [F(-1)] . . . 263

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \frac{c^2\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b} - \frac{c^2\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cosh(2*a/b)*Shi(2*a/b+2*arccsch(c*x))/b+1/2*c^2*Chi(2*a/b+2*arccsc
h(c*x))*sinh(2*a/b)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5556, 12, 3384, 3379, 3382}

$$\int \frac{1}{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = \frac{c^2\sinh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)}{2b} - \frac{c^2\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\operatorname{csch}^{-1}(cx)\right)}{2b}$$

```
[In] Int[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*Cos
h[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^2 \text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\left(c^2 \text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\left(\frac{1}{2}c^2 \text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right)
\end{aligned}$$

$$= -\left(\frac{1}{2}\left(c^2 \cosh\left(\frac{2a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$
$$+ \frac{1}{2}\left(c^2 \sinh\left(\frac{2a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)$$
$$= \frac{c^2 \text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{2b} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$$
$$= \frac{c^2\left(\text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right) - \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\right)}{2b}$$

```
[In] Integrate[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^2*(CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*
SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]]))/(2*b)
```

Maple [F]

$$\int \frac{1}{x^3\left(a+b\,\text{arccsch}\left(cx\right)\right)}dx$$

```
[In] int(1/x^3/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x^3/(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\text{arcsch}\left(cx\right)+a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^3*arccsch(c*x) + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^3 \left(a + b\operatorname{acsch}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x**3/(a+b*acsch(c*x)),x)
[Out] Integral(1/(x**3*(a + b*acsch(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsch}\left(cx\right) + a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsch}\left(cx\right) + a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^3 \left(a + b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}\,dx$$

```
[In] int(1/(x^3*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x^3*(a + b*asinh(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx$

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Rubi [A] (verified) . . . 264
Mathematica [A] (verified) . . . 266
Maple [F] . . . 267
Fricas [F] . . . 267
Sympy [F] . . . 267
Maxima [F] . . . 267
Giac [F] . . . 268
Mupad [F(-1)] . . . 268

Optimal result

Integrand size = 14, antiderivative size = 117

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx = &\frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{4b}\\ &-\frac{c^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{csch}^{-1}(cx)\right)}{4b}\\ &-\frac{c^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{csch}^{-1}(cx)\right)}{4b}\\ &+\frac{c^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{csch}^{-1}(cx)\right)}{4b}\end{aligned}$$

```
[Out] 1/4*c^3*Chi(a/b+arccsch(c*x))*cosh(a/b)/b-1/4*c^3*Chi(3*a/b+3*arccsch(c*x))
*cosh(3*a/b)/b-1/4*c^3*Shi(a/b+arccsch(c*x))*sinh(a/b)/b+1/4*c^3*Shi(3*a/b+
3*arccsch(c*x))*sinh(3*a/b)/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used

$= \{6421, 5556, 3384, 3379, 3382\}$

$$\int \frac{1}{x^4\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b} - \frac{c^3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b} - \frac{c^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b} + \frac{c^3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b}$$

```
[In] Int[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^3*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]
*CoshIntegral[(3*a)/b + 3*ArcCsch[c*x]])/(4*b) - (c^3*Sinh[a/b]*SinhIntegra
l[a/b + ArcCsch[c*x]])/(4*b) + (c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcCsch[c*x]])/(4*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3 \text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= -\left(c^3 \text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \text{csch}^{-1}(cx)\right)\right) \\
&= \frac{1}{4}c^3 \text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) - \frac{1}{4}c^3 \text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{1}{4}\left(c^3 \cosh\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad - \frac{1}{4}\left(c^3 \cosh\left(\frac{3a}{b}\right)\right) \text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad - \frac{1}{4}\left(c^3 \sinh\left(\frac{a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{4}\left(c^3 \sinh\left(\frac{3a}{b}\right)\right) \text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right) \\
&= \frac{c^3 \cosh\left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{3a}{b}\right) \text{Chi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b} \\
&\quad - \frac{c^3 \sinh\left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b} + \frac{c^3 \sinh\left(\frac{3a}{b}\right) \text{Shi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = -\frac{c^3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right.}{4b}$$

```
[In] Integrate[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -1/4*(c^3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]]) + Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcCsch[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcCsch[c
*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCsch[c*x])]))/b
```

Maple [F]

$$\int \frac{1}{x^4 \left(a + b \operatorname{arccsch}\left(cx\right)\right)} dx$$

```
[In] int(1/x^4/(a+b*arccsch(c*x)),x)
[Out] int(1/x^4/(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a + b\operatorname{csch}^{-1}(cx)\right)} dx = \int \frac{1}{(b \operatorname{arccsch}\left(cx\right) + a)x^4} dx$$

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^4*arccsch(c*x) + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a + b\operatorname{csch}^{-1}(cx)\right)} dx = \int \frac{1}{x^4 \left(a + b \operatorname{acsch}\left(cx\right)\right)} dx$$

```
[In] integrate(1/x**4/(a+b*acsch(c*x)),x)
[Out] Integral(1/(x**4*(a + b*acsch(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a + b\operatorname{csch}^{-1}(cx)\right)} dx = \int \frac{1}{(b \operatorname{arccsch}\left(cx\right) + a)x^4} dx$$

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a + b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + b\operatorname{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^4 \left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)}\,dx$$

```
[In] int(1/(x^4*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x^4*(a + b*asinh(1/(c*x)))), x)
```

3.39 $\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx$

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Mathematica [N/A] 270
Maple [N/A] (verified) 270
Fricas [N/A] 270
Sympy [N/A] 270
Maxima [N/A] 271
Giac [N/A] 272
Mupad [N/A] 272

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx$$

Mathematica [N/A]

Not integrable

Time = 5.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a + b \operatorname{arccsch}(cx)\right)^3 dx$$

```
[In] int((d*x)^m*(a+b*arccsch(c*x))^3,x)
[Out] int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 22.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a + b \operatorname{acsch}(cx)\right)^3 \, dx$$

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**3,x)
[Out] Integral((d*x)**m*(a + b*acsch(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 7.73 (sec) , antiderivative size = 1351, normalized size of antiderivative = 84.44

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*d^m*x*x^m*log(sqrt(c^2*x^2 + 1) + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(
m + 1)) - integrate((3*((b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (a*b^
2*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^3*c^2)*x^2 + (b^3*c^2*d^m*
(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^3*d^m*(m
 + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d
^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x))*x^m)*
log(sqrt(c^2*x^2 + 1) + 1)^2 + (b^3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d^m*(m +
 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*
d^m*(m + 1))*log(x)^3 + (b^3*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m
 + 1)*log(c)^2 + 3*a^2*b*c^2*d^m*(m + 1)*log(c))*x^2 + 3*(b^3*d^m*(m + 1)*lo
g(c) - a*b^2*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m +
 1))*x^2)*log(x)^2 + 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(
c) + a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m
 + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*log(x))*sqrt(c^2*x^2 + 1)*x^m +
(b^3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m +
 1)*log(c) + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x)^3 + (b^3*c^
2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*d^m
*(m + 1)*log(c))*x^2 + 3*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3
*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*log(x)^2 + 3*(b^3*d^m
*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^2*b*d^m*(m + 1) + (b^3*c
^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m
 + 1))*x^2)*log(x))*x^m - 3*((b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1
)*log(c) + a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*
d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2
 + b^3*d^m*(m + 1))*log(x)^2 + 2*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1)
 + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*log(x))*sqrt(c
^2*x^2 + 1)*x^m + (b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) +
a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)
*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(
m + 1))*log(x)^2 + 2*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3*c^2
*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*log(x))*x^m)*log(sqrt(c^2
*x^2 + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sqrt(c^2*x^2 +
 1) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((d*x)^m*(a + b*asinh(1/(c*x)))^3,x)
[Out] int((d*x)^m*(a + b*asinh(1/(c*x)))^3, x)
```

3.40 $\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 dx$

Optimal result . . . 273
Rubi [N/A] . . . 273
Mathematica [N/A] . . . 274
Maple [N/A] (verified) . . . 274
Fricas [N/A] . . . 274
Sympy [N/A] . . . 274
Maxima [N/A] . . . 275
Giac [N/A] . . . 275
Mupad [N/A] . . . 276

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 dx = \text{Int}\left((dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 dx = \int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 dx$$

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 dx$$

Mathematica [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a + b \operatorname{arccsch}(cx)\right)^2 dx$$

```
[In] int((d*x)^m*(a+b*arccsch(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 10.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a + b \operatorname{acsch}(cx)\right)^2 \, dx$$

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*acsch(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 644, normalized size of antiderivative = 40.25

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d^m*x*x^m*log(sqrt(c^2*x^2 + 1) + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(
m + 1)) - integrate(-((b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c)
+ (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2 + (b^2*
c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x)^2 + 2*(b^2*d^m*(m + 1)*log(c)
 - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2
)*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m
+ 1)*log(c) + (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))
*x^2 + (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x)^2 + 2*(b^2*d^m*(m
 + 1)*log(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*
(m + 1))*x^2)*log(x))*x^m - 2*((b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) -
(a*b*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^2*c^2)*x^2 + (b^2*c^2*d
^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^2*d^m*
(m + 1)*log(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^
m*(m + 1))*x^2 + (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x))*x^m)*l
og(sqrt(c^2*x^2 + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sqr
t(c^2*x^2 + 1) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 5.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

```
[In] int((d*x)^m*(a + b*asinh(1/(c*x)))^2,x)
[Out] int((d*x)^m*(a + b*asinh(1/(c*x)))^2, x)
```

3.41 $\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)\, dx$

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Rubi [A] (verified) . . . 277
Mathematica [A] (verified) . . . 278
Maple [F] . . . 279
Fricas [F] . . . 279
Sympy [F] . . . 279
Maxima [F] . . . 279
Giac [F] . . . 280
Mupad [F(-1)] . . . 280

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \frac{(dx)^{1+m}\left(a + b\text{csch}^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m \,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, -\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arccsch(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[1
-1/2*m],-1/c^2/x^2)/c/m/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6419, 346, 371}

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \frac{(dx)^{m+1}\left(a + b\text{csch}^{-1}(cx)\right)}{d(m+1)} + \frac{b(dx)^m \,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, -\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometri
c2F1[1/2, -1/2*m, 1 - m/2, -(1/(c^2*x^2))])/(c*m*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
```

```
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(dx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d(1+m)} + \frac{(bd)\int \frac{(dx)^{-1+m}}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c(1+m)} \\ &= \frac{(dx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d(1+m)} - \frac{\left(b\left(\frac{1}{x}\right)^m(dx)^m\right)\operatorname{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c(1+m)} \\ &= \frac{(dx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, -\frac{1}{c^2x^2}\right)}{cm(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\begin{aligned}&\int (dx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\ &= \frac{x(dx)^m\left((1+m)\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2x^2\right)}{\sqrt{1+c^2x^2}}\right)}{(1+m)^2}\end{aligned}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(d*x)^m*((1 + m)*(a + b*ArcCsch[c*x]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/Sqrt[1 + c^2*x^2]))/
(1 + m)^2
```

Maple [F]

$$\int (dx)^m \left(a + b \operatorname{arccsch}(cx)\right) dx$$

```
[In] int((d*x)^m*(a+b*arccsch(c*x)),x)
[Out] int((d*x)^m*(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsch}(cx) + a)(dx)^m\, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int (dx)^m \left(a + b \operatorname{acsch}(cx)\right)\, dx$$

```
[In] integrate((d*x)**m*(a+b*acsch(c*x)),x)
[Out] Integral((d*x)**m*(a + b*acsch(c*x)), x)
```

Maxima [F]

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsch}(cx) + a)(dx)^m\, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] (c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sqr
t(c^2*x^2 + 1) + m + 1), x) - (d^m*x*x^m*log(x) - d^m*x*x^m*log(sqrt(c^2*x^
2 + 1) + 1))/(m + 1) - integrate((c^2*d^m*(m + 1)*x^2*log(c) + d^m*(m + 1)*
log(c) - d^m)*x^m/(c^2*(m + 1)*x^2 + m + 1), x))*b + (d*x)^(m + 1)*a/(d*(m
+ 1))
```

Giac [F]

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \int (b\,\text{arcsch}\,(cx) + a)(dx)^m\, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \int (d\,x)^m \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int((d*x)^m*(a + b*asinh(1/(c*x))),x)
[Out] int((d*x)^m*(a + b*asinh(1/(c*x))), x)
```

3.42 $\int \frac{(dx)^m}{a+b\mathbf{csch}^{-1}(cx)}\,dx$

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Mathematica [N/A] 282
Maple [N/A] (verified) 282
Fricas [N/A] 282
Sympy [N/A] 282
Maxima [N/A] 283
Giac [N/A] 283
Mupad [N/A] 283

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)}\,dx = \operatorname{Int}\left(\frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)}, x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a + b\text{csch}^{-1}(cx)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a + b\ \text{arccsch}\,(cx)}dx$$

```
[In] int((d*x)^m/(a+b*arccsch(c*x)),x)
[Out] int((d*x)^m/(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{arcsch}\,(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arccsch(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a + b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a + b\,\text{acsch}\,(cx)}\,dx$$

```
[In] integrate((d*x)**m/(a+b*acsch(c*x)),x)
[Out] Integral((d*x)**m/(a + b*acsch(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\operatorname{arcsch}(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\operatorname{arcsch}(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a + b\operatorname{csch}^{-1}(cx)}\,dx = \int \frac{(d\,x)^m}{a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int((d*x)^m/(a + b*asinh(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*asinh(1/(c*x))), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\mathbf{csch}^{-1}(cx)\right)^2}\,dx$

Optimal result . . . 284
Rubi [N/A] . . . 284
Mathematica [N/A] . . . 285
Maple [N/A] (verified) . . . 285
Fricas [N/A] . . . 285
Sympy [N/A] . . . 286
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Giac [N/A] . . . 287
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Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}, x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{\left(a+b\ \text{arccsch}\left(cx\right)\right)^2}dx$$

```
[In] int((d*x)^m/(a+b*arccsch(c*x))^2,x)
[Out] int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(b\,\text{arcsch}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\,\operatorname{acsch}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate((d*x)**m/(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*acsch(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 566, normalized size of antiderivative = 35.38

$$\int \frac{(dx)^m}{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(b\,\operatorname{arcsch}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*d^m*x^3 + d^m*x)*sqrt(c^2*x^2 + 1)*x^m + (c^2*d^m*x^3 + d^m*x)*x^m)/
((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^2)*lo
g(x) - (b^2*c^2*x^2 + sqrt(c^2*x^2 + 1)*b^2 + b^2)*log(sqrt(c^2*x^2 + 1) +
1) + sqrt(c^2*x^2 + 1)*(b^2*log(c) + b^2*log(x) - a*b)) - integrate(-((c^2*
d^m*(m + 3)*x^2 + d^m*(m + 1))*(c^2*x^2 + 1)*x^m + (c^4*d^m*(m + 2)*x^4 + c
^2*d^m*(3*m + 5)*x^2 + 2*d^m*(m + 1))*sqrt(c^2*x^2 + 1)*x^m + (c^4*d^m*(m +
 1)*x^4 + 2*c^2*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c) - a*b*
c^4)*x^4 + 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) + (c^2*x^2 + 1)*(b
^2*log(c) + b^2*log(x) - a*b) - a*b + (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*l
og(x) - (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + (c^2*x^2 + 1)*b^2 + b^2 + 2*(b^2*c^2
*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*sqrt(c^2*x^2
+ 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^
2)*log(x))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(b\,\text{arcsch}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(d\,x)^m}{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int((d*x)^m/(a + b*asinh(1/(c*x)))^2,x)
[Out] int((d*x)^m/(a + b*asinh(1/(c*x)))^2, x)
```

3.44 $\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\, dx$

Optimal result . . . 288
Rubi [A] (verified) . . . 288
Mathematica [A] (verified) . . . 292
Maple [A] (verified) . . . 292
Fricas [B] (verification not implemented) . . . 293
Sympy [F] . . . 293
Maxima [A] (verification not implemented) . . . 293
Giac [F] . . . 294
Mupad [F(-1)] . . . 295

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\text{csch}^{-1}(cx)}{4e} + \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e} + \frac{bd(2c^2d^2-e^2)\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c^3}$$

```
[Out] -1/4*b*d^4*arccsch(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsch(c*x))/e+1/2*b*d*(2*c^2
*d^2-e^2)*arctanh((1+1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2-e^2)*x*(1+1/c
^2/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1+1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(1+1
/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used

$= \{6425, 1582, 1489, 1821, 858, 221, 272, 65, 214\}$

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \frac{(d+ex)^4 \left(a+b\text{csch}^{-1}(cx)\right)}{4e} + \frac{bd\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)(2c^2d^2-e^2)}{2c^3} + \frac{bde^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c} + \frac{be^3x^3\sqrt{\frac{1}{c^2x^2}+1}}{12c} + \frac{bex\sqrt{\frac{1}{c^2x^2}+1}(9c^2d^2-e^2)}{6c^3} - \frac{bd^4\text{csch}^{-1}(cx)}{4e}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*(9*c^2*d^2 - e^2)*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 +
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsch[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsch[c*x]))/(4*e) + (b*d*(2*
c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(2*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} + \frac{b\int \frac{(d+ex)^4}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{4ce} \\
&= \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^4 x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{4ce} \\
&= \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{b\operatorname{Subst}\left(\int \frac{(e+dx)^4}{x^4\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce}
\end{aligned}$$

$$= \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} + \frac{b\operatorname{Subst}\left(\int \frac{-12de^3-2e^2\left(9d^2-\frac{e^2}{c^2}\right)x-12d^3ex^2-3d^4x^3}{x^3\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{12ce}$$

$$= \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{b\operatorname{Subst}\left(\int \frac{4e^2\left(9d^2-\frac{e^2}{c^2}\right)+12de\left(2d^2-\frac{e^2}{c^2}\right)x+6d^4x^2}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{24ce}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} + \frac{b\operatorname{Subst}\left(\int \frac{-12de\left(2d^2-\frac{e^2}{c^2}\right)-6d^4x}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{24ce}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{(bd^4)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4ce} - \frac{\left(bd\left(2d^2-\frac{e^2}{c^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\operatorname{csch}^{-1}(cx)}{4e} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{(bd(2c^2d^2-e^2))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{4c^3}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\operatorname{csch}^{-1}(cx)}{4e} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{(bd(2c^2d^2-e^2))\operatorname{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\operatorname{csch}^{-1}(cx)}{4e} + \frac{(d+ex)^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} + \frac{bd(2c^2d^2-e^2)\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int (d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+be\sqrt{1+\frac{1}{c^2x^2}}x(-2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2e}{12c^3}$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 + 1/(c^
2*x^2)]*x*(-2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 +
6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsch[c*x] + 6*b*d*(2*c^2*d^2 - e^2)*L
og[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(12*c^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.50

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\left(\frac{c\,e^3\,\text{arccsch}(cx)x^4}{4}+c\,e^2\,\text{arccsch}(cx)x^3d+\frac{3ce\,\text{arccsch}(cx)x^2d^2}{2}+\text{arccsch}(cx)xc\,d^3+\frac{c\,\text{arccsch}(cx)d^4}{4e}-\frac{\sqrt{c^2x^2+}\right.}{}$
derivativedivides	$\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\text{arccsch}(cx)c^4d^4}{4e}+\text{arccsch}(cx)c^4d^3x+\frac{3e\,\text{arccsch}(cx)c^4d^2x^2}{2}+e^2\,\text{arccsch}(cx)c^4d\,x^3+\frac{e^3\,\text{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+1}\right.}{}$
default	$\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\text{arccsch}(cx)c^4d^4}{4e}+\text{arccsch}(cx)c^4d^3x+\frac{3e\,\text{arccsch}(cx)c^4d^2x^2}{2}+e^2\,\text{arccsch}(cx)c^4d\,x^3+\frac{e^3\,\text{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+1}\right.}{}$

```
[In] int((e*x+d)^3*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arccsch(c*x)*x^4+c*e^2*arccsch(c*x)*x^3*d+
3/2*c*e*arccsch(c*x)*x^2*d^2+arccsch(c*x)*x*c*d^3+1/4*c/e*arccsch(c*x)*d^4-
1/12/c^4/e*(c^2*x^2+1)^(1/2)*(3*c^4*d^4*arctanh(1/(c^2*x^2+1)^(1/2))-12*c^3
*d^3*e*arcsinh(c*x)-e^4*c^2*x^2*(c^2*x^2+1)^(1/2)-6*c^2*d*e^3*x*(c^2*x^2+1)
^(1/2)-18*c^2*d^2*e^2*(c^2*x^2+1)^(1/2)+6*c*d*e^3*arcsinh(c*x)+2*e^4*(c^2*x
^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(147) = 294$.

Time $= 0.33$ (sec) , antiderivative size $= 419$, normalized size of antiderivative $= 2.51$

$$\int (d+ex)^3 \left(a + b\text{csch}^{-1}(cx)\right)\, dx$$

$$= \frac{3\,ac^3e^3x^4 + 12\,ac^3de^2x^3 + 18\,ac^3d^2ex^2 + 12\,ac^3d^3x + 3\left(4\,bc^3d^3 + 6\,bc^3d^2e + 4\,bc^3de^2 + bc^3e^3\right)\log\left(cx\sqrt{\frac{c^2}{c}}\right.}{}$$

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3*
d^3*x + 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*e^3)*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 6*(2*b*c^2*d^3 - b*d*e^2)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e +
4*b*c^3*d*e^2 + b*c^3*e^3)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1)
 + 3*(b*c^3*e^3*x^4 + 4*b*c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3*d^3*x
 - 4*b*c^3*d^3 - 6*b*c^3*d^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e^3*x^3 + 6*b*c^2*d*e^2*x^2 +
2*(9*b*c^2*d^2*e - b*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]

$$\int (d+ex)^3 \left(a + b\text{csch}^{-1}(cx)\right)\, dx = \int \left(a + b\,\text{acsch}\left(cx\right)\right)\left(d+ex\right)^3\, dx$$

```
[In] integrate((e*x+d)**3*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int (d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\
&= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2e \\
&\quad + \frac{1}{4}\left(4x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2 \\
&\quad + \frac{1}{12}\left(3x^4\operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be^3 + ad^3x \\
&\quad + \frac{\left(2cx\operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^3}{2c}
\end{aligned}$$

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arccsch(c*x) + x*s
qrt(1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*
x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^
2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arccsch(c*
x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e
^3 + a*d^3*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - l
og(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^3/c
```

Giac [F]

$$\int (d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int (ex+d)^3(b\operatorname{arcsch}(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^3\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^3,x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^3, x)
```

3.45 $\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx$

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Optimal result

Integrand size = 16, antiderivative size = 122

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2-e^2)\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] -1/3*b*d^3*arccsch(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsch(c*x))/e+1/6*b*(6*c^2*d
^2-e^2)*arctanh((1+1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1+1/c^2/x^2)^(1/2)/c+1/6*
b*e^2*x^2*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6425, 1582, 1489, 1821, 858, 221, 272, 65, 214}

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)(6c^2d^2-e^2)}{6c^3} + \frac{bdex\sqrt{\frac{1}{c^2x^2}+1}}{c} + \frac{be^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*d*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsch[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsch[c*x]))/(3*e) +
(b*(6*c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
```

```
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{b\int \frac{(d+ex)^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{3ce} \\
&= \frac{(d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^3 x}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{3ce} \\
&= \frac{(d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} - \frac{b\operatorname{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{3ce} \\
&= \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{b\operatorname{Subst}\left(\int \frac{-6de^2-e\left(6d^2-\frac{e^2}{c^2}\right)x-2d^3x^2}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6ce} \\
&= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{e\left(6d^2-\frac{e^2}{c^2}\right)+2d^3x}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6ce} \\
&= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad - \frac{(bd^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{3ce} - \frac{\left(b\left(6d^2-\frac{e^2}{c^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c}
\end{aligned}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e}$$
$$+ \frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{(b(6c^2d^2-e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{12c^3}$$
$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$
$$- \frac{(b(6c^2d^2-e^2))\,\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)}{6c}$$
$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e}$$
$$+ \frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2-e^2)\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{c^2x\left(be\sqrt{1+\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\,\text{csch}^{-1}(cx)+b(6c^2d^2}{6c^3}$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x*(b*e*Sqrt[1 + 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e^
2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsch[c*x] + b*(6*c^2*d^2
 - e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.52

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\left(\frac{c\,e^2\,\operatorname{arccsch}(cx)x^3}{3}+ce\,\operatorname{arccsch}(cx)x^2d+\operatorname{arccsch}(cx)xc\,d^2+\frac{c\,\operatorname{arccsch}(cx)d^3}{3e}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\,\operatorname{arctanh}\left(\frac{}{\sqrt{c}}\right.\right.\right.}{c}$
derivativedivides	$\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^3d^3}{3e}+\operatorname{arccsch}(cx)c^3d^2x+e\,\operatorname{arccsch}(cx)c^3d\,x^2+\frac{e^2\,\operatorname{arccsch}(cx)c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2}}\right.\right.\right.}{c^2}}{c}$
default	$\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^3d^3}{3e}+\operatorname{arccsch}(cx)c^3d^2x+e\,\operatorname{arccsch}(cx)c^3d\,x^2+\frac{e^2\,\operatorname{arccsch}(cx)c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2}}\right.\right.\right.}{c^2}}{c}$

```
[In] int((e*x+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arccsch(c*x)*x^3+c*e*arccsch(c*x)*x^2*d+ar
ccsch(c*x)*x*c*d^2+1/3*c/e*arccsch(c*x)*d^3+1/6/c^3/e*(c^2*x^2+1)^(1/2)*(-2
*c^3*d^3*arctanh(1/(c^2*x^2+1)^(1/2))+6*c^2*d^2*e*arcsinh(c*x)+6*c*d*e^2*(c
^2*x^2+1)^(1/2)+e^3*c*x*(c^2*x^2+1)^(1/2)-e^3*arcsinh(c*x))/((c^2*x^2+1)/c^
2/x^2)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(108) = 216$.

Time $= 0.31$ (sec) , antiderivative size $= 328$, normalized size of antiderivative $= 2.69$

$$\int (d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x+2\left(3\,bc^3d^2+3\,bc^3de+bc^3e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-\left(6\,bc^2d^2-be\right.}{}$$

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(3*b*c^3*d^2 + 3
*b*c^3*d*e + b*c^3*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) -
(6*b*c^2*d^2 - b*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(3*b
*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) -
 c*x - 1) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^
2 - 3*b*c^3*d*e - b*c^3*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) + (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]

$$\int (d+ex)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)\, dx = \int \left(a+b\,\operatorname{acsch}(cx)\right)(d+ex)^2\, dx$$

```
[In] integrate((e*x+d)**2*(a+b*acsch(c*x)),x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.57

$$\begin{aligned}
&\int (d+ex)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)\, dx \\
&= \frac{1}{3}\,ae^2x^3 + adex^2 + \left(x^2\operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bde \\
&\quad + \frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 \\
&\quad + ad^2x + \frac{\left(2\,cx\operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^2}{2\,c}
\end{aligned}$$

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*
b*d*e + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^
2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2)
+ 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c
^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^2/c
```

Giac [F]

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (ex+d)^2 (b\,\text{arcsch}\,(cx)+a)\, dx$$

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, (d+e\,x)^2\, dx$$

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^2,x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^2, x)
```

3.46 $\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 81

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{bd^2\text{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{bd\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] -1/2*b*d^2*arccsch(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsch(c*x))/e+b*d*arctanh((1
+1/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6425, 1582, 1410, 1821, 858, 221, 272, 65, 214}

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{bd\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c} + \frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c} - \frac{bd^2\text{csch}^{-1}(cx)}{2e}$$

```
[In] Int[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsch[c*x])/(2*e) + ((d + e*
x)^2*(a + b*ArcCsch[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1410

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} + \frac{b\int \frac{(d+ex)^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{2ce} \\
&= \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{2ce} \\
&= \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^2}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} + \frac{b\text{Subst}\left(\int \frac{-2de-d^2x}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - \frac{(bd)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(bd^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{bd^2\operatorname{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - \frac{(bd)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{bd^2\operatorname{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - (bcd)\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)
\end{aligned}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{bd^2\text{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{bd\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\begin{aligned}\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = {} & adx + \frac{1}{2}aex^2 + \frac{bex\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c} \\ & + bdx\text{csch}^{-1}(cx) + \frac{1}{2}bex^2\text{csch}^{-1}(cx) \\ & + \frac{2bd\sqrt{1+\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*A
rcCsch[c*x] + (b*e*x^2*ArcCsch[c*x])/2 + (2*b*d*Sqrt[1 + 1/(c^2*x^2)]*x*Arc
Tanh[(-1 + Sqrt[1 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\,\text{arccsch}(cx)e\,x^2}{2}+\text{arccsch}(cx)dxc+\frac{\sqrt{c^2x^2+1}\left(2dc\,\text{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2c^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x}\right)}{c}$	98
derivativedivides	$\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\text{arccsch}(cx)d\,c^2x+\frac{\text{arccsch}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2+1}\left(2dc\,\text{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c}}{c}$	115
default	$\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\text{arccsch}(cx)d\,c^2x+\frac{\text{arccsch}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2+1}\left(2dc\,\text{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c}}{c}$	115

```
[In] int((e*x+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arccsch(c*x)*e*x^2+arccsch(c*x)*d*x*c+1/2/c^2/
((c^2*x^2+1)/c^2/x^2)^(1/2)/x*(c^2*x^2+1)^(1/2)*(2*d*c*arcsinh(c*x)+e*(c^2*
x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(71) = 142$.

Time $= 0.28$ (sec) , antiderivative size $= 207$, normalized size of antiderivative $= 2.56$

$$\int (d+ex)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{acex^2 + 2\,acdx + bex\sqrt{\frac{c^2x^2+1}{c^2x^2}} - 2\,bd\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (2\,bcd + bce)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) -}{2\,c}$$

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*e*x^2 + 2*a*c*d*x + b*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 2*b*d*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + (2*b*c*d + b*c*e)*log(c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (2*b*c*d + b*c*e)*log(c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*e*x^2 + 2*b*c*d*x - 2*b*c*d - b*c*e)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

Sympy [F]

$$\int (d+ex)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int (a+b\operatorname{acsch}(cx))\,(d+ex)\,dx$$

```
[In] integrate((e*x+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.20$ (sec) , antiderivative size $= 87$, normalized size of antiderivative $= 1.07$

$$\int (d+ex)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{1}{2}\,aex^2 + \frac{1}{2}\left(x^2\operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)be + adx$$

$$+ \frac{\left(2\,cx\operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd}{2\,c}$$

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b*e + a*d*
x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(
c^2*x^2) + 1) - 1))*b*d/c
```

Giac [F]

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (ex+d)(b\,\text{arcsch}\,(cx)+a)\,dx$$

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x),x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x), x)
```

3.47 $\int \left(a + b\text{csch}^{-1}(cx)\right)\, dx$

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Mupad [F(-1)] . . . 312

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\, dx = ax + bx\text{csch}^{-1}(cx) + \frac{b\text{arctanh}\left(\sqrt{1 + \frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsch(c*x)+b*arctanh((1+1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6413, 272, 65, 214}

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\, dx = ax + \frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2} + 1}\right)}{c} + bx\text{csch}^{-1}(cx)$$

```
[In] Int[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6413

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \operatorname{csch}^{-1}(cx)\,dx \\
&= ax + bx\operatorname{csch}^{-1}(cx) + \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\,dx}{c} \\
&= ax + bx\operatorname{csch}^{-1}(cx) - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx\operatorname{csch}^{-1}(cx) - (bc)\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right) \\
&= ax + bx\operatorname{csch}^{-1}(cx) + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(30) = 60$.

Time $= 0.05$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 2.13$

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = ax + bx\operatorname{csch}^{-1}(cx) + \frac{2b\sqrt{1+\frac{1}{c^2x^2}}x\operatorname{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

```
[In] Integrate[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (2*b*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(-1 + Sqrt[1
+ c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

method	result	size
default	$ax + bx \operatorname{arccsch}(cx) + \frac{b \ln\left(cx + cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
parts	$ax + bx \operatorname{arccsch}(cx) + \frac{b \ln\left(cx + cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
derivativedivides	$\frac{acx + b\left(cx \operatorname{arccsch}(cx) + \ln\left(cx + cx\sqrt{1+\frac{1}{c^2x^2}}\right)\right)}{c}$	39

```
[In] int(a+b*arccsch(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.77

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx$$

$$= \frac{acx + bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (bcx -}{c}$$

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)))/c
```

Sympy [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int \left(a + b\operatorname{acsch}(cx)\right)\, dx$$

```
[In] integrate(a+b*acsch(c*x),x)
```

```
[Out] Integral(a + b*acsch(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx$$
$$= ax + \frac{\left(2\,cx\,\operatorname{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b/c
```

Giac [F]

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int b\,\operatorname{arcsch}\left(cx\right) + a\, dx$$

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arccsch(c*x) + a, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\operatorname{csch}^{-1}(cx)\right)\, dx = \int a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\, dx$$

```
[In] int(a + b*asinh(1/(c*x)),x)
```

```
[Out] int(a + b*asinh(1/(c*x)), x)
```

3.48 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex}\,dx$

Optimal result . . . 313
Rubi [A] (verified) . . . 314
Mathematica [C] (verified) . . . 315
Maple [F] . . . 316
Fricas [F] . . . 316
Sympy [F] . . . 316
Maxima [F] . . . 317
Giac [F] . . . 317
Mupad [F(-1)] . . . 317

Optimal result

Integrand size = 16, antiderivative size = 215

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2,\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)}{2e}$$

```
[Out] -(a+b*arccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsch(c*x)
)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))*(e-(c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*arcc
sch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))*(e+(c^2*d^2+e^2)^(1/2))/c/d)/e-1
/2*b*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e+b*polylog(2,(1/c/x+(1+1/c^2
/x^2)^(1/2))*(e-(c^2*d^2+e^2)^(1/2))/c/d)/e+b*polylog(2,(1/c/x+(1+1/c^2/x^2
)^(1/2))*(e+(c^2*d^2+e^2)^(1/2))/c/d)/e
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6424, 2598}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{b\,\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c
*d)])/e + ((a + b*ArcCsch[c*x])*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsc
h[c*x])/(c*d)])/e - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e +
(b*PolyLog[2, ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e + (b*Pol
yLog[2, ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e - (b*PolyLog[2
, E^(2*ArcCsch[c*x])])/(2*e)
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 6424

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[(a + b*ArcCsch[c*x])*(Log[1 - (e - Sqrt[c^2*d^2 + e^2])*(E^ArcCsch[c*x
]/(c*d))]/e), x] + (Dist[b/(c*e), Int[Log[1 - (e - Sqrt[c^2*d^2 + e^2])*(E^
ArcCsch[c*x]/(c*d))]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Dist[b/(c*e), In
t[Log[1 - (e + Sqrt[c^2*d^2 + e^2])*(E^ArcCsch[c*x]/(c*d))]/(x^2*Sqrt[1 + 1
/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2*ArcCsch[c*x])]/(x^2*S
```

```
qrt[1 + 1/(c^2*x^2)]), x], x] + Simp[(a + b*ArcCsch[c*x])*(Log[1 - (e + Sqr
t[c^2*d^2 + e^2])*(E^ArcCsch[c*x]/(c*d))]/e), x] - Simp[(a + b*ArcCsch[c*x]
)*(Log[1 - E^(2*ArcCsch[c*x])]/e), x]) /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{e} + \frac{b\int \frac{\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{ce} \\
&\quad + \frac{b\int \frac{\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{ce} - \frac{b\int \frac{\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{ce} \\
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} \\
&\quad + \frac{b\operatorname{PolyLog}\left(2,\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)}{2e}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.35

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex}\,dx &= \frac{a\log(d+ex)}{e} \\
&\quad + \frac{b\left(\pi^2 - 4i\pi\operatorname{csch}^{-1}(cx) - 8\operatorname{csch}^{-1}(cx)^2 - 32\arcsin\left(\frac{\sqrt{1+\frac{ie}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(icd+e)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right)}{\sqrt{c^2d^2+e^2}}\right) - 8c\right.}{}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 -
32*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*ArcTan[((I*c*d + e)*Cot[(Pi + (2*I
)*ArcCsch[c*x])/4])/Sqrt[c^2*d^2 + e^2]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*Arc
Csch[c*x])] + (4*I)*Pi*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/
(c*d)] + 8*ArcCsch[c*x]*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])
/(c*d)] + (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*Log[1 + ((-e + Sqrt[
c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + (4*I)*Pi*Log[1 - ((e + Sqrt[c^2*d^
2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*ArcCsch[c*x]*Log[1 - ((e + Sqrt[c^2*d^
2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt
[2]]*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (4*I)*Pi*L
og[e + d/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((e - Sqrt[c
^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*PolyLog[2, ((e + Sqrt[c^2*d^2 + e
^2])*E^ArcCsch[c*x])/(c*d)]))/(8*e)
```

Maple [F]

$$\int \frac{a + b\ \mathrm{arccsch}\left(cx\right)}{ex + d}dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x+d),x)
```

Fricas [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{d + ex}\,dx = \int \frac{b\,\mathrm{arcsch}\left(cx\right) + a}{ex + d}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{d + ex}\,dx = \int \frac{a + b\,\mathrm{acsch}\left(cx\right)}{d + ex}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x + d), x) + a*log(e*x
+ d)/e
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{d+e\,x}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x), x)
```

3.49 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal result . . . 318
Rubi [A] (verified) . . . 318
Mathematica [A] (verified) . . . 320
Maple [A] (verified) . . . 320
Fricas [B] (verification not implemented) . . . 321
Sympy [F] . . . 322
Maxima [F] . . . 322
Giac [F] . . . 322
Mupad [F(-1)] . . . 323

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = \frac{b\operatorname{csch}^{-1}(cx)}{de} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2+e^2}}$$

```
[Out] b*arccsch(c*x)/d/e+(-a-b*arccsch(c*x))/e/(e*x+d)+b*arctanh((c^2*d-e/x)/c/(c
^2*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d/(c^2*d^2+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6425, 1582, 1489, 858, 221, 739, 212}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{d\sqrt{c^2d^2+e^2}} + \frac{b\operatorname{csch}^{-1}(cx)}{de}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*ArcCsch[c*x])/(d*e) - (a + b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c
^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2
 + e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a + b\text{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)}\,dx}{ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}\,dx}{ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cde}$$

$$= \frac{b\text{csch}^{-1}(cx)}{de} - \frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d-\frac{e}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{cd}$$

$$= \frac{b\text{csch}^{-1}(cx)}{de} - \frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\text{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2+e^2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a}{e(d+ex)} - \frac{b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\text{arcsinh}\left(\frac{1}{cx}\right)}{de} + \frac{b\log(d+ex)}{d\sqrt{c^2d^2+e^2}} - \frac{b\log\left(e+c\left(-cd+\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2+e^2}}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcSinh[1/(c*x)])/
(d*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 + e^2]) - (b*Log[e + c*(-(c*d) + S
qrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 + e^2])
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\operatorname{arccsch}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{\frac{c^2d^2+e^2}{e^2}}-\ln\left(\frac{2\sqrt{c^2x^2+1}\sqrt{\frac{c^2d^2+e^2}{e^2}}\,e-2d\,c^2x+2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+bc^2\left(-\frac{\operatorname{arccsch}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{\frac{c^2d^2+e^2}{e^2}}-\ln\left(\frac{2\sqrt{c^2x^2+1}\sqrt{\frac{c^2d^2+e^2}{e^2}}\,e-2d\,c^2x+2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+bc^2\left(-\frac{\operatorname{arccsch}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{\frac{c^2d^2+e^2}{e^2}}-\ln\left(\frac{2\sqrt{c^2x^2+1}\sqrt{\frac{c^2d^2+e^2}{e^2}}\,e-2d\,c^2x+2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2+e^2}{e^2}}}\right)}{c}$

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arccsch(c*x)+1/e*(c^2*x^2+1)^(1/2)*(ar
ctanh(1/(c^2*x^2+1)^(1/2))*((c^2*d^2+e^2)/e^2)^(1/2)-ln(2*((c^2*x^2+1)^(1/2
)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d)))/((c^2*x^2+1)/c^2/x^2
)^(1/2)/x/d/((c^2*d^2+e^2)/e^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(92) = 184$.

Time = 0.29 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.61

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{ac^2d^3+ade^2-\sqrt{c^2d^2+e^2}(be^2x+bde)\log\left(-\frac{c^3d^2x-cde+(c^3d^2+ce^2)x\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\left(c^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+c^2dx-e\right)\sqrt{c^2d^2+e^2}}{ex+d}\right.}{}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*e^2 - sqrt(c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*log(-(c^3*d^2
*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + (c^2*d*x*s
qrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 + e^2))/(e*x + d))
 - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*log(c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*
log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^2*d^3 + b*d*e^2)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e + d^2*e^3 + (c
^2*d^3*e^2 + d*e^4)*x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a + b\,\operatorname{acsch}(cx)}{(d+ex)^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**2,x)
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b\,\operatorname{arcsch}(cx) + a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="maxima")
[Out] -1/2*(2*c^2*integrate(x/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e + (c^2*e^2
*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt(c^2*x^2 + 1)), x) + I*c*(log(I*c*x +
 1) - log(-I*c*x + 1))/(c^2*d^2 + e^2) - 2*e*log(e*x + d)/(c^2*d^3 + d*e^2)
 - (2*c^2*d^3*log(c) + 2*d*e^2*log(c) - 2*(c^2*d^2*e + e^3)*x*log(x) + (c^2
*d^2*e*x + c^2*d^3)*log(c^2*x^2 + 1) - 2*(c^2*d^3 + d*e^2)*log(sqrt(c^2*x^2
 + 1) + 1))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d*e^4)*x))*b - a/(e^2*x +
 d*e)
```

Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b\,\operatorname{arcsch}(cx) + a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^2,x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^2, x)
```

3.50 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal result . . . 324
Rubi [A] (verified) . . . 324
Mathematica [A] (verified) . . . 327
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Optimal result

Integrand size = 16, antiderivative size = 163

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\left(2c^2d^2+e^2\right)\operatorname{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] 1/2*b*arccsch(c*x)/d^2/e+1/2*(-a-b*arccsch(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d
^2+e^2)*arctanh((c^2*d-e/x)/c/(c^2*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^2/
(c^2*d^2+e^2)^(3/2)-1/2*b*c*e*(1+1/c^2/x^2)^(1/2)/d/(c^2*d^2+e^2)/(e+d/x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6425, 1582, 1489, 1665, 858, 221, 739, 212}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\left(2c^2d^2+e^2\right)\operatorname{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}} - \frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{2d\left(c^2d^2+e^2\right)\left(\frac{d}{x}+e\right)} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(d*(c^2*d^2 + e^2)*(e + d/x)) + (b*ArcCs
ch[c*x])/(2*d^2*e) - (a + b*ArcCsch[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2
 + e^2)*ArcTanh[(c^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])
])/(2*d^2*(c^2*d^2 + e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^2}\,dx}{2ce} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2x^4}\,dx}{2ce} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\text{Subst}\left(\int \frac{x^2}{(e+dx)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce} \\
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{(bc)\text{Subst}\left(\int \frac{e-\left(d+\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2e\left(c^2d^2+e^2\right)} \\
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2e} \\
&\quad - \frac{\left(bc\left(2+\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2\left(c^2d^2+e^2\right)} \\
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} \\
&\quad + \frac{\left(bc\left(2+\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int \frac{1}{d^2+\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d-\frac{e}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2\left(c^2d^2+e^2\right)}
\end{aligned}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\text{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b(2c^2d^2+e^2)\arctanh\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.25

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bce\sqrt{1+\frac{1}{c^2x^2}}x}{d\left(c^2d^2+e^2\right)(d+ex)} - \frac{b\text{csch}^{-1}(cx)}{e(d+ex)^2} + \frac{b\text{arcsinh}\left(\frac{1}{cx}\right)}{d^2e} + \frac{b(2c^2d^2+e^2)\log(d+ex)}{d^2\left(c^2d^2+e^2\right)^{3/2}} - \frac{b(2c^2d^2+e^2)\log\left(e+c\left(-cd+\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{d^2\left(c^2d^2+e^2\right)^{3/2}}\right)$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d^2 + e^2)*
(d + e*x)) - (b*ArcCsch[c*x])/(e*(d + e*x)^2) + (b*ArcSinh[1/(c*x)])/(d^2*e
) + (b*(2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c^2*d^2 + e^2)^(3/2)) - (b*(2*
c^2*d^2 + e^2)*Log[e + c*(-(c*d) + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)
])*x])/(d^2*(c^2*d^2 + e^2)^(3/2)))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(150) = 300.

Time = 2.62 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.34

method	result
parts	$-\frac{a}{2(ex+d)^2e}+b\left(-\frac{c^3\,\mathrm{arccsch}(cx)}{2(cex+cd)^2e}-\frac{\sqrt{c^2x^2+1}\left(-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^3-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^2\right.\right.$
derivativedivides	$-\frac{a\,c^3}{2(cex+cd)^2e}+b\,c^3\left(-\frac{\mathrm{arccsch}(cx)}{2(cex+cd)^2e}-\frac{\sqrt{c^2x^2+1}\left(-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^3-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^2\right.\right.$
default	$-\frac{a\,c^3}{2(cex+cd)^2e}+b\,c^3\left(-\frac{\mathrm{arccsch}(cx)}{2(cex+cd)^2e}-\frac{\sqrt{c^2x^2+1}\left(-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^3-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^2\right.\right.$

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arccsch(c*x)-1/2/e*(c^2*x^
2+1)^(1/2)*(-((c^2*d^2+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*c^3*d^3
-((c^2*d^2+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*c^3*d^2*e*x+2*ln(2*
((c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))*c^3*
d^3+2*ln(2*((c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x
+c*d))*c^3*d^2*e*x+(c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*c*d*e^2-((c^
2*d^2+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*c*d*e^2-((c^2*d^2+e^2)/e
^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*e^3*c*x+ln(2*((c^2*x^2+1)^(1/2)*((c^
2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))*c*d*e^2+ln(2*((c^2*x^2+1)^(
1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))*e^3*c*x)/((c^2*x^2
+1)/c^2/x^2)^(1/2)/x/d^2/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(147) = 294$.

Time $= 0.45$ (sec) , antiderivative size $= 745$, normalized size of antiderivative $= 4.57$

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx =$$

$$-\frac{ac^4d^6 + bc^3d^5e + 2\,ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 + (bc^3d^3e^3 + bcde^5)x^2 - (2\,bc^2d^4e + bd^2e^3 + (2\,bc^2d^2e^3 + b$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^4*d^6 + b*c^3*d^5*e + 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4 +
 (b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e + b*d^2*e^3 + (2*b*c^2*d^
2*e^3 + b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 + b*d*e^4)*x)*sqrt(c^2*d^2 + e^2)*l
og(-(c^3*d^2*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + (c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 + e^2)
)/(e*x + d)) + 2*(b*c^3*d^4*e^2 + b*c*d^2*e^4)*x - (b*c^4*d^6 + 2*b*c^2*d^4
*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4
*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) - c*x + 1) + (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 +
2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*
x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^4*d^6 + 2*b*c^2*
d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (
(b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 + b*c*d^2*e^4)*x)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3
+ 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*
x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a + b\,\operatorname{acsch}(cx)}{(d+ex)^3}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*I*c^3*d*(log(I*c*x + 1) - log(-I*c*x + 1))/(c^4*d^4 + 2*c^2*d^2*e^2
 + e^4) + 4*c^2*integrate(1/2*x/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x
+ d^2*e + (c^2*d^2*e + e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*
x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(3*c^2*d^2*e
+ e^3)*log(e*x + d)/(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4) - (2*c^4*d^6*log(c)
 + 2*d^2*e^4*log(c) - 2*d^2*e^4 + 2*(2*d^4*e^2*log(c) - d^4*e^2)*c^2 - 2*(c
^2*d^3*e^3 + d*e^5)*x + (c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4
)*x^2 + 2*(c^4*d^5*e - c^2*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*((c^4*d^4*e^2 +
 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*x)*log(x)
 - 2*(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d
^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^
2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x))*b - 1/2*a/(e^3*x^2 + 2*d*
e^2*x + d^2*e)
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{(d+e\,x)^3}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^3, x)
```

3.51 $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 918

$$\begin{aligned}
&\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= -\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad - \frac{32bcd^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{4bc\left(c^2d^2-3e^2\right)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{32bcd^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4bcd\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{32bd^4\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

```
[Out] 2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arccs
ch(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsch(c*x))/e^3+4/35*b*(c^2*x^2+1)*(e
```

```
*x+d)^(1/2)/c^3/(1+1/c^2/x^2)^(1/2)+8/105*b*d*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3
/e/x/(1+1/c^2/x^2)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/
2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+
d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+
d)^(1/2)-4/35*b*c*d^2*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*
(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)
/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2))
)^(1/2)+4/105*b*c*(2*c^2*d^2+9*e^2)*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*
2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^
2*x^2+1)^(1/2)/(-c^2)^(5/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2)+32/105*b*c*d^3*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2
)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)-4/105*b*c*d*(c^2*d^2+e^2)*EllipticF(1/2*(1-x*(-c^2)^
(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2
*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^2/x
/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules

used = {45, 6445, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551, 847}

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$= -\frac{32b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{32bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d^3}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)d^2}{3e^3}$$

$$-\frac{32bc\sqrt{d+ex}\sqrt{c^2x^2+1}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d^2}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

$$-\frac{4b\sqrt{d+ex}(c^2x^2+1)\,d}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{4(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)d}{5e^3}$$

$$-\frac{4bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d}{105\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{4b(d+ex)^{3/2}\left(c^2x^2+1\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}$$

$$-\frac{4bc(c^2d^2-3e^2)\sqrt{d+ex}\sqrt{c^2x^2+1}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (-4*b*d*Sqrt[d + e*x]*(1 + c^2*x^2))/(105*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) +
(4*b*(d + e*x)^(3/2)*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(
a + b*ArcCsch[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*
e^3) - (32*b*c*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(
-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[
-c^2]*e)]) - (4*b*c*(c^2*d^2 - 3*e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqr
t[-c^2]*e)])/(35*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*
x))/(c^2*d - Sqrt[-c^2]*e)]) + (32*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sq
rt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt
[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(3/2)*e^2*Sqrt
```

```
[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-
c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(Sqr
t[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(105*c*e^3*S
qrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
```

```
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} + \frac{b\int \frac{2(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{105e^3\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} + \frac{(2b)\int \frac{(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{105ce^3} \\
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{x\sqrt{1+c^2x^2}}\,dx}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(-\frac{12de(d+ex)^{3/2}}{\sqrt{1+c^2x^2}} + \frac{8d^2(d+ex)^{3/2}}{x\sqrt{1+c^2x^2}} + \frac{15e^2x(d+ex)^{3/2}}{\sqrt{1+c^2x^2}}\right)dx}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int\frac{(d+ex)^{3/2}}{x\sqrt{1+c^2x^2}}dx}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int\frac{(d+ex)^{3/2}}{\sqrt{1+c^2x^2}}dx}{35ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x(d+ex)^{3/2}}{\sqrt{1+c^2x^2}}dx}{7ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{16bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int\left(\frac{2de}{\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)dx}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(16bd\sqrt{1+c^2x^2}\right)\int\frac{\frac{1}{2}\left(3c^2d^2-e^2\right)+2c^2dex}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\int\frac{\left(-\frac{3e}{2}+\frac{3}{2}c^2dx\right)\sqrt{d+ex}}{\sqrt{1+c^2x^2}}dx}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} + \frac{\left(16bd^4\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(32bd^2\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{105ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(8b\sqrt{1+c^2x^2}\right)\int \frac{-3c^2de+\frac{3}{4}c^2\left(c^2d^2-3e^2\right)x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105c^5e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(8bd(-c^2d^2-e^2)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad + \frac{\left(16bd^4\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{105ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(16bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2b(c^2d^2-3e^2)\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2bd(c^2d^2+e^2)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(64b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(64b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(16b\sqrt{-c^2}d(-c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad - \frac{64b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{64b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{16bd(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3\sqrt{-c^2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(32bd^4\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right)}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(4b\sqrt{-c^2}(c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{35c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{\left(32b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(4b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{35c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad - \frac{32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{4b\sqrt{-c^2}(c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{32b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{16bd(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3\sqrt{-c^2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(32bd^4\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{4b(d+ex)^{3/2}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad - \frac{32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{4b\sqrt{-c^2}(c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{32b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{16bd(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^3\sqrt{-c^2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{32bd^4\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 45.60 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.19

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = -\frac{ad^3\sqrt{d+ex}B_{-\frac{ex}{d}}\left(3,\frac{3}{2}\right)}{e^3\sqrt{1+\frac{ex}{d}}}$$

$$+\left(b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4\left(5c^2d^2+9e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{105e^2}-\frac{16c^3d^3\text{csch}^{-1}(cx)}{105e^3}-\frac{2}{7}c^3x^3\text{csch}^{-1}(cx)-\frac{2c^2x^2\left(2e\sqrt{1+\frac{1}{c^2x^2}}+cd\text{csch}^{-1}(cx)\right)}{35e}-\frac{8cx\left(cde\sqrt{1+\frac{1}{c^2x^2}}\right.}{}\right.}{\sqrt{d+ex}}\right.\right.$$

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]

[Out] -((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d])) +
 (b*(-((c*(e + d/x)*x*((4*(5*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*e
^2) - (16*c^3*d^3*ArcCsch[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsch[c*x])/7 - (2
*c^2*x^2*(2*e*Sqrt[1 + 1/(c^2*x^2)] + c*d*ArcCsch[c*x]))/(35*e) - (8*c*x*(c
*d*e*Sqrt[1 + 1/(c^2*x^2)] - c^2*d^2*ArcCsch[c*x]))/(105*e^2)))/Sqrt[d + e*
x]) - (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(9*c^3*d^3*e + c*d*e^3)*Sqrt[1
 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*S
qrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(
c*d - I*e)*(8*c^4*d^4 - 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I +
 c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-5*c^3*d^3*e - 9*c*d*e^3)*Cosh[2*ArcCsch[
c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I +
c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/
(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*
(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d +
e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)
*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^
2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 2515, normalized size of antiderivative = 2.74

method	result	size
derivativedivides	Expression too large to display	2515
default	Expression too large to display	2515
parts	Expression too large to display	2518

```
[In] int(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(1
/7*arccsch(c*x)*(e*x+d)^(7/2)-2/5*arccsch(c*x)*d*(e*x+d)^(5/2)+1/3*arccsch(
c*x)*d^2*(e*x+d)^(3/2)+2/105/c^4*(-7*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^
3*d*e*(e*x+d)^(5/2)-3*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d*(e*x+d)^(7/2)
-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e*(e*x+d)^(1/2)+7*((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c^4*d^2*(e*x+d)^(5/2)+5*I*((c*d+I*e)*c/(c^2*d^2+e^2))
^(1/2)*c^3*d^2*e*(e*x+d)^(3/2)-4*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-5*(-(I*c*e*(e*x+d)+c^2*d
*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-I*((
c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^3*(e*x+d)^(1/2)+8*(-(I*c*e*(e*x+d)+c^
2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)
+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c
^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2
))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4+I*(-(I*c*e*(e*x+d)+c^2*
d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*
d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3-5*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*(e*x+d)^(3/2)+9*I*(-(I*c*e*(e*x+d)
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x
+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^
3*e+((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^4*(e*x+d)^(1/2)+3*I*((c*d+I*e)*
c/(c^2*d^2+e^2))^(1/2)*c^3*e*(e*x+d)^(7/2)+13*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d
)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2-14*(-(I
*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d
)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(
```

```
1/2))*c^2*d^2*e^2-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+
e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)
*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(
c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2))*c^3*d^3*e-3*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e^2*(e*x+d)^(
3/2)+3*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*e^3*(e*x+d)^(3/2)+((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e^2*(e*x+d)^(1/2)+9*(-(I*c*e*(e*x+d)+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4-9*(-(I*c*e*
(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2
*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+
I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*e^4)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c
*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F]

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x + d), x)
```

Sympy [F]

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^2(a+b\operatorname{acsch}(cx))\sqrt{d+ex}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))*sqrt(d + e*x), x)
```

Maxima [F]

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/11025*(1157625*c^2*e^3*integrate(1/105*sqrt(e*x + d)*x^4*log(x)/(c^2*e^3
*x^2 + e^3), x) + 1680*c^2*d^3*(integrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)
/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/
c^2 + 2*sqrt(e*x + d)/(c^2*e))/e^2 + 1157625*e^3*integrate(1/105*sqrt(e*x +
 d)*x^2*log(x)/(c^2*e^3*x^2 + e^3), x) + 280*c^2*d^2*(3*e^2*integrate(sqrt(
e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*
(e*x + d)^(3/2)/(c^2*e))/e^2 - 42*c^2*d*(15*e^2*integrate(((e*x + d)*c^2*d
 - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqr
t(e*x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2 - 5*(e*x + d)^(3/2)*c^2*d -
15*sqrt(e*x + d)*e^2)/(c^4*e))/e^2 - 3675*(3*e^2*integrate(sqrt(e*x + d)/((
e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*(e*x + d)^(
3/2)/(c^2*e))*log(c) + 105*c^2*(105*e^4*integrate(sqrt(e*x + d)/((e*x + d)^
2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^4 + 2*(15*(e*x + d)^(7/2)*
c^2 - 42*(e*x + d)^(5/2)*c^2*d + 35*(c^2*d^2 - e^2)*(e*x + d)^(3/2))/(c^4*e
))*log(c)/e^2 + 30*c^2*(105*e^4*integrate(sqrt(e*x + d)/((e*x + d)^2*c^2 -
2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^4 + 2*(15*(e*x + d)^(7/2)*c^2 - 42
*(e*x + d)^(5/2)*c^2*d + 35*(c^2*d^2 - e^2)*(e*x + d)^(3/2))/(c^4*e))/e^2 -
 210*(15*e^3*x^3 + 3*d*e^2*x^2 - 4*d^2*e*x + 8*d^3)*sqrt(e*x + d)*log(sqrt(
c^2*x^2 + 1) + 1)/e^3 - 11025*integrate(2/105*(15*c^2*e^3*x^4 + 3*c^2*d*e^2
*x^3 - 4*c^2*d^2*e*x^2 + 8*c^2*d^3*x)*sqrt(e*x + d)/(c^2*e^3*x^2 + e^3 + (c
^2*e^3*x^2 + e^3)*sqrt(c^2*x^2 + 1)), x))*b + 2/105*a*(15*(e*x + d)^(7/2)/e
^3 - 42*(e*x + d)^(5/2)*d/e^3 + 35*(e*x + d)^(3/2)*d^2/e^3)
```

Giac [F]

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^2\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\sqrt{d+e\,x}\,dx$$

```
[In] int(x^2*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int(x^2*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.52 $\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 347
Rubi [A] (verified) . . . 348
Mathematica [C] (verified) . . . 356
Maple [C] (verified) . . . 356
Fricas [F(-1)] . . . 357
Sympy [F] . . . 358
Maxima [F] . . . 358
Giac [F] . . . 359
Mupad [F(-1)] . . . 359

Optimal result

Integrand size = 19, antiderivative size = 679

$$
\begin{aligned}
&\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{8bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8bcd^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{8bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] -2/3*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsch(
c*x))/e^2+4/15*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1+1/c^2/x^2)^(1/2)+8/15*b
*d^3*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)
^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e
))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*c*d*EllipticE(1/2
```

```
*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2))
)^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/
2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/15*b*c*d^2*EllipticF(1/2*(1
-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(
1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(
3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*c*(c^2*d^2+e^2)*EllipticF
(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(
-c^2)^(5/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {45, 6445, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551}

$$
\begin{aligned}
&\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{8bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{4bc\sqrt{c^2x^2+1}(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad - \frac{8bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{8bcd\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} + \frac{4b(c^2x^2+1)\sqrt{d+ex}}{15c^3x\sqrt{\frac{1}{c^2x^2}+1}}
\end{aligned}
$$

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d*(
d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
cCsch[c*x]))/(5*e^2) + (8*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[A
rcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2
]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2
*d - Sqrt[-c^2]*e)]) - (8*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*
e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2
*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*
```

```
x^2)]*x*Sqrt[d + e*x]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]
/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{b\int \frac{2(d+ex)^{3/2}(-2d+3ex)}{15e^2\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{(2b)\int \frac{(d+ex)^{3/2}(-2d+3ex)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{15ce^2} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{(d+ex)^{3/2}(-2d+3ex)}{x\sqrt{1+c^2x^2}}\,dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(\frac{3e(d+ex)^{3/2}}{\sqrt{1+c^2x^2}} - \frac{2d(d+ex)^{3/2}}{x\sqrt{1+c^2x^2}}\right)\,dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= -\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}$$
$$- \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{1+c^2x^2}}\,dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{(d+ex)^{3/2}}{\sqrt{1+c^2x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}$$
$$+ \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}$$
$$- \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{\frac{1}{2}\left(3c^2d^2-e^2\right)+2c^2dex}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}$$
$$+ \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15c\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{15ce\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(8bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15ce\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(2b(-c^2d^2-e^2)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} - \frac{\left(4bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{15ce\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(16b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{\left(16b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(4b\sqrt{-c^2}(-c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^5e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{16b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{16b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4b(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3\sqrt{-c^2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(8bd^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right)}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4b(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3\sqrt{-c^2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(8bd^3\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4b(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3\sqrt{-c^2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{8bd^3\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.33 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.62

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{15}\left(\frac{4b\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}\right.$$
$$+\frac{2a\sqrt{d+ex}(-2d^2+dex+3e^2x^2)}{e^2}+\frac{2b\sqrt{d+ex}(-2d^2+dex+3e^2x^2)\operatorname{csch}^{-1}(cx)}{e^2}$$
$$+\frac{4ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(2cd(cd+ie)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)\middle|\frac{cd-ie}{cd+ie}\right)+(c^2d^2-2icde+e^2)\,\mathrm{Ell}\right.}{c^3\sqrt{-\frac{c}{cd-}}}$$

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] ((4*b*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2
 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2
)*ArcCsch[c*x])/e^2 + ((4*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((
e*(I + c*x))/(c*d - I*e))]*(2*c*d*(c*d + I*e)*EllipticE[I*ArcSinh[Sqrt[-(c/
(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (c^2*d^2 - (2*I)*c
*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d
 - I*e)/(c*d + I*e)] - 2*c^2*d^2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt
[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^3*Sqrt[-(c
/(c*d - I*e))]*e^2*Sqrt[1 + 1/(c^2*x^2)]*x))/15
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.06 (sec) , antiderivative size = 1963, normalized size of antiderivative = 2.89

method	result	size
derivativedivides	Expression too large to display	1963
default	Expression too large to display	1963
parts	Expression too large to display	1967

```
[In] int(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/5*(e*x+d)^(5/2)+1/3*(e*x+d)^(3/2)*d)-b*(-1/5*arccsch(c*x)*(e*
x+d)^(5/2)+1/3*arccsch(c*x)*(e*x+d)^(3/2)*d+2/15/c^3*(-2*I*(-(I*c*e*(e*x+d)
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x
+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+
```

```
e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+((c*d+I*e)*c/(c^2*
d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*e^3
*(e*x+d)^(1/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*e*(e*x+d)^(5/2)+2*I*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^(3/2)-(-(I*c*e*(e*x+d)+c^
2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)
+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-2
*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(
e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1
/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^
2))^(1/2))*c^3*d^3+2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e
^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*
EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c
^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2))*c^3*d^3-2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+
3*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*
e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)
^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2))*c^2*d^2*e+((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(
1/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e*(e*x+d)^(1/2)+(-(I*c*e*(
e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*
d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*
c*d*e^2-2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*
((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((
e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2))*c*d*e^2+I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(
c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c
*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3+((c*d+I*e)*c/(c^2*d^2+e^2))^(1/
2)*c*d*e^2*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/
e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x(a+b\,\operatorname{acsch}(cx))\,\sqrt{d+ex}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
[Out] Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x), x)
```

Maxima [F]

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\,\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] -1/225*(3375*c^2*e^2*integrate(1/15*sqrt(e*x + d)*x^3*log(x)/(c^2*e^2*x^2 +
 e^2), x) - 60*c^2*d^2*(e*integrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*
x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^2 +
2*sqrt(e*x + d)/c^2)/e^2 + 3375*e^2*integrate(1/15*sqrt(e*x + d)*x*log(x)/(
c^2*e^2*x^2 + e^2), x) - 10*(3*e^3*integrate(sqrt(e*x + d)/((e*x + d)^2*c^2
 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*(e*x + d)^(3/2)/c^2)*c^2*
d/e^2 + 225*(e*integrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^
2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^2 + 2*sqrt(e*x
+ d)/c^2)*log(c) - 15*c^2*(15*e^3*integrate(((e*x + d)*c^2*d - c^2*d^2 - e^
2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x
)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2 - 5*(e*x + d)^(3/2)*c^2*d - 15*sqrt(e*x +
d)*e^2)/c^4)*log(c)/e^2 - 6*c^2*(15*e^3*integrate(((e*x + d)*c^2*d - c^2*d^
2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x +
d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2 - 5*(e*x + d)^(3/2)*c^2*d - 15*sqrt(
e*x + d)*e^2)/c^4)/e^2 - 30*(3*e^2*x^2 + d*e*x - 2*d^2)*sqrt(e*x + d)*log(s
qrt(c^2*x^2 + 1) + 1)/e^2 - 225*integrate(2/15*(3*c^2*e^2*x^3 + c^2*d*e*x^2
 - 2*c^2*d^2*x)*sqrt(e*x + d)/(c^2*e^2*x^2 + e^2 + (c^2*e^2*x^2 + e^2)*sqrt
(c^2*x^2 + 1)), x))*b + 2/15*a*(3*(e*x + d)^(5/2)/e^2 - 5*(e*x + d)^(3/2)*d
/e^2)
```

Giac [F]

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\sqrt{d+e\,x}\,dx$$

```
[In] int(x*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int(x*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.53 $\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 360
Rubi [A] (verified) . . . 361
Mathematica [C] (warning: unable to verify) . . . 366
Maple [C] (verified) . . . 367
Fricas [F(-1)] . . . 368
Sympy [F] . . . 368
Maxima [F] . . . 368
Giac [F] . . . 369
Mupad [F(-1)] . . . 369

Optimal result

Integrand size = 18, antiderivative size = 429

$$\begin{aligned}
&\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} \\
&\quad + \frac{4bc\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&\quad + \frac{4bcd\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e+4/3*b*c*EllipticE(1/2*(1-x*(-c^2)^(1
/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d
)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/
(-c^2)^(1/2)))^(1/2)+4/3*b*c*d*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*((e*
x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(
1/2)-4/3*b*d^2*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e
/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^
2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6425, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435}

$$\begin{aligned}
&\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad - \frac{4bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{4bcd\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{4bc\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}
\end{aligned}$$

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (4*b*c*Sqrt[d + e*x]*Sqrt[
1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^
2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*c*d*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2]
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e
)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
(2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{(2b)\int \frac{(d+ex)^{3/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{3ce} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{\left(4bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2bd^2\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2be\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(8b\sqrt{-c^2}d\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad + \frac{8b\sqrt{-c^2}d\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}} \\
&\quad - \frac{\left(4b\sqrt{-c^2}d\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&\quad + \frac{4b\sqrt{-c^2}d\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(4bd^2\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{3ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&\quad + \frac{4b\sqrt{-c^2}d\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4bd^2\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.85 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.16

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{2a(d+ex)^{3/2}}{3e}$$

$$+b\left(-\frac{(cd+cex)\left(-\frac{4}{3}\sqrt{1+\frac{1}{c^2x^2}}-\frac{2cd\operatorname{csch}^{-1}(cx)}{3e}-\frac{2}{3}cx\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}-\frac{2(cd+cex)\left(-\frac{\sqrt{2}cde\sqrt{1+icx}(i+cx)\sqrt{\frac{cd+cex}{cd-ie}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{e(i+cx}{cd-i}\right.\right.\right.}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}\right.\right.\right.$$

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) + (b*(-(((c*d + c*e*x)*((-4*Sqrt[1 + 1/(c^2*x^2
)])/3 - (2*c*d*ArcCsch[c*x])/(3*e) - (2*c*x*ArcCsch[c*x])/3))/Sqrt[d + e*x]
) - (2*(c*d + c*e*x)*(-((Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d
+ c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
 (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[
(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(c^2*d^2 + e^2)*Sqr
t[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
 (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])
/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cosh[2*ArcCsch[
c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I +
c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/
(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*
(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d +
e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)
*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2)))
)/(3*e*Sqrt[e + d/x]*Sqrt[c*x]*Sqrt[d + e*x])))/c^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.60 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{2a(ex+d)^{\frac{3}{2}}}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(i\,\text{EllipticF}\left(\sqrt{}\right.\right.\right.$
default	$\frac{2a(ex+d)^{\frac{3}{2}}}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(i\,\text{EllipticF}\left(\sqrt{}\right.\right.\right.$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(i\,\text{Elliptic}\right.\right.\right.$

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/3*a*(e*x+d)^(3/2)+b*(1/3*(e*x+d)^(3/2)*arccsch(c*x)+2/3/c^2*(-(I*c*e
*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^
2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(I*EllipticF((e*x+d)^(1/2)*((
c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2))*c*d*e-I*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(
c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/
(c^2*d^2+e^2))^(1/2))*c*d*e-2*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+Elliptic
E((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2
*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))
^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticF((e*x+d)^(1/2)*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2))*e^2+EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*
c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+
d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e
-c*d)))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acsch}\left(cx\right)\right)\sqrt{d+ex}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}\left(b\,\text{arcsch}\left(cx\right)+a\right)dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] -1/9*(27*c^2*e*integrate(1/3*sqrt(e*x + d)*x^2*log(x)/(c^2*e*x^2 + e), x) +
 9*e^2*integrate(sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d
^2 + e^2), x)*log(c) + 6*c^2*d*(e^2*integrate(((e*x + d)*c^2*d - c^2*d^2 -
e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)),
 x)/c^2 + 2*sqrt(e*x + d)*e/c^2)/e^2 + 27*e*integrate(1/3*sqrt(e*x + d)*log
(x)/(c^2*e*x^2 + e), x) - 3*(3*e^4*integrate(sqrt(e*x + d)/((e*x + d)^2*c^2
 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*(e*x + d)^(3/2)*e/c^2)*c^
2*log(c)/e^2 - 6*(e*x + d)^(3/2)*log(sqrt(c^2*x^2 + 1) + 1)/e - 2*(3*e^4*in
tegrate(sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)
, x)/c^2 - 2*(e*x + d)^(3/2)*e/c^2)*c^2/e^2 - 9*integrate(2/3*(c^2*e*x^2 +
c^2*d*x)*sqrt(e*x + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1) + e),
 x))*b + 2/3*(e*x + d)^(3/2)*a/e
```

Giac [F]

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}\left(b\,\operatorname{arcsch}(cx)+a\right)dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\sqrt{d+e\,x}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.54 $\int \frac{\sqrt{d+ex}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal result . . . 370
Rubi [N/A] . . . 370
Mathematica [N/A] . . . 371
Maple [N/A] (verified) . . . 371
Fricas [N/A] . . . 371
Sympy [N/A] . . . 371
Maxima [N/A] . . . 372
Giac [N/A] . . . 372
Mupad [N/A] . . . 372

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 43.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\ \text{arccsch}\left(cx\right))\sqrt{ex+d}}{x}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
[Out] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{ex+d}(b\,\text{arcsch}\left(cx\right)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 13.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\sqrt{d+ex}}{x}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x,x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.57

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 2*sqrt(
e*x + d))*a - ((sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt
(d))) + 2*sqrt(e*x + d))*log(c) + integrate(sqrt(e*x + d)*log(x)/x, x) - in
tegrate(sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/x, x))*b
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)\sqrt{d+e\,x}}{x}\,dx$$

```
[In] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x,x)
```

```
[Out] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

3.55 $\int \frac{\sqrt{d+ex}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result . . . 373
Rubi [N/A] . . . 373
Mathematica [N/A] . . . 374
Maple [N/A] (verified) . . . 374
Fricas [N/A] . . . 374
Sympy [N/A] . . . 374
Maxima [N/A] . . . 375
Giac [N/A] . . . 375
Mupad [N/A] . . . 375

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \operatorname{Int}\left(\frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\,\text{arccsch}\,(cx))\,\sqrt{ex+d}}{x^2}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
[Out] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 14.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex}}{x^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x**2,x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 7.10

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d) - 2
*sqrt(e*x + d)/x)*a - 1/2*((e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d)
+ sqrt(d)))/sqrt(d) - 2*sqrt(e*x + d)/x)*log(c) + 2*integrate(sqrt(e*x + d)
*log(x)/x^2, x) - 2*integrate(sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/x^2,
 x))*b
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)\,\sqrt{d+e\,x}}{x^2}\,dx$$

```
[In] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

3.56 $\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 486

$$\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$
$$+\frac{28bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$
$$-\frac{4bc(2c^2d^2-e^2)\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{4bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arccsch(c*x))/e+4/15*b*e*(c^2*x^2+1)*(e*x+d)^(1/2)/c
^3/x/(1+1/c^2/x^2)^(1/2)+28/15*b*c*d*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c
^2*x^2+1)^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/(-c^2)^(1/
2)))^(1/2)-4/15*b*c*(2*c^2*d^2-e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*
2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)
*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/x/(1+1/c^2/x^2)^(1/2)/(e*x
+d)^(1/2)-4/5*b*d^3*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/
2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d
*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6425, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435, 945, 1598}

$$\int (d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e}$$
$$-\frac{4bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$-\frac{4bc\sqrt{c^2x^2+1}(2c^2d^2-e^2)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$+\frac{28bcd\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}$$
$$+\frac{4be(c^2x^2+1)\sqrt{d+ex}}{15c^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*e*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*(
d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (28*b*c*d*Sqrt[d + e*x]*Sqrt[1
 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2
]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) - (4*b*c*(2*c^2*d^2 - e^2)*Sqrt[(d + e*x)/(
d + e/Sqrt[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*Sqr
t[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/
(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(5*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
```

```
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{(2b)\int \frac{(d+ex)^{5/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{5ce}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{3de^2x}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{e^3x^2}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right)dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{\left(6bd^2\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(2bd^3\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(6bde\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2be^2\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{x^2}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e}$$
$$+ \frac{\left(6bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(6bd^2\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(2be\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\frac{ex}{c^2}+2dx^2}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(2bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(12b\sqrt{-c^2}d^2\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} \\
&+ \frac{12b\sqrt{-c^2}d^2\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(2be\sqrt{\frac{1}{c^2}+x^2}\right)\int\frac{\frac{e}{c^2}+2dx}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{\left(4bd^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right)}{5ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}} \\
&- \frac{\left(12b\sqrt{-c^2}d^2\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} \\
&+ \frac{12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&- \frac{\left(4bd\sqrt{\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\left(-2d^2+\frac{e^2}{c^2}\right)\sqrt{\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{\left(4bd^3\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right.}{5ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} \\
&+ \frac{12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&- \frac{4bd^3\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}} \\
&- \frac{\left(4b\sqrt{-c^2}\left(-2d^2+\frac{e^2}{c^2}\right)\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} \\
&+ \frac{28b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&+ \frac{4b\sqrt{-c^2}\left(2d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{4bd^3\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.78

$$\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{2\left(\frac{2be^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}+3a(d+ex)^{5/2}+3b(d+ex)^{5/2}\text{csch}^{-1}(cx)+\frac{2ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx}{cd-ie}}}\right.}{}$$

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*((2*b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + 3*a*(d + e*x)^(5/2)
 + 3*b*(d + e*x)^(5/2)*ArcCsch[c*x] + ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d +
 I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(7*c*d*(c*d + I*e)*EllipticE[I*A
rcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (-
9*c^2*d^2 - (7*I)*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*S
qrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + 3*c^2*d^2*EllipticPi[1 - (I*e)/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*
e)]))/(c^3*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(15*e)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 1939, normalized size of antiderivative = 3.99

method	result	size
derivativedivides	Expression too large to display	1939
default	Expression too large to display	1939
parts	Expression too large to display	1941

```
[In] int((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*arccsch(c*x)*(e*x+d)^(5/2)+2/15/c^3*(-2*I*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+2*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),
(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-3*I*(-(I*c*e*(e*x
+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(
e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d
^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+I*((c*d+I*e)*c/
```

```
(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*e^3+2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*
c^3*d^2*(e*x+d)^(3/2)-9*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^
2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+7*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^
(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+3*(-(I*c*e*(e
*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d
*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2
*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3+I*((c*d+I*e)*c/
(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^2*e-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*
c^3*d^3*(e*x+d)^(1/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2
*d^2*e-6*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e
*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2))*c*d*e^2+7*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-I*(-(I*c*e*(e*x+d)+c^2*d*(e*
x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3-((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d
)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-
c*d)))
```

Fricas [F]

$$\int (d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x + a*d + (b*e*x + b*d)*arccsch(c*x))*sqrt(e*x + d), x)
```

Sympy [F]

$$\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acsch}\left(cx\right)\right)(d+ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\,\text{arcsch}\left(cx\right)+a)\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 2/5*(e*x + d)^(5/2)*a/e - 1/75*(375*c^2*e^2*integrate(1/5*sqrt(e*x + d)*x^3
*log(x)/(c^2*e*x^2 + e), x) + 375*c^2*d*e*integrate(1/5*sqrt(e*x + d)*x^2*l
og(x)/(c^2*e*x^2 + e), x) + 75*d*e^2*integrate(sqrt(e*x + d)/((e*x + d)^2*c
^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)*log(c) + 30*c^2*d^2*(e^2*integr
ate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d
 + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^2 + 2*sqrt(e*x + d)*e/c^2)/e^2 + 375
*e^2*integrate(1/5*sqrt(e*x + d)*x*log(x)/(c^2*e*x^2 + e), x) + 375*d*e*int
egrate(1/5*sqrt(e*x + d)*log(x)/(c^2*e*x^2 + e), x) - 25*(3*e^4*integrate(s
qrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2
 - 2*(e*x + d)^(3/2)*e/c^2)*c^2*d*log(c)/e^2 - 20*(3*e^4*integrate(sqrt(e*x
 + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*(e*x
 + d)^(3/2)*e/c^2)*c^2*d/e^2 + 75*(e^2*integrate(((e*x + d)*c^2*d - c^2*d^2
 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d
)), x)/c^2 + 2*sqrt(e*x + d)*e/c^2)*log(c) - 5*c^2*(15*e^4*integrate(((e*x
 + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2
 + e^2)*sqrt(e*x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2*e - 5*(e*x + d)^(
3/2)*c^2*d*e - 15*sqrt(e*x + d)*e^3)/c^4)*log(c)/e^2 - 30*(e^2*x^2 + 2*d*e*
x + d^2)*sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e - 2*c^2*(15*e^4*integra
te(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d
 + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2*e - 5*(e
*x + d)^(3/2)*c^2*d*e - 15*sqrt(e*x + d)*e^3)/c^4)/e^2 - 75*integrate(2/5*(
c^2*e^2*x^3 + 2*c^2*d*e*x^2 + c^2*d^2*x)*sqrt(e*x + d)/(c^2*e*x^2 + (c^2*e*
x^2 + e)*sqrt(c^2*x^2 + 1) + e), x))*b
```

Giac [F]

$$\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^{3/2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2),x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.57 $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal result . . . 387
Rubi [A] (verified) . . . 388
Mathematica [C] (warning: unable to verify) . . . 398
Maple [C] (verified) . . . 399
Fricas [F] . . . 401
Sympy [F] . . . 401
Maxima [F] . . . 401
Giac [F] . . . 402
Mupad [F(-1)] . . . 402

Optimal result

Integrand size = 21, antiderivative size = 939

$$
\begin{aligned}
&\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4} \\
&+ \frac{24bcd^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{4bc(2c^2d^2+9e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{64bcd^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{32bcd(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{64bd^4\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] 2*d^2*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsch
(c*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsch(c*x))/e^4-2*d^3*(a+b*arccsch(c*x)
)*(e*x+d)^(1/2)/e^4+4/35*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1+1/c^2/x^2)^(1
/2)-4/21*b*d*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1+1/c^2/x^2)^(1/2)+64/35*
b*d^4*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2
)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+
e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+24/35*b*c*d^2*EllipticE
(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^
2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+4/105*b*c*(2*c^2*d^2+9*
e^2)*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(5/2)/e^3
/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-64/35*b*c
*d^3*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(
1/2)))^(1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/105*b*
c*d*(c^2*d^2+e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^
2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1
/2)
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used

$= \{45, 6445, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858, 956, 1668\}$

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$$

$$= \frac{64b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$- \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d^3}{e^4}$$

$$- \frac{64bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d^3}{35\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+ \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4}$$

$$+ \frac{24bc\sqrt{d+ex}\sqrt{c^2x^2+1}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d^2}{35\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

$$- \frac{4b\sqrt{d+ex}(c^2x^2+1)\,d}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{6(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)d}{5e^4}$$

$$- \frac{32bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d}{105\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+ \frac{4b\sqrt{d+ex}(c^2x^2+1)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} + \frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}$$

$$+ \frac{4bc(2c^2d^2+9e^2)\sqrt{d+ex}\sqrt{c^2x^2+1}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]) - (4*b*d
*Sqrt[d + e*x]*(1 + c^2*x^2))/(21*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d^3
*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) + (2*
(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^4) + (24*b*c*d^2*Sqrt[d + e*x]*S
qrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt
[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2
)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*(2*c^2*d^2 + 9*
e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^
```

```
3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
64*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d -
 Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (32*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x
^2)]*x*Sqrt[d + e*x]) + (64*b*d^4*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
 + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2
]], (2*e)/(Sqrt[-c^2]*d + e)])/(35*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}\\
&\quad-\frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4}\\
&\quad+\frac{b\int\frac{2\sqrt{d+ex}\left(-16d^3+8d^2ex-6de^2x^2+5e^3x^3\right)}{35e^4\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c}\\
&=-\frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}\\
&\quad-\frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4}\\
&\quad+\frac{(2b)\int\frac{\sqrt{d+ex}\left(-16d^3+8d^2ex-6de^2x^2+5e^3x^3\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{35ce^4}\\
&=-\frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}\\
&\quad-\frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4}\\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}\left(-16d^3+8d^2ex-6de^2x^2+5e^3x^3\right)}{x\sqrt{1+c^2x^2}}\,dx}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x}\\
&=-\frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}\\
&\quad-\frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4}\\
&\quad+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(\frac{8d^2e\sqrt{d+ex}}{\sqrt{1+c^2x^2}}-\frac{16d^3\sqrt{d+ex}}{x\sqrt{1+c^2x^2}}-\frac{6de^2x\sqrt{d+ex}}{\sqrt{1+c^2x^2}}+\frac{5e^3x^2\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\right)dx}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4} \\
&\quad - \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{1+c^2x^2}}\,dx}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{35ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(12bd\sqrt{1+c^2x^2}\right)\int \frac{x\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{35ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{7ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{8bd\sqrt{d+ex}(1+c^2x^2)}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4} \\
&\quad - \frac{\left(32bd^4\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{35ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int \frac{-\frac{e}{2}+\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{2d+3ex-c^2dx^2}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4} \\
&\quad + \frac{32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{\left(32bd^4\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{\frac{7}{2}c^2de^2+\frac{1}{2}c^2e\left(2c^2d^2+9e^2\right)x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{35ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4bd(c^2d^2+e^2)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(64b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4} \\
&+ \frac{32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{64b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{\left(64bd^4\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(4bd(c^2d^2+e^2)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{105c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{\left(2b(2c^2d^2+9e^2)\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{105c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{\left(8b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{\left(8b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{35c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4} \\
&+ \frac{24b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{64b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{8b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(4b\sqrt{-c^2}(2c^2d^2+9e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{\left(8b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{105c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{\left(64bd^4\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right.}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}$$
$$- \frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}$$
$$+ \frac{24b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$- \frac{4b\sqrt{-c^2}(2c^2d^2+9e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$- \frac{64b\sqrt{-c^2}d^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{32b\sqrt{-c^2}d(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{64bd^4\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.76 (sec) , antiderivative size = 1098, normalized size of antiderivative = 1.17

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \frac{ad^4\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(4,\frac{1}{2}\right)}{e^4\sqrt{d+ex}}$$
$$+ \frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4\left(-16c^2d^2+9e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{105e^3}+\frac{32c^3d^3\text{csch}^{-1}(cx)}{35e^4}-\frac{2c^3x^3\text{csch}^{-1}(cx)}{7e}-\frac{4c^2x^2\left(e\sqrt{1+\frac{1}{c^2x^2}}-3cd\text{csch}^{-1}(cx)\right)}{35e^2}+\frac{4cx\left(5cde\sqrt{1+\frac{1}{c^2x}}\right.}{}\right.}{\sqrt{d+ex}}\right.}{}$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x)*x*((4*(-16*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsch[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsch[c*x])/(7*e) -
(4*c^2*x^2*(e*Sqrt[1 + 1/(c^2*x^2)] - 3*c*d*ArcCsch[c*x]))/(35*e^2) + (4*c*
x*(5*c*d*e*Sqrt[1 + 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsch[c*x]))/(105*e^3)))/Sq
rt[d + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(40*c^3*d^3*e - 8*c*d
*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[A
rcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/
(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) +
(I*Sqrt[2]*(c*d - I*e)*(48*c^4*d^4 - 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[1 + I*c*x
]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e,
 ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1
+ 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-16*c^3*d^3*e + 9*c*d*e^3)*
Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (
2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))
/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellip
ticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*
e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*
(I + c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*x])))/c^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 2545, normalized size of antiderivative = 2.71

method	result	size
derivativedivides	Expression too large to display	2545
default	Expression too large to display	2545
parts	Expression too large to display	2546

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/7*(e*x+d)^(7/2)+3/5*d*(e*x+d)^(5/2)-d^2*(e*x+d)^(3/2)+d^3*(e*
x+d)^(1/2))-b*(-1/7*arccsch(c*x)*(e*x+d)^(7/2)+3/5*arccsch(c*x)*d*(e*x+d)^(
5/2)-arccsch(c*x)*d^2*(e*x+d)^(3/2)+arccsch(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^
4*(14*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d*e*(e*x+d)^(5/2)+3*((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2)*c^4*d*(e*x+d)^(7/2)+40*I*(-(I*c*e*(e*x+d)+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d
```

```
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e-14*((
c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^2*(e*x+d)^(5/2)-3*I*((c*d+I*e)*c/(c^2
*d^2+e^2))^(1/2)*c*e^3*(e*x+d)^(3/2)-3*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*
c^3*e*(e*x+d)^(7/2)-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^
2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3+19*((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2)*c^4*d^3*(e*x+d)^(3/2)-24*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-16*(-(I*c*e*(e*x+d)+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4+48*(-(I
*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d
)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)
*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4+8*I*((c*
d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e*(e*x+d)^(1/2)-8*((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2)*c^4*d^4*(e*x+d)^(1/2)+8*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*
c*d*e^3*(e*x+d)^(1/2)-48*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2
*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^
(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e
)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2))*c^3*d^3*e+3*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e^2*(e*x
+d)^(3/2)+15*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/
2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Elliptic
F((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2-7*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(
-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2-19*I*((c*d+I*e)*
c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*e*(e*x+d)^(3/2)-8*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2)*c^2*d^2*e^2*(e*x+d)^(1/2)-9*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-
(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4+9*(-(I*c*e*(e*x+d)+c^2*d*
(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4)/((c^2*(e
*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2
*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{x^3(a+b\operatorname{acsch}(cx))}{\sqrt{d+ex}}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] 2/35*a*(5*(e*x + d)^(7/2)/e^4 - 21*(e*x + d)^(5/2)*d/e^4 + 35*(e*x + d)^(3/
2)*d^2/e^4 - 35*sqrt(e*x + d)*d^3/e^4) + 1/35*b*(2*(5*e^4*x^4 - d*e^3*x^3 +
 2*d^2*e^2*x^2 - 8*d^3*e*x - 16*d^4)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x +
 d)*e^4) + 35*integrate(2/35*(5*c^2*e^4*x^5 - c^2*d*e^3*x^4 + 2*c^2*d^2*e^2
*x^3 - 8*c^2*d^3*e*x^2 - 16*c^2*d^4*x)/((c^2*e^4*x^2 + e^4)*sqrt(c^2*x^2 +
1)*sqrt(e*x + d) + (c^2*e^4*x^2 + e^4)*sqrt(e*x + d)), x) - 35*integrate(-1
/35*(2*c^2*d*e^3*x^4 + 16*c^2*d^3*e*x^2 - 5*(7*e^4*log(c) + 2*e^4)*c^2*x^5
+ 32*c^2*d^4*x - (4*c^2*d^2*e^2 + 35*e^4*log(c))*x^3 - 35*(c^2*e^4*x^5 + e^
4*x^3)*log(x))/((c^2*e^4*x^2 + e^4)*sqrt(e*x + d)), x))
```

Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{x^3\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.58 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal result . . . 403
Rubi [A] (verified) . . . 404
Mathematica [C] (warning: unable to verify) . . . 412
Maple [C] (verified) . . . 413
Fricas [F] . . . 414
Sympy [F] . . . 414
Maxima [F] . . . 414
Giac [F] . . . 415
Mupad [F(-1)] . . . 415

Optimal result

Integrand size = 21, antiderivative size = 707

$$
\begin{aligned}
&\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx \\
&= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad - \frac{4bcd\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{32bcd^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&\quad + \frac{4bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&\quad - \frac{32bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] -4/3*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsch(
c*x))/e^3+2*d^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+4/15*b*(c^2*x^2+1)*(e*
x+d)^(1/2)/c^3/e/x/(1+1/c^2/x^2)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(1-x*(-c^
2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)
```

```
^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^2/x^2
)^(1/2)/(e*x+d)^(1/2)-4/5*b*c*d*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1
/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^
2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c
^2)^(1/2)))^(1/2)+32/15*b*c*d^2*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1
/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^
2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1
/2)/(e*x+d)^(1/2)+4/15*b*c*(c^2*d^2+e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^
(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^2/x/(1+1/c^
2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 6445, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858}

$$
\begin{aligned}
&\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx \\
&= \frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad - \frac{32bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{32bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{4bc\sqrt{c^2x^2+1}(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad - \frac{4bcd\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} + \frac{4b(c^2x^2+1)\sqrt{d+ex}}{15c^3ex\sqrt{\frac{1}{c^2x^2}+1}}
\end{aligned}
$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*d
^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^3) - (
```

```
4*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2
]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(5*(-c^2)^(3/2)*e
^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) +
(32*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*
EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d
- Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x
]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sq
rt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[
-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (32*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{b\int \frac{2\sqrt{d+ex}\left(8d^2-4dex+3e^2x^2\right)}{15e^3\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{(2b)\int \frac{\sqrt{d+ex}\left(8d^2-4dex+3e^2x^2\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{15ce^3} \\
&= \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}\left(8d^2-4dex+3e^2x^2\right)}{x\sqrt{1+c^2x^2}}\,dx}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(-\frac{4de\sqrt{d+ex}}{\sqrt{1+c^2x^2}} + \frac{8d^2\sqrt{d+ex}}{x\sqrt{1+c^2x^2}} + \frac{3e^2x\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\right)dx}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}}{x\sqrt{1+c^2x^2}}dx}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x\sqrt{d+ex}}{\sqrt{1+c^2x^2}}dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{\left(16bd^3\sqrt{1+c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\int\frac{-\frac{e}{2}+\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(16b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}dx, x\right.}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad - \frac{16b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(16bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2bd\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b(c^2d^2+e^2)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&- \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&- \frac{16b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{32b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(4b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{\left(4b\sqrt{-c^2}(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&- \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&- \frac{4b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{32b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{4b\sqrt{-c^2}(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(32bd^3\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&- \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&- \frac{4b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{32b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{4b\sqrt{-c^2}(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^5e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{32bd^3\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.98 (sec) , antiderivative size = 1012, normalized size of antiderivative = 1.43

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{ad^3\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(3,\frac{1}{2}\right)}{e^3\sqrt{d+ex}}$$

$$+\frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4cd\sqrt{1+\frac{1}{c^2x^2}}}{5e^2}-\frac{16c^2d^2\text{csch}^{-1}(cx)}{15e^3}-\frac{2c^2x^2\text{csch}^{-1}(cx)}{5e}-\frac{4cx\left(e\sqrt{1+\frac{1}{c^2x^2}}-2cd\text{csch}^{-1}(cx)\right)}{15e^2}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{e+\frac{d}{x}}\sqrt{cx}\left(-\frac{\sqrt{2}\left(7c^2d^2e\right.}{}\right.}$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]

[Out] -((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x])) +
 (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^2*
ArcCsch[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsch[c*x])/(5*e) - (4*c*x*(e*Sqrt[1
 + 1/(c^2*x^2)] - 2*c*d*ArcCsch[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[e
 + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(7*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)
*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d
 - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^
(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(8*c^3*d
^3 - 3*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (
6*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sq
rt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSi
n[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I
 + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*
e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*
e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c
*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I
*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d
)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sq
rt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x])))/c^3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.41 (sec) , antiderivative size = 1991, normalized size of antiderivative = 2.82

method	result	size
derivativedivides	Expression too large to display	1991
default	Expression too large to display	1991
parts	Expression too large to display	1994

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*a
rccsch(c*x)*(e*x+d)^(5/2)-2/3*arccsch(c*x)*(e*x+d)^(3/2)*d+arccsch(c*x)*d^2
*(e*x+d)^(1/2)+2/15/c^3*(7*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2)*c^3*d*(e*x+d)^(5/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/
2)*c^2*e-4*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)
*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF(
(e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2))*c^3*d^3-3*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/
(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^
2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*
c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-I*(-(I*c*e*(e*x+d)+c^2*d*(
e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3+8*(-(I*c*e
*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^
2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*
d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(
c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3-2*I*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e+2*((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^
(1/2)*e^3-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+I*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2*e+4*(-(I*c*e*(e*x+d)+c^2*d
*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-3*(-
(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x
+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)
*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2))*c*d*e^2-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^
2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*E
```

```
llipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^
2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))
^(1/2))*c^2*d^2*e-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/
((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{x^2(a+b\operatorname{acsch}(cx))}{\sqrt{d+ex}}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*a*(3*(e*x + d)^(5/2)/e^3 - 10*(e*x + d)^(3/2)*d/e^3 + 15*sqrt(e*x + d)
*d^2/e^3) + 1/15*b*(2*(3*e^3*x^3 - d*e^2*x^2 + 4*d^2*e*x + 8*d^3)*log(sqrt(
c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^3) + 15*integrate(2/15*(3*c^2*e^3*x^4 -
c^2*d*e^2*x^3 + 4*c^2*d^2*e*x^2 + 8*c^2*d^3*x)/((c^2*e^3*x^2 + e^3)*sqrt(c^
2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^2 + e^3)*sqrt(e*x + d)), x) - 15*inte
grate(-1/15*(2*c^2*d*e^2*x^3 - 3*(5*e^3*log(c) + 2*e^3)*c^2*x^4 - 16*c^2*d^
3*x - (8*c^2*d^2*e + 15*e^3*log(c))*x^2 - 15*(c^2*e^3*x^4 + e^3*x^2)*log(x)
)/((c^2*e^3*x^2 + e^3)*sqrt(e*x + d)), x))
```

Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{x^2\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.59 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal result . . . 416
Rubi [A] (verified) . . . 417
Mathematica [C] (verified) . . . 422
Maple [C] (verified) . . . 422
Fricas [F] . . . 424
Sympy [F] . . . 424
Maxima [F] . . . 424
Giac [F] . . . 424
Mupad [F(-1)] . . . 425

Optimal result

Integrand size = 19, antiderivative size = 474

$$
\begin{aligned}
&\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx \\
&= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{4bc\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8bcd\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{8bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^2-2*d*(a+b*arccsch(c*x))*(e*x+d)^(1/
2)/e^2+8/3*b*d^2*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*
(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-
c^2)^(1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*Elli
pticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^
2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e/x/(1+1/c^2
/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/3*b*c*d*EllipticF(
1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/
2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-
c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {45, 6445, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551}

$$\begin{aligned}
&\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx \\
&= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{8bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad - \frac{8bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad + \frac{4bc\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}
\end{aligned}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*A
rcCsch[c*x]))/(3*e^2) + (4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[Ar
cSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]
*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]) - (8*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)
]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (
2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{b\int \frac{2(-2d+ex)\sqrt{d+ex}}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(2b)\int \frac{(-2d+ex)\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{3ce^2}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(\frac{e\sqrt{d+ex}}{\sqrt{1+c^2x^2}} - \frac{2d\sqrt{d+ex}}{x\sqrt{1+c^2x^2}}\right)\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} - \frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(8b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(8bd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(8bd^2\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{8b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{8bd^2\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.88

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$$

$$= \frac{\frac{2a(-2d+ex)(d+ex)}{e^2} + \frac{2b(-2d+ex)(d+ex)\text{csch}^{-1}(cx)}{e^2} + \frac{2b(icd+e)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\left(2i(cd+ie)e\sqrt{-\frac{e(-i+cx)}{cd+ie}}E\left(\arcsin\left(\sqrt{\frac{c(d+e\right.\right.\right.}{cd-i}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] ((2*a*(-2*d + e*x)*(d + e*x))/e^2 + (2*b*(-2*d + e*x)*(d + e*x)*ArcCsch[c*x
])/e^2 + (2*b*(I*c*d + e)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c*e*(I + c*x)*(d +
e*x))/(I*c*d + e)^2]*((2*I)*(c*d + I*e)*e*Sqrt[-((e*(-I + c*x))/(c*d + I*e)
)]*EllipticE[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d + I*
e)] - I*c*d*e*Sqrt[2 + (2*I)*c*x]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*
d - I*e))]], (I*c*d + e)/(2*e)] + 2*e^2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]
*EllipticF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)
] + 2*c^2*d^2*Sqrt[2 + (2*I)*c*x]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((
e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(e^3*(c + c^3*x^2)))/(3*S
qrt[d + e*x])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.00 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.83

method	result
derivativedivides	$-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\operatorname{arccsch}(cx)d\sqrt{ex+d}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{i}$
default	$-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\operatorname{arccsch}(cx)d\sqrt{ex+d}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{i}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-d\sqrt{ex+d}\right)}{e^2}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(}{}}\right.}{}$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/3*(e*x+d)^(3/2)+d*(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc
h(c*x)+arccsch(c*x)*d*(e*x+d)^(1/2)+2/3/c^2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2)*(2*I*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-2*I*Ellipt
icPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2
+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2
))*c*d*e-EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c
*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((c
*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2))*c^2*d^2+2*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/
(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE
((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2
/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x*arccsch(c*x) + a*x)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{x(a+b\operatorname{acsch}(cx))}{\sqrt{d+ex}}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
[Out] Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] 2/3*a*((e*x + d)^(3/2)/e^2 - 3*sqrt(e*x + d)*d/e^2) + 1/3*b*(2*(e^2*x^2 - d
*e*x - 2*d^2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2) + 3*integrate(
2/3*(c^2*e^2*x^3 - c^2*d*e*x^2 - 2*c^2*d^2*x)/((c^2*e^2*x^2 + e^2)*sqrt(c^2
*x^2 + 1)*sqrt(e*x + d) + (c^2*e^2*x^2 + e^2)*sqrt(e*x + d)), x) - 3*integr
ate(-1/3*(2*c^2*d*e*x^2 - (3*e^2*log(c) + 2*e^2)*c^2*x^3 + (4*c^2*d^2 - 3*e
^2*log(c))*x - 3*(c^2*e^2*x^3 + e^2*x)*log(x))/((c^2*e^2*x^2 + e^2)*sqrt(e*
x + d)), x))
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \int \frac{x\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.60 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal result . . . 426
Rubi [A] (verified) . . . 427
Mathematica [C] (verified) . . . 430
Maple [C] (verified) . . . 431
Fricas [F] . . . 431
Sympy [F] . . . 432
Maxima [F] . . . 432
Giac [F] . . . 432
Mupad [F(-1)] . . . 432

Optimal result

Integrand size = 18, antiderivative size = 284

$$\begin{aligned}
&\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx \\
&= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} \\
&\quad + \frac{4bc\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4bd\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

```
[Out] 2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e+4*b*c*EllipticF(1/2*(1-x*(-c^2)^(1/2))
^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1
)^(1/2)*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/
2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^
(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)
/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6425, 1588, 958, 733, 430, 947, 174, 552, 551}

$$\begin{aligned}
&\int \frac{a + b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx \\
&= \frac{2\sqrt{d+ex}\left(a + b\text{csch}^{-1}(cx)\right)}{e} \\
&\quad + \frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}\,x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \\
&\quad - \frac{4bd\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), \frac{2e}{\sqrt{-c^2}d+e}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}
\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e + (4*b*c*Sqrt[(d + e*x)/(d + e/Sqr
t[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*Sqrt[1 + 1/(c^2
*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
+ e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
```

```
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{ce} \\
&= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4bd\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.45 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.08

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\left(ae(d+ex) - \frac{b\left(e+\frac{d}{x}\right)\left(-cex\operatorname{csch}^{-1}(cx) + \frac{\sqrt{2}\sqrt{1+icx}\left(-e^2(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}\,\operatorname{EllipticF}\left(\arcsin\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right),\frac{icd+e}{2e}\right)+cd(icd+e)\sqrt{-\frac{e(i+cx)}{cd-ie}}\sqrt{\cdots}\right.}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{-\frac{e(i+cx)}{cd-ie}}(cd+cex)}}{c}\right.}{e^2\sqrt{d+ex}}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]

[Out] (2*(a*e*(d + e*x) - (b*(e + d/x)*(-(c*e*x*ArcCsch[c*x]) + (Sqrt[2]*Sqrt[1 +
 I*c*x]*(-(e^2*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin[S
qrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]) + c*d*(I*c*d + e)*S
qrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)]))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[-((e*(I + c*x))/(c*d - I*e
))]*(c*d + c*e*x))))/c))/(e^2*Sqrt[d + e*x])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\dfrac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\dfrac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\text{EllipticF}\left(\sqrt{}\right.\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c}{c}}}\right)}{e}$
default	$\dfrac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\dfrac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\text{EllipticF}\left(\sqrt{}\right.\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c}{c}}}\right)}{e}$
parts	$\dfrac{2a\sqrt{ex+d}}{e}+\dfrac{2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\dfrac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\text{EllipticF}\left(\right.\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-}{}}}\right)}{e}$

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arccsch(c*x)+2/c*(-(I*c*e*(e*x+d)+c^2
*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-EllipticP
i((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)))
/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{a + b\,\operatorname{acsch}(cx)}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\operatorname{arcsch}(cx) + a}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] b*(2*sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate(2*(c^2*e*x^2 +
c^2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e*x^2 + e)
*sqrt(e*x + d)), x) - integrate(((e*log(c) + 2*e)*c^2*x^2 + 2*c^2*d*x + e*l
og(c) + (c^2*e*x^2 + e)*log(x))/((c^2*e*x^2 + e)*sqrt(e*x + d)), x)) + 2*sq
rt(e*x + d)*a/e
```

Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\operatorname{arcsch}(cx) + a}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.61 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

Optimal result 433
Rubi [N/A] 433
Mathematica [N/A] 434
Maple [N/A] (verified) 434
Fricas [N/A] 434
Sympy [N/A] 434
Maxima [N/A] 435
Giac [N/A] 435
Mupad [N/A] 435

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x\sqrt{ex+d}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\sqrt{ex+d}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e*x^2 + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 8.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.52

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{ex+d}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(log(c)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d) +
 integrate(log(x)/(sqrt(e*x + d)*x), x) - integrate(log(sqrt(c^2*x^2 + 1) +
 1)/(sqrt(e*x + d)*x), x))*b + a*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x +
d) + sqrt(d)))/sqrt(d)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{ex+d}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 4.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

3.62 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

Optimal result . . . 436
Rubi [N/A] . . . 436
Mathematica [N/A] . . . 437
Maple [N/A] (verified) . . . 437
Fricas [N/A] . . . 437
Sympy [N/A] . . . 437
Maxima [N/A] . . . 438
Giac [N/A] . . . 438
Mupad [N/A] . . . 438

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x^2\sqrt{ex+d}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\sqrt{ex+d}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e*x^3 + d*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 17.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x^2\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x**2*sqrt(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 8.33

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{\sqrt{ex+d}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*sqrt(e*x + d)*e/((e*x + d)*d - d^2) + e*log((sqrt(e*x + d) - sqrt(d
))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2))*log(c) - 2*integrate(log(x)/(sqrt(e*
x + d)*x^2), x) + 2*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*x^2
), x))*b - 1/2*a*(2*sqrt(e*x + d)*e/((e*x + d)*d - d^2) + e*log((sqrt(e*x +
 d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{\sqrt{ex+d}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

3.63 $\int \frac{x^3\left(a+b\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal result . . . 439
Rubi [A] (verified) . . . 440
Mathematica [C] (warning: unable to verify) . . . 448
Maple [C] (verified) . . . 449
Fricas [F] . . . 450
Sympy [F] . . . 451
Maxima [F] . . . 451
Giac [F] . . . 451
Mupad [F(-1)] . . . 452

Optimal result

Integrand size = 21, antiderivative size = 731

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4} - \frac{32bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} + \frac{8bcd^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4bc(2c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{64bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] -2*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsch(c*
x))/e^4+2*d^3*(a+b*arccsch(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsch(c*x))
*(e*x+d)^(1/2)/e^4+4/15*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1+1/c^2/x^2)
^(1/2)-64/5*b*d^3*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)
```

```
*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(
-c^2)^(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/15*b*c*d
*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^3/x/(
1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+8*b*c*d^2*Ell
ipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c
^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(
1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*c*(2*c^2*d
^2-e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(
c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2
)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 6445, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435, 945, 1598}

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}$$
$$- \frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$
$$- \frac{64bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce^4x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$+ \frac{8bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$- \frac{4bc\sqrt{c^2x^2+1}(2c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$- \frac{32bcd\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} + \frac{4b(c^2x^2+1)\sqrt{d+ex}}{15c^3e^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^3*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsch[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/e^4 + (2*
(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) - (32*b*c*d*Sqrt[d + e*x]*Sqr
t[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]
```

```
*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*c*d^2*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)
^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*(2*c^2*d^2 - e^2
)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[
ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^
2]*e)])/(15*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b
*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)]
)/(5*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{b\int \frac{2\left(16d^3+8d^2ex-2de^2x^2+e^3x^3\right)}{5e^4\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c} \\
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{(2b)\int \frac{16d^3+8d^2ex-2de^2x^2+e^3x^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{5ce^4} \\
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{16d^3+8d^2ex-2de^2x^2+e^3x^3}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(\frac{8d^2e}{\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{16d^3}{x\sqrt{d+ex}\sqrt{1+c^2x^2}} - \frac{2de^2x}{\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{e^3x^2}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)dx}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&\quad + \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{5ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int\frac{x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{5ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^2}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&\quad + \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int\frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}dx}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}dx}{5ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4bd^2\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{5ce^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{ex+2c^2dx^2}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}dx}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&\quad + \frac{32b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(64bd^3\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{e+2c^2dx}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&- \frac{8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b(-2c^2d^2+e^2)\sqrt{1+c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{15c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{\left(64bd^3\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&- \frac{8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{64bd^3\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(8b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{\left(4b\sqrt{-c^2}(-2c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} \\
&- \frac{32b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{8b\sqrt{-c^2}d^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{4b\sqrt{-c^2}(2c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15c^5e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{64bd^3\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 35.09 (sec) , antiderivative size = 1042, normalized size of antiderivative = 1.43

$$
\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(4,-\frac{1}{2}\right)}{e^4(d+ex)^{3/2}}
$$

$$
+ b\left(- \frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(\frac{32cd\sqrt{1+\frac{1}{c^2x^2}}}{15e^3} - \frac{32c^2d^2\operatorname{csch}^{-1}(cx)}{5e^4} + \frac{2c^2d^2\operatorname{csch}^{-1}(cx)}{e^3\left(e+\frac{d}{x}\right)} - \frac{2c^2x^2\operatorname{csch}^{-1}(cx)}{5e^2} - \frac{2cx\left(2e\sqrt{1+\frac{1}{c^2x^2}} - 9cd\operatorname{csch}^{-1}(cx)\right)}{15e^3}\right)}{(d+ex)^{3/2}} - \frac{2\left(e+\right.}{}\right.
$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]

[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
 + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 + 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsch[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsch[c*x])/(e^3*(e + d/x))
```

```
- (2*c^2*x^2*ArcCsch[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 + 1/(c^2*x^2)] - 9*
c*d*ArcCsch[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(
3/2)*(-((Sqrt[2]*(32*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d +
 c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(
e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(48*c^3*d^3 - 8*c*d*
e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (16*c*d*e*Co
sh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*
I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((
e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(
c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellipti
cE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*
EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]
) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt
[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSi
n[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I
 + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2
 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.98 (sec) , antiderivative size = 2021, normalized size of antiderivative = 2.76

method	result	size
derivativedivides	Expression too large to display	2021
default	Expression too large to display	2021
parts	Expression too large to display	2022

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/5*(e*x+d)^(5/2)+(e*x+d)^(3/2)*d-3*d^2*(e*x+d)^(1/2)-d^3/(e*x+
d)^(1/2))-b*(-1/5*arccsch(c*x)*(e*x+d)^(5/2)+arccsch(c*x)*(e*x+d)^(3/2)*d-3
*arccsch(c*x)*d^2*(e*x+d)^(1/2)-arccsch(c*x)*d^3/(e*x+d)^(1/2)-2/15/c^3*(-2
*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(
e*x+d)^(1/2)*e^3-I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*El
lipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d
)^(1/2)*c^2*d^2*e+2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^(3/2)
-8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e
```

```
*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^
(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2))*c^3*d^3+48*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^
2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c
*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2))*c^3*d^3-24*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+32*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d
)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)-48*I*EllipticPi((e*x+d)^(1/2
)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*
d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*e*(e*x+
d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e
*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e-8*(-(I*c*e*(e*x+d)+c^2*d*
(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2+9*(-(
I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+
d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^
(1/2))*c*d*e^2+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^2*e-((c*
d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*
d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/
2)/(I*e-c*d)))
```

Fricas [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^
2), x)
```

Sympy [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x^3(a+b\,\text{acsch}\,(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] 2/5*a*((e*x + d)^(5/2)/e^4 - 5*(e*x + d)^(3/2)*d/e^4 + 15*sqrt(e*x + d)*d^2
/e^4 + 5*d^3/(sqrt(e*x + d)*e^4)) + 1/5*b*(2*(e^3*x^3 - 2*d*e^2*x^2 + 8*d^2
*e*x + 16*d^3)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^4) + 5*integrate
(2/5*(c^2*e^3*x^4 - 2*c^2*d*e^2*x^3 + 8*c^2*d^2*e*x^2 + 16*c^2*d^3*x)/((c^2
*e^4*x^2 + e^4)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^4*x^2 + e^4)*sqrt(
e*x + d)), x) - 5*integrate(-1/5*(2*c^2*d*e^3*x^4 - 48*c^2*d^3*e*x^2 - (5*e
^4*log(c) + 2*e^4)*c^2*x^5 - 32*c^2*d^4*x - (12*c^2*d^2*e^2 + 5*e^4*log(c))
*x^3 - 5*(c^2*e^4*x^5 + e^4*x^3)*log(x))/((c^2*e^5*x^3 + c^2*d*e^4*x^2 + e^
5*x + d*e^4)*sqrt(e*x + d)), x))
```

Giac [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x^3\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.64 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal result . . . 453
Rubi [A] (verified) . . . 454
Mathematica [C] (verified) . . . 459
Maple [C] (verified) . . . 460
Fricas [F(-1)] . . . 461
Sympy [F] . . . 461
Maxima [F] . . . 461
Giac [F] . . . 462
Mupad [F(-1)] . . . 462

Optimal result

Integrand size = 21, antiderivative size = 499

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{4bc\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} - \frac{20bcd\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{32bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2*d^2*(a+b*arccsch(c*x))/e^3/(e*x+
d)^(1/2)-4*d*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+32/3*b*d^2*EllipticPi(1/2
*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(
c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1
+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1
/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)
*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2)-20/3*b*c*d*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2
)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {45, 6445, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435}

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{32bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} - \frac{20bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} + \frac{4bc\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
 b*ArcCsch[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) +
(4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]
*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^
2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
20*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*Ell
ipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - S
qrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) +
 (32*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2
]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d
 + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\ &\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{b\int \frac{2\left(-8d^2-4dex+e^2x^2\right)}{3e^3\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c} \\ &= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\ &\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(2b)\int \frac{-8d^2-4dex+e^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{3ce^3}\end{aligned}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{-8d^2-4dex+e^2x^2}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(-\frac{4de}{\sqrt{d+ex}\sqrt{1+c^2x^2}} - \frac{8d^2}{x\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{1}\right.}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}$$
$$- \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2bd\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(16b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{16b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right.}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(32bd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{\left(4b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{20b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(32bd^2\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$
$$+ \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$- \frac{20b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{32bd^2\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 35.03 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.96

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(3,-\frac{1}{2}\right)}{e^3(d+ex)^{3/2}}$$
$$+ b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(-\frac{4\sqrt{1+\frac{1}{c^2x^2}}}{3e^2}+\frac{16cd\,\text{csch}^{-1}(cx)}{3e^3}-\frac{2cd\,\text{csch}^{-1}(cx)}{e^2\left(e+\frac{d}{x}\right)}-\frac{2cx\,\text{csch}^{-1}(cx)}{3e^2}\right)}{(d+ex)^{3/2}}+\frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx)^{3/2}\left(-\frac{5\sqrt{2}cde\sqrt{1+icx}(i+cx)\sqrt{\frac{cd+}{cd-}}}{\sqrt{1+\frac{1}{c^2x^2}}}\right.\right.$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]

[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2
))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2) + (16*
c*d*ArcCsch[c*x])/(3*e^3) - (2*c*d*ArcCsch[c*x])/(e^2*(e + d/x)) - (2*c*x*A
rcCsch[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*((
-5*Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*
EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(
Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d
```

```
 + e)]) + (I*Sqrt[2]*(c*d - I*e)*(8*c^2*d^2 - e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*
(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sq
rt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x
^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x
)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*
x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*
d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d +
 c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
 - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x
)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x
]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*
d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e)
)]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1
 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2))))/(3*e^3*(d + e*x)^(3
/2))))/c^3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.03 (sec) , antiderivative size = 896, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(}{c^2d}}}{}\right.}{}$
default	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(}{c^2d}}}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)}{e^3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c}{c}}}{}\right.$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e*
x+d)^(3/2)*arccsch(c*x)-2*arccsch(c*x)*d*(e*x+d)^(1/2)-arccsch(c*x)*d^2/(e*
x+d)^(1/2)-2/3/c^2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(5
*I*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c
```

```
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-8*I*EllipticPi((e*x+d)^(1/2)*((c*d+
I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^
2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c*d*e-4*EllipticF((e*x
+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+8*EllipticP
i((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*
c^2*d^2+EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE((e*x+d)^(1/2)*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e
^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x^2(a+b\operatorname{acsch}(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] 2/3*a*((e*x + d)^(3/2)/e^3 - 6*sqrt(e*x + d)*d/e^3 - 3*d^2/(sqrt(e*x + d)*e
^3)) + 1/3*b*(2*(e^2*x^2 - 4*d*e*x - 8*d^2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqr
```

```
t(e*x + d)*e^3) + 3*integrate(2/3*(c^2*e^2*x^3 - 4*c^2*d*e*x^2 - 8*c^2*d^2*
x)/((c^2*e^3*x^2 + e^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^2 + e^
3)*sqrt(e*x + d)), x) - 3*integrate(-1/3*(6*c^2*d*e^2*x^3 - (3*e^3*log(c) +
 2*e^3)*c^2*x^4 + 16*c^2*d^3*x + 3*(8*c^2*d^2*e - e^3*log(c))*x^2 - 3*(c^2*
e^3*x^4 + e^3*x^2)*log(x))/((c^2*e^4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3)*s
qrt(e*x + d)), x))
```

Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x^2\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.65 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 318

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{4bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}\,e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{8bd\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2*d*(a+b*arccsch(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)
/e^2-8*b*d*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*
(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(
1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4*b*c*EllipticF(1/
2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)
))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^
2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {45, 6445, 12, 6853, 6874, 733, 430, 946, 174, 552, 551}

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{8bd\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} + \frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Ar
cCsch[c*x]))/e^2 + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt
[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c
^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqr
t[d + e*x]) - (8*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1
 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sq
rt[-c^2]*d + e)])/(c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c} \\
&= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{(2b)\int \frac{2d+ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce^2} \\
&= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{2d+ex}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(\frac{e}{\sqrt{d+ex}\sqrt{1+c^2x^2}} + \frac{2d}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(8bd\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(8bd\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right.}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{8bd\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), \frac{2e}{\sqrt{-c^2}d+e}\right)}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.03 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.83

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}} + \frac{b(2d+ex)\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}} - \frac{2ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd-ie}}\right.\right.\right.\right.}{c\sqrt{}}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsch[c*x])/Sqrt[d + e
*x] - ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*
d - I*e))]*(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d
 - I*e)/(c*d + I*e)] - 2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*
d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c*Sqrt[-(c/(c*d - I*
e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/e^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.32

method	result
parts	$\frac{2a\left(\sqrt{ex+d}+\frac{d}{\sqrt{ex+d}}\right)}{e^2}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex-}{c^2d^2+e}}\right.}{}$
derivativedivides	$-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsch}(cx)-\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(}{c^2d^2}}\right.}{}$
default	$-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsch}(cx)-\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(}{c^2d^2}}\right.}{}$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/e^2*((e*x+d)^(1/2)+d/(e*x+d)^(1/2))+2*b/e^2*((e*x+d)^(1/2)*arccsch(c*x)
+arccsch(c*x)*d/(e*x+d)^(1/2)+2/c*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^
2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(
2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*((c
*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/
(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)))/((c^2*(e*x+d)^2-2*
c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))
^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x(a+b\,\operatorname{acsch}(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] b*(2*(e*x + 2*d)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2) + integrate
(2*(c^2*e*x^2 + 2*c^2*d*x)/((c^2*e^2*x^2 + e^2)*sqrt(c^2*x^2 + 1)*sqrt(e*x
+ d) + (c^2*e^2*x^2 + e^2)*sqrt(e*x + d)), x) - integrate((6*c^2*d*e*x^2 +
(e^2*log(c) + 2*e^2)*c^2*x^3 + (4*c^2*d^2 + e^2*log(c))*x + (c^2*e^2*x^3 +
e^2*x)*log(x))/((c^2*e^3*x^3 + c^2*d*e^2*x^2 + e^3*x + d*e^2)*sqrt(e*x + d)
), x)) + 2*a*(sqrt(e*x + d)/e^2 + d/(sqrt(e*x + d)*e^2))
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.66 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

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Maxima [F] . . . 475
Giac [F] . . . 476
Mupad [F(-1)] . . . 476

Optimal result

Integrand size = 18, antiderivative size = 149

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arccsch(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(1-x*(-c^2)^(1/2))
^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((
e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6425, 1588, 947, 174, 552, 551}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCsch[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*
x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*
x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
```

```
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} \\
&\quad + \frac{\left(4b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} \\
&\quad + \frac{\left(4b\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} \\
&\quad + \frac{4b\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right), \frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \frac{-2e(1+c^2x^2)\left(a+b\text{csch}^{-1}(cx)\right)+2bc(icd+e)\sqrt{2+\frac{2}{c^2x^2}}x\sqrt{1+icx}\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}$$
$$e^2\sqrt{d+ex}\,(1+c^2x^2)$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*e*(1 + c^2*x^2)*(a + b*ArcCsch[c*x]) + 2*b*c*(I*c*d + e)*Sqrt[2 + 2/(c^
2*x^2)]*x*Sqrt[1 + I*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*Ell
ipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d +
 e)/(2*e)])/(e^2*Sqrt[d + e*x]*(1 + c^2*x^2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.20

method	result
derivativedivides	$-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\text{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\,\text{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(cd+$ / $c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2+e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}$ / e
default	$-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\text{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\,\text{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(cd+$ / $c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2+e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}$ / e
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+2b\left(-\frac{\text{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\,\text{EllipticPi}\left(\sqrt{ex+d}\right.\right.$ / $c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2+e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}$ / e

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsch(c*x)+2/c/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/d/((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2)*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)
*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi
```

```
((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2
)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a + b\,\text{acsch}\,(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(3/2),x)
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] -(2*c^2*integrate(x/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2
*e*x^2 + e)*sqrt(e*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d
)*e) + integrate(((e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + e*log(c) + (c^2*e*
x^2 + e)*log(x))/((c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt(e*x + d)),
 x))*b - 2*a/(sqrt(e*x + d)*e)
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

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Maple [N/A] (verified) 478
Fricas [N/A] 478
Sympy [N/A] 479
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Giac [N/A] 479
Mupad [N/A] 480

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x\left(ex+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x),
x)
```

Sympy [N/A]

Not integrable

Time = 32.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x\,(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 8.52

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*((e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2) + 2
*e/(sqrt(e*x + d)*d))*log(c)/e + integrate(log(x)/(sqrt(e*x + d)*e*x^2 + sq
rt(e*x + d)*d*x), x) - integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*
e*x^2 + sqrt(e*x + d)*d*x), x)) + a*(log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*
x + d) + sqrt(d)))/d^(3/2) + 2/(sqrt(e*x + d)*d))
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

3.68 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

Optimal result 481
Rubi [N/A] 481
Mathematica [N/A] 482
Maple [N/A] (verified) 482
Fricas [N/A] 482
Sympy [N/A] 483
Maxima [N/A] 483
Giac [N/A] 483
Mupad [N/A] 484

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x^2\left(ex+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2)
, x)
```

Sympy [N/A]

Not integrable

Time = 59.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x^2\,(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 252, normalized size of antiderivative = 12.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*((3*e^2*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5
/2) + 2*(3*(e*x + d)*e^2 - 2*d*e^2)/((e*x + d)^(3/2)*d^2 - sqrt(e*x + d)*d^
3))*log(c)/e - 2*integrate(log(x)/(sqrt(e*x + d)*e*x^3 + sqrt(e*x + d)*d*x^
2), x) + 2*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e*x^3 + sqrt
(e*x + d)*d*x^2), x)) - 1/2*a*(2*(3*(e*x + d)*e - 2*d*e)/((e*x + d)^(3/2)*d
^2 - sqrt(e*x + d)*d^3) + 3*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d)
+ sqrt(d)))/d^(5/2))
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x^2\,(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

3.69 $\int \frac{x^3\left(a+b\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal result . . . 485
Rubi [A] (verified) . . . 486
Mathematica [C] (warning: unable to verify) . . . 496
Maple [C] (verified) . . . 497
Fricas [F(-1)] . . . 499
Sympy [F] . . . 499
Maxima [F] . . . 499
Giac [F] . . . 500
Mupad [F(-1)] . . . 500

Optimal result

Integrand size = 21, antiderivative size = 777

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+\frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}$$
$$-\frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}$$
$$-\frac{8b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+\frac{4bc(2c^2d^2+e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$-\frac{32bcd\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+\frac{64bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/3*d^3*(a+b*arccsch(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsch
(c*x))/e^4-6*d^2*(a+b*arccsch(c*x))/e^4/(e*x+d)^(1/2)+4/3*b*d^2*(c^2*x^2+1)
/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsch(c
*x))*(e*x+d)^(1/2)/e^4+64/3*b*d^2*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2
```

```
^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(
-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(
1/2)+4/3*b*c*(2*c^2*d^2+e^2)*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2)
,(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1
)^(1/2)/(-c^2)^(3/2)/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(
c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/3*b*d^2*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/
2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(
e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c
^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-32/3*b*c*d*EllipticF(1/2*(1-x*(-c^
2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(
c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^
3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {45, 6445, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430, 1665}

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}$$
$$- \frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$
$$+ \frac{64bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$- \frac{8b\sqrt{-c^2}d^2\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{4bc\sqrt{c^2x^2+1}(2c^2d^2+e^2)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$- \frac{32bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$+ \frac{4bd^2(c^2x^2+1)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*d^2*(1 + c^2*x^2))/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
rt[d + e*x]) + (2*d^3*(a + b*ArcCsch[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*Arc
Csch[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^4) - (8*b*S
qrt[-c^2]*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*e^3*(c
^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c
^2]*e)]) + (4*b*c*(2*c^2*d^2 + e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[
-c^2]*e)])/(3*(-c^2)^(3/2)*e^3*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (32*b*c*d*Sqrt[(c^2*(d + e*x))/
(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)
*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (64*b*d^2*Sqrt[(Sqrt[-c^2]*(d
 + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^4*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} + \frac{b\int \frac{2\left(-16d^3-24d^2ex-6de^2x^2+e^3x^3\right)}{3e^4\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c} \\
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} + \frac{(2b)\int \frac{-16d^3-24d^2ex-6de^2x^2+e^3x^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^4} \\
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{-16d^3-24d^2ex-6de^2x^2+e^3x^3}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} \\
&\quad - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(-\frac{24d^2e}{(d+ex)^{3/2}\sqrt{1+c^2x^2}} - \frac{16d^3}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}} - \frac{6de^2x}{(d+ex)^{3/2}\sqrt{1+c^2x^2}} + \frac{e^3x^2}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\right)dx}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} - \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{x}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{68bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} \\
&\quad - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} \\
&\quad - \frac{\left(32bd^3\sqrt{1+c^2x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1+c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)\,dx}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32bcd^2\sqrt{1+c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int \frac{-\frac{e}{2}+\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{\frac{d}{2}-\frac{\left(2c^2d^2+e^2\right)x}{2e}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{68bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} \\
&\quad - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} - \frac{\left(32bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(32bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4bcd^2\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(16bcd^2\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2b\left(1+\frac{2c^2d^2}{e^2}\right)\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4b\left(\frac{de}{2}+\frac{d\left(2c^2d^2+e^2\right)}{2e}\right)\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} \\
&- \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} \\
&- \frac{\left(32bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&- \frac{\left(64bcd^2\sqrt{1+c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(8b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{\left(32b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{\left(4b\sqrt{-c^2}\left(1+\frac{2c^2d^2}{e^2}\right)\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3c^3e\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{\left(8b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(8b\sqrt{-c^2}\left(\frac{de}{2}+\frac{d\left(2c^2d^2+e^2\right)}{2e}\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}}}{\sqrt{2}}\right.}{3c^3e^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} \\
&\quad - \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} \\
&\quad - \frac{24b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{4b\sqrt{-c^2}(2c^2d^2+e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{32b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(64bd^2\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right)}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32bcd^2\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{3e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} \\
&- \frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4} \\
&- \frac{24b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{4b\sqrt{-c^2}(2c^2d^2+e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{32b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{\left(64b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{\left(64bd^2\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right.}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}$$
$$- \frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$
$$- \frac{8b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{4b\sqrt{-c^2}(2c^2d^2+e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$- \frac{32b\sqrt{-c^2}d\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3c^3e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{64bd^2\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.88 (sec) , antiderivative size = 1108, normalized size of antiderivative = 1.43

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(4,-\frac{3}{2}\right)}{e^4(d+ex)^{5/2}}$$
$$+ \frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{4\sqrt{1+\frac{1}{c^2x^2}}}{3e\left(c^2d^2+e^2\right)}+\frac{32cd\text{csch}^{-1}(cx)}{3e^4}-\frac{2cd\text{csch}^{-1}(cx)}{3e^2\left(e+\frac{d}{x}\right)^2}-\frac{2cx\text{csch}^{-1}(cx)}{3e^3}-\frac{2\left(2c^2d^2e\sqrt{1+\frac{1}{c^2x^2}}+7c^3d^3\text{csch}^{-1}(cx)+7cde^2\text{csch}^{-1}(\right.}{3e^3\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right.}{(d+ex)^{5/2}}\right.}{}$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
 + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e*(c^2*d^2 + e
^2)) + (32*c*d*ArcCsch[c*x])/(3*e^4) - (2*c*d*ArcCsch[c*x])/(3*e^2*(e + d/x
)^2) - (2*c*x*ArcCsch[c*x])/(3*e^3) - (2*(2*c^2*d^2*e*Sqrt[1 + 1/(c^2*x^2)]
 + 7*c^3*d^3*ArcCsch[c*x] + 7*c*d*e^2*ArcCsch[c*x]))/(3*e^3*(c^2*d^2 + e^2)
*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]
*(8*c^3*d^3*e + 8*c*d*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*
d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*
x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(16*c^4*d^4 + 16*c^2*d^2*e^2 -
e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e^3*Cosh[
2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*
c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[
ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*Ell
ipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) +
 (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e
*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[S
qrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I +
c*x))/(c*d - I*e))]))))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 +
c^2*x^2))))/(3*e^4*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.29 (sec) , antiderivative size = 2728, normalized size of antiderivative = 3.51

method	result	size
derivativedivides	Expression too large to display	2728
default	Expression too large to display	2728
parts	Expression too large to display	2729

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/3*(e*x+d)^(3/2)+3*d*(e*x+d)^(1/2)-1/3*d^3/(e*x+d)^(3/2)+3*d^2
/(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsch(c*x)+3*arccsch(c*x)*d*(e*x+d)
^(1/2)-1/3*arccsch(c*x)*d^3/(e*x+d)^(3/2)+3*arccsch(c*x)*d^2/(e*x+d)^(1/2)-
2/3/c^2*(-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1
/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipti
cF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2))*c*d*e^3*(e*x+d)^(1/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))
```

```
^(1/2)*c^3*d^4*e-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e
^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*
EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2)*c^4*d^3*(e*x+d)^2+8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^
2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2
*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4*(e*x+d)^(1/2)-16*(-(I*c
*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((
c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c
/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4*(e*x+d)^(1
/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*e*(e*x+d)^2+2*((c*d+I*e)*c/
(c^2*d^2+e^2))^(1/2)*c^4*d^4*(e*x+d)+16*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^
2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(
1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2))*c*d*e^3*(e*x+d)^(1/2)+I*((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2)*c*d^2*e^3+16*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^
2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*
e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2))*c^3*d^3*e*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c
^4*d^5+7*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e
*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2))*c^2*d^2*e^2*(e*x+d)^(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)
-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))
^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2*(e*x+d)^
(1/2)-16*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((
e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/
d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2
*d^2*e^2*(e*x+d)^(1/2)-2*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e*(e*x
+d)-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e^2-(-(I*c*e*(e*x+d)+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4*(e*x+d)^(1/
2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e
*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^
(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2))*e^4*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2
)/c^2/e^2/x^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^
2))^(1/2)/(I*e-c*d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{x^3(a+b\,\operatorname{acsch}(cx))}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*b*(2*(e^3*x^3 - 6*d*e^2*x^2 - 24*d^2*e*x - 16*d^3)*log(sqrt(c^2*x^2 + 1
) + 1)/((e^5*x + d*e^4)*sqrt(e*x + d)) + 3*integrate(2/3*(c^2*e^3*x^4 - 6*c
^2*d*e^2*x^3 - 24*c^2*d^2*e*x^2 - 16*c^2*d^3*x)/((c^2*e^5*x^3 + c^2*d*e^4*x
^2 + e^5*x + d*e^4)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^5*x^3 + c^2*d*
e^4*x^2 + e^5*x + d*e^4)*sqrt(e*x + d)), x) - 3*integrate(-1/3*(10*c^2*d*e^
3*x^4 + 80*c^2*d^3*e*x^2 - (3*e^4*log(c) + 2*e^4)*c^2*x^5 + 32*c^2*d^4*x +
3*(20*c^2*d^2*e^2 - e^4*log(c))*x^3 - 3*(c^2*e^4*x^5 + e^4*x^3)*log(x))/((c
^2*e^6*x^4 + 2*c^2*d*e^5*x^3 + 2*d*e^5*x + d^2*e^4 + (c^2*d^2*e^4 + e^6)*x^
2)*sqrt(e*x + d)), x)) + 2/3*a*((e*x + d)^(3/2)/e^4 - 9*sqrt(e*x + d)*d/e^4
 - 9*d^2/(sqrt(e*x + d)*e^4) + d^3/((e*x + d)^(3/2)*e^4))
```

Giac [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{x^3\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.70 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal result . . . 501
Rubi [A] (verified) . . . 502
Mathematica [C] (warning: unable to verify) . . . 509
Maple [C] (verified) . . . 510
Fricas [F] . . . 511
Sympy [F] . . . 512
Maxima [F] . . . 512
Giac [F] . . . 512
Mupad [F(-1)] . . . 513

Optimal result

Integrand size = 21, antiderivative size = 569

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+\frac{4b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+\frac{4bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{32bd\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] -2/3*d^2*(a+b*arccsch(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsch(c*x))/e^3/(e
*x+d)^(1/2)-4/3*b*d*(c^2*x^2+1)/c/e/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)+2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3-32/3*b*d*EllipticPi(1/2*(
1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^
2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1
/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*d*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(e
*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c^
2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+4*b*c*EllipticF(1/2*(1-x*(-c^2)^(1/
```

```
2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^
2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1
+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {45, 6445, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430}

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}$$
$$+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$
$$+\frac{4b\sqrt{-c^2}d\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$-\frac{32bd\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$+\frac{4bc\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$-\frac{4bd(c^2x^2+1)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*d*(1 + c^2*x^2))/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a
+ b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*
x]))/e^3 + (4*b*Sqrt[-c^2]*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e
)])/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(
c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
```

```
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} \\
&\quad + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{b\int \frac{2\left(8d^2+12dex+3e^2x^2\right)}{3e^3\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c} \\
&= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} \\
&\quad + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{(2b)\int \frac{8d^2+12dex+3e^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^3}
\end{aligned}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}$$
$$+ \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{8d^2+12dex+3e^2x^2}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(\frac{12de}{(d+ex)^{3/2}\sqrt{1+c^2x^2}} + \frac{8d^2}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}} + \frac{3e^2x}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\right)dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}$$
$$+ \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(8bd\sqrt{1+c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{12bd(1+c^2x^2)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}$$
$$+ \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{\left(16bd^2\sqrt{1+c^2x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1+c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(16bcd\sqrt{1+c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{e^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{-\frac{e}{2}+\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$
\begin{aligned}
&= -\frac{12bd(1+c^2x^2)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} \\
&\quad + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(16bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(16bd\sqrt{1+c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2bcd\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{e^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(8bcd\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{e^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} \\
&\quad + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{\left(16bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(32bcd\sqrt{1+c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3e^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(16b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{\left(4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} \\
&\quad + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(32bd\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(16bcd\sqrt{1+c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{3e^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} \\
&\quad + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{12b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(32b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad - \frac{\left(32bd\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right.}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} \\
&\quad + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{4b\sqrt{-c^2}d\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{32bd\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 35.05 (sec) , antiderivative size = 1076, normalized size of antiderivative = 1.89

$$
\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(3,-\frac{3}{2}\right)}{e^3(d+ex)^{5/2}}
$$

$$
+ b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{\frac{4cd\sqrt{1+\frac{1}{c^2x^2}}}{3e^2\left(c^2d^2+e^2\right)}-\frac{16\operatorname{csch}^{-1}(cx)}{3e^3}+\frac{2\operatorname{csch}^{-1}(cx)}{3e\left(e+\frac{d}{x}\right)^2}+\frac{4\left(cde\sqrt{1+\frac{1}{c^2x^2}}+2c^2d^2\operatorname{csch}^{-1}(cx)+2e^2\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}} - \frac{2\left(e+\frac{d}{x}\right)^{5/2}(
$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]

[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2
))) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2*(c^
2*d^2 + e^2)) - (16*ArcCsch[c*x])/(3*e^3) + (2*ArcCsch[c*x])/(3*e*(e + d/x)
^2) + (4*(c*d*e*Sqrt[1 + 1/(c^2*x^2)] + 2*c^2*d^2*ArcCsch[c*x] + 2*e^2*ArcC
sch[c*x]))/(3*e^2*(c^2*d^2 + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e + d
/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]*(3*c^2*d^2*e + 3*e^3)*Sqrt[1 + I*c*x]*(I
```

```
+ c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x)
)/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*
(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(8
*c^3*d^3 + 9*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c
*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e
))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2
)) - (2*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(
c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF
[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-(
(e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*
d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*
d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I
*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c
*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
(I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))
/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(2 + c^2*x^2))))/(3*e^3*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c^
3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.88 (sec) , antiderivative size = 2492, normalized size of antiderivative = 4.38

method	result	size
derivativedivides	Expression too large to display	2492
default	Expression too large to display	2492
parts	Expression too large to display	2500

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2)+2*d/(e*x+d)^(1/2))+b*((e*x+d)
^(1/2)*arccsch(c*x)-1/3*arccsch(c*x)*d^2/(e*x+d)^(3/2)+2*arccsch(c*x)*d/(e*
x+d)^(1/2)+2/3/c*(-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*
(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^
2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)-4*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(
-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)+(-(I*c
*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c
*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
```

```
2))*c^3*d^3*(e*x+d)^(1/2)-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(
c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+
I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2
*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1/2)+8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)
,1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2)*d*e^3+((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^2+3*I*(-(I
*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d
)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2))*e^3*(e*x+d)^(1/2)-2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)
-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e-4*(-(I*c*e*(e*x+d)+c^2*d*(e*
x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^
(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*
c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+
d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c
^2*d*e*(e*x+d)^2+8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*El
lipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2
*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^
(1/2))*c*d*e^2*(e*x+d)^(1/2)+3*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)+((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4+2*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2
*d^2*e*(e*x+d)+((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^2)/((c^2*(e*x+d)^2
-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1
/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2
+ 3*d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{x^2(a+b\,\text{acsch}\,(cx))}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*b*(2*(3*e^2*x^2 + 12*d*e*x + 8*d^2)*log(sqrt(c^2*x^2 + 1) + 1)/((e^4*x
+ d*e^3)*sqrt(e*x + d)) + 3*integrate(2/3*(3*c^2*e^2*x^3 + 12*c^2*d*e*x^2 +
 8*c^2*d^2*x)/((c^2*e^4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3)*sqrt(c^2*x^2 +
 1)*sqrt(e*x + d) + (c^2*e^4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3)*sqrt(e*x
+ d)), x) - 3*integrate(1/3*(30*c^2*d*e^2*x^3 + 3*(e^3*log(c) + 2*e^3)*c^2*
x^4 + 16*c^2*d^3*x + (40*c^2*d^2*e + 3*e^3*log(c))*x^2 + 3*(c^2*e^3*x^4 + e
^3*x^2)*log(x))/((c^2*e^5*x^4 + 2*c^2*d*e^4*x^3 + 2*d*e^4*x + d^2*e^3 + (c^
2*d^2*e^3 + e^5)*x^2)*sqrt(e*x + d)), x)) + 2/3*a*(3*sqrt(e*x + d)/e^3 + 6*
d/(sqrt(e*x + d)*e^3) - d^2/((e*x + d)^(3/2)*e^3))
```

Giac [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{x^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.71 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal result . . . 514
Rubi [A] (verified) . . . 515
Mathematica [C] (verified) . . . 520
Maple [C] (verified) . . . 521
Fricas [F] . . . 522
Sympy [F] . . . 522
Maxima [F] . . . 522
Giac [F] . . . 523
Mupad [F(-1)] . . . 523

Optimal result

Integrand size = 19, antiderivative size = 393

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$
$$-\frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+\frac{8b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/3*d*(a+b*arccsch(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsch(c*x))/e^2/(e*x+d)
^(1/2)+4/3*b*(c^2*x^2+1)/c/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2
)+8/3*b*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c
^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2
)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(1/2*(
1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^
(1/2))*(-c^2)^(1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e/(c^2*d^2+e^2)/x/(1+
1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {45, 6445, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551}

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} + \frac{8b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} + \frac{4b(c^2x^2+1)}{3cx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*(1 + c^2*x^2))/(3*c*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x]) + (2*d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
ch[c*x]))/(e^2*Sqrt[d + e*x]) - (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c
^2*d - Sqrt[-c^2]*e)])/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{2d\big(a+b\operatorname{csch}^{-1}(cx)\big)}{3e^2(d+ex)^{3/2}} - \frac{2\big(a+b\operatorname{csch}^{-1}(cx)\big)}{e^2\sqrt{d+ex}} + \frac{b\int \frac{2(-2d-3ex)}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c}$$

$$= \frac{2d\big(a+b\operatorname{csch}^{-1}(cx)\big)}{3e^2(d+ex)^{3/2}} - \frac{2\big(a+b\operatorname{csch}^{-1}(cx)\big)}{e^2\sqrt{d+ex}} + \frac{(2b)\int \frac{-2d-3ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^2}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1+c^2x^2}} - \frac{2d}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\right)\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b(1+c^2x^2)}{c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1+c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1+c^2x^2}}\right)\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4bc\sqrt{1+c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{e\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b(1+c^2x^2)}{c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1+c^2x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{e\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$
$$- \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(8bc\sqrt{1+c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1+c^2x^2}}\,dx}{3e\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x\right.}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

$$= \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$
$$- \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{\left(8b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(4bc\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1+c^2x^2}}\,dx}{3e\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$
$$- \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{\left(8b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2d-\sqrt{-c^2}e}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{\left(8b\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$
$$- \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{8b\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.93 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.99

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2}{3}\left(\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}x}{\left(c^2d^2+e^2\right)\sqrt{d+ex}} - \frac{a(2d+3ex)}{e^2(d+ex)^{3/2}} - \frac{b(2d+3ex)\operatorname{csch}^{-1}(cx)}{e^2(d+ex)^{3/2}}\right.$$
$$\left. + \frac{2ib\sqrt{-\frac{c}{cd-ie}}\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(cdE\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right) - cd\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\right.\right.\right.\right.}}{c^2de^2\sqrt{1+\frac{1}{c^2x^2}}}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/((c^2*d^2 + e^2)*Sqrt[d + e*x]) - (a*(2
*d + 3*e*x))/(e^2*(d + e*x)^(3/2)) - (b*(2*d + 3*e*x)*ArcCsch[c*x])/(e^2*(d
 + e*x)^(3/2)) + ((2*I)*b*Sqrt[-(c/(c*d - I*e))]*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(c*d*EllipticE[I*ArcSinh[Sqrt[
-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - c*d*EllipticF[
I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] +
 2*(c*d - I*e)*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]
*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^2*d*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x)))/3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.47 (sec) , antiderivative size = 2106, normalized size of antiderivative = 5.36

method	result	size
derivativedivides	Expression too large to display	2106
default	Expression too large to display	2106
parts	Expression too large to display	2110

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(1/(e*x+d)^(1/2)-1/3*d/(e*x+d)^(3/2))-b*(1/(e*x+d)^(1/2)*arccsch(
c*x)-1/3*arccsch(c*x)*d/(e*x+d)^(3/2)-2/3/c*(I*((c*d+I*e)*c/(c^2*d^2+e^2))^
(1/2)*(e*x+d)^2*c^2*d*e+2*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^
2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*
e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*
d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2
),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-(-(
I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+
d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^
(1/2))*c^3*d^3*(e*x+d)^(1/2)-2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/
(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^
2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d
+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*
c^3*d^2*(e*x+d)^2+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e+2*((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2
)*d*e^3+2*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2
)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticP
i((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*
e^3*(e*x+d)^(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2)
)^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ell
ipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2
+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(-(I*c*e*(e*x+d)+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)
^(1/2)-2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((
```

```
e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/
d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c*d
*e^2*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4-2*I*((c*d+I*e)
*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^2*d^2*e-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)
*c*d^2*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)
/x/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)/
d))
```

Fricas [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*
d^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{x(a+b\operatorname{acsch}(cx))}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b*(2*(3*e*x + 2*d)*log(sqrt(c^2*x^2 + 1) + 1)/((e^3*x + d*e^2)*sqrt(e*
x + d)) + 3*integrate(2/3*(3*c^2*e*x^2 + 2*c^2*d*x)/((c^2*e^3*x^3 + c^2*d*e
^2*x^2 + e^3*x + d*e^2)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^3 + c^
2*d*e^2*x^2 + e^3*x + d*e^2)*sqrt(e*x + d)), x) + 3*integrate(-1/3*(10*c^2*
d*e*x^2 - 3*(e^2*log(c) - 2*e^2)*c^2*x^3 + (4*c^2*d^2 - 3*e^2*log(c))*x - 3
*(c^2*e^2*x^3 + e^2*x)*log(x))/((c^2*e^4*x^4 + 2*c^2*d*e^3*x^3 + 2*d*e^3*x
+ d^2*e^2 + (c^2*d^2*e^2 + e^4)*x^2)*sqrt(e*x + d)), x)) - 2/3*a*(3/(sqrt(e
*x + d)*e^2) - d/((e*x + d)^(3/2)*e^2))
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \int \frac{x\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.72 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal result . . . 524
Rubi [A] (verified) . . . 525
Mathematica [C] (verified) . . . 529
Maple [C] (verified) . . . 530
Fricas [F] . . . 531
Sympy [F] . . . 531
Maxima [F] . . . 531
Giac [F] . . . 532
Mupad [F(-1)] . . . 532

Optimal result

Integrand size = 18, antiderivative size = 369

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$
$$+ \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$
$$+ \frac{4b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3cde\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] -2/3*(a+b*arccsch(c*x))/e/(e*x+d)^(3/2)-4/3*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e^
2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*EllipticE(1/2*(1-x*(-c^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(
1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2
)/((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)+4/3*b*EllipticPi(1/2*(1-x*(-c^2)^(1/2)
)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*(
(e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/d/e/x/(1+1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6425, 1588, 972, 759, 21, 733, 435, 947, 174, 552, 551}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}} + \frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3cdex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} - \frac{4be(c^2x^2+1)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*e*(1 + c^2*x^2))/(3*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(3*e*(d + e*x)^(3/2)) + (4*b*Sqrt[-c^2
]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/S
qrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*d*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*Sqrt[(Sq
rt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*d*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right)\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3cde\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}-\frac{\left(4bc\sqrt{\frac{1}{c^2}+x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3d\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$-\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{3cde\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$
$$+\frac{\left(2bc\sqrt{\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3d\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{\left(4b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx,x,\sqrt{1-\sqrt{}\right.}{3cde\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$
$$+\frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}}$$
$$+\frac{\left(4b\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx,x,\sqrt{1-\sqrt{-c^2}x}\right)}{3cde\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$
$$+\frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$
$$+\frac{4b\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3cde\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.43 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.42

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2a}{3e(d+ex)^{3/2}}$$

$$+\frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3 x^3\left(-\frac{\sqrt[4]{1+\frac{1}{c^2x^2}}}{3cd\left(c^2d^2+e^2\right)}+\frac{2\text{csch}^{-1}(cx)}{3c^2d^2e}+\frac{2e\text{csch}^{-1}(cx)}{3c^2d^2\left(e+\frac{d}{x}\right)^2}-\frac{4\left(-cde\sqrt{1+\frac{1}{c^2x^2}}+c^2d^2\text{csch}^{-1}(cx)+e^2\text{csch}^{-1}(cx)\right)}{3c^2d^2\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}}+\frac{2\left(e+\frac{d}{x}\right)^{5/2}(}{}\right.}{}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]

[Out] (-2*a)/(3*e*(d + e*x)^(3/2)) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/(
c^2*x^2)])/(3*c*d*(c^2*d^2 + e^2)) + (2*ArcCsch[c*x])/(3*c^2*d^2*e) + (2*e*
ArcCsch[c*x])/(3*c^2*d^2*(e + d/x)^2) - (4*(-(c*d*e*Sqrt[1 + 1/(c^2*x^2)])
+ c^2*d^2*ArcCsch[c*x] + e^2*ArcCsch[c*x]))/(3*c^2*d^2*(c^2*d^2 + e^2)*(e +
 d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((I*Sqrt[2]*c*d*
(c*d - I*e)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]
*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c
*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e*
Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (
2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))
/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellip
ticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*
e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*
(I + c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2))))/(3*e*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.24 (sec) , antiderivative size = 2079, normalized size of antiderivative = 5.63

method	result	size
derivativedivides	Expression too large to display	2079
default	Expression too large to display	2079
parts	Expression too large to display	2081

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsch(c*x)-2/3/c*(I*((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2)*d*e^3-I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),
1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)
*c/(c^2*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2
)*c^3*d^2*(e*x+d)^2+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^
2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*E
llipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-(-(I*c*e*(e*x+d)+c^2*d*
(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+
d)^(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((
e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/
d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^3
*d^3*(e*x+d)^(1/2)-I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e
^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*
EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c
^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)+2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^
3*(e*x+d)-2*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^2*d^2*e+I*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*
d^4+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*
e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)
^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-
(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)+(-(I*c*
e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c
^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c
```

```
*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/
(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/
2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^2*d*e-((c*d+I*e)*c/(c^2*
d^2+e^2))^(1/2)*c*d^2*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2
/e^2/x^2)^(1/2)/x/d^2/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2)/(I*e-c*d)))
```

Fricas [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*
e*x + d^3), x)
```

Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a + b\,\text{acsch}\,(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*c^2*integrate(1/3*x/((c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt
(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqr
t(e*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((e^2*x + d*e)*sqrt(e*x + d)
) + 3*integrate(1/3*((3*e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + 3*e*log(c) +
3*(c^2*e*x^2 + e)*log(x))/((c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2
*e + (c^2*d^2*e + e^3)*x^2)*sqrt(e*x + d)), x))*b - 2/3*a/((e*x + d)^(3/2)*
e)
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{(d+e\,x)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.73 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

Optimal result 533
Rubi [N/A] 533
Mathematica [N/A] 534
Maple [N/A] (verified) 534
Fricas [N/A] 534
Sympy [N/A] 535
Maxima [N/A] 535
Giac [N/A] 535
Mupad [N/A] 536

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 38.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x\left(ex+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*
e*x^2 + d^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 162.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x\,(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 239, normalized size of antiderivative = 11.38

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b*((3*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5/
2) + 2*(3*(e*x + d)*e + d*e)/((e*x + d)^(3/2)*d^2))*log(c)/e + 3*integrate(
log(x)/(sqrt(e*x + d)*e^2*x^3 + 2*sqrt(e*x + d)*d*e*x^2 + sqrt(e*x + d)*d^2
*x), x) - 3*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2*x^3 + 2
*sqrt(e*x + d)*d*e*x^2 + sqrt(e*x + d)*d^2*x), x)) + 1/3*a*(3*log((sqrt(e*x
 + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5/2) + 2*(3*e*x + 4*d)/((e*x
 + d)^(3/2)*d^2))
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

3.74 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

Optimal result . . . 537
Rubi [N/A] . . . 537
Mathematica [N/A] . . . 538
Maple [N/A] (verified) . . . 538
Fricas [N/A] . . . 538
Sympy [F(-1)] . . . 539
Maxima [N/A] . . . 539
Giac [N/A] . . . 539
Mupad [N/A] . . . 540

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 34.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x^2\left(ex+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*
e*x^3 + d^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(5/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 316, normalized size of antiderivative = 15.05

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] 1/6*b*((2*(15*(e*x + d)^2*e^2 - 10*(e*x + d)*d*e^2 - 2*d^2*e^2)/((e*x + d)^
(5/2)*d^3 - (e*x + d)^(3/2)*d^4) + 15*e^2*log((sqrt(e*x + d) - sqrt(d))/(sq
rt(e*x + d) + sqrt(d)))/d^(7/2))*log(c)/e - 6*integrate(log(x)/(sqrt(e*x +
d)*e^2*x^4 + 2*sqrt(e*x + d)*d*e*x^3 + sqrt(e*x + d)*d^2*x^2), x) + 6*integ
rate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2*x^4 + 2*sqrt(e*x + d)*d*
e*x^3 + sqrt(e*x + d)*d^2*x^2), x)) - 1/6*a*(2*(15*(e*x + d)^2*e - 10*(e*x
+ d)*d*e - 2*d^2*e)/((e*x + d)^(5/2)*d^3 - (e*x + d)^(3/2)*d^4) + 15*e*log(
(sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(7/2))
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{cx}\right)}{x^2\,(d+e\,x)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

3.75 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal result . . . 541
Rubi [A] (verified) . . . 542
Mathematica [C] (warning: unable to verify) . . . 549
Maple [C] (verified) . . . 550
Fricas [F(-1)] . . . 552
Sympy [F(-1)] . . . 552
Maxima [F] . . . 553
Giac [F] . . . 553
Mupad [F(-1)] . . . 553

Optimal result

Integrand size = 18, antiderivative size = 648

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}}$$
$$-\frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$
$$-\frac{4bc(7c^2d^2+3e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2\sqrt{-c^2}e}{-c^2d+\sqrt{-c^2}e}\right)}{15\sqrt{-c^2}d^2\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$
$$-\frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+\frac{4b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] -2/5*(a+b*arccsch(c*x))/e/(e*x+d)^(5/2)-4/15*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e
^2)/x/(e*x+d)^(3/2)/(1+1/c^2/x^2)^(1/2)-16/15*b*c*e*(c^2*x^2+1)/(c^2*d^2+e^
2)^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*e*(c^2*x^2+1)/c/d^2/(c^2*d^2
+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*c*(7*c^2*d^2+3*e^2)*Ellipt
icE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(e*(-c^2)^(1/2)/(-c^2*d+e*
(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d^2+e^2)^2/x
```

```
/(-c^2)^(1/2)/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)-4/15*b
*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*((e*x+d)/(d+e/(-c^2)^
(1/2)))^(1/2)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*E
llipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)
+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/
2)/c/d^2/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6425, 1588, 972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$
$$-\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$
$$+\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}$$
$$+\frac{16bc\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)^2\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}$$
$$+\frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cd^2ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$
$$-\frac{4be(c^2x^2+1)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}-\frac{16bce(c^2x^2+1)}{15x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)^2\sqrt{d+ex}}$$
$$-\frac{4be(c^2x^2+1)}{15cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)(d+ex)^{3/2}}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-4*b*e*(1 + c^2*x^2))/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
 e*x)^(3/2)) - (16*b*c*e*(1 + c^2*x^2))/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*e*(1 + c^2*x^2))/(5*c*d^2*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(5*e*(d +
 e*x)^(5/2)) + (16*b*c*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE
[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c
```

```
^2]*e)])/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d +
e/Sqrt[-c^2])]) + (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE
[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c
^2]*e)])/(5*c*d^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d
 + e/Sqrt[-c^2])]) - (4*b*Sqrt[-c^2]*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]*Sqr
t[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]
*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2
*e)/(Sqrt[-c^2]*d + e)])/(5*c*d^2*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
```

```
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}\,dx}{5ce} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\
&\quad - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}} + \frac{1}{d^2x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right)dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5cd\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} \\
&\quad - \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\
&\quad - \frac{\left(4bc\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5d^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{3d}{2}+\frac{ex}{2}}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2+\frac{e^2}{c^2}\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}x}\sqrt{1+\sqrt{-c^2}x}\sqrt{d+ex}}\,dx}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\
&\quad + \frac{\left(2bc\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5d^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(8bc\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\frac{1}{4}\left(3d^2-\frac{e^2}{c^2}\right)+dex}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15d\left(c^2d^2+e^2\right)\left(d^2+\frac{e^2}{c^2}\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{\sqrt{-c^2}}-\frac{ex^2}{\sqrt{-c^2}}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\
&\quad - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2+\frac{e^2}{c^2}\right)\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(8bc\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{15\left(c^2d^2+e^2\right)\left(d^2+\frac{e^2}{c^2}\right)\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}} \\
&\quad + \frac{\left(4b\sqrt{1+c^2x^2}\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{\sqrt{-c^2}\left(d+\frac{e}{\sqrt{-c^2}}\right)}}}\,dx, x, \sqrt{1-\sqrt{-c^2}x}\right)}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&\quad + \frac{4b\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(16b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c\left(c^2d^2+e^2\right)\left(d^2+\frac{e^2}{c^2}\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}} \\
&\quad - \frac{\left(4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d-\frac{\sqrt{-c^2}e}{c^2}}}\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-c^2}ex^2}{c^2\left(d-\frac{\sqrt{-c^2}e}{c^2}\right)}}}dx,x,\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)}{15c^3d\left(d^2+\frac{e^2}{c^2}\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\
&\quad + \frac{16bc\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&\quad + \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&\quad - \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4b\sqrt{1+c^2x^2}\sqrt{1-\frac{e\left(1-\sqrt{-c^2}x\right)}{\sqrt{-c^2}d+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.91 (sec) , antiderivative size = 1217, normalized size of antiderivative = 1.88

$$
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2a}{5e(d+ex)^{5/2}}
$$

$$
+ b\left(-\frac{c^4\left(e+\frac{d}{x}\right)^4x^4\left(-\frac{4\left(7c^2d^2+3e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{15c^2d^2\left(c^2d^2+e^2\right)^2}+\frac{2\operatorname{csch}^{-1}(cx)}{5c^3d^3e}-\frac{2e^2\operatorname{csch}^{-1}(cx)}{5c^3d^3\left(e+\frac{d}{x}\right)^3}+\frac{2\left(-2cde^2\sqrt{1+\frac{1}{c^2x^2}}+9c^2d^2e\operatorname{csch}^{-1}(cx)+9e^3\operatorname{csch}^{-1}(cx)\right)}{15c^3d^3\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)^2}-\frac{2\left(\right.\right.}{(d+ex)^{7/2}}\right.
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]

[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((-4*(7*c^2*d^2 +
 3*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(15*c^2*d^2*(c^2*d^2 + e^2)^2) + (2*ArcCsch[
```

```
c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsch[c*x])/(5*c^3*d^3*(e + d/x)^3) + (2*(-2
*c*d*e^2*Sqrt[1 + 1/(c^2*x^2)] + 9*c^2*d^2*e*ArcCsch[c*x] + 9*e^3*ArcCsch[c
*x]))/(15*c^3*d^3*(c^2*d^2 + e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 +
 1/(c^2*x^2)] - 8*c*d*e^3*Sqrt[1 + 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsch[c*x] +
18*c^2*d^2*e^2*ArcCsch[c*x] + 9*e^4*ArcCsch[c*x]))/(15*c^3*d^3*(c^2*d^2 + e
^2)^2*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*(-((Sq
rt[2]*(c^2*d^2*e + e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d -
 I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2
*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))
/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(3*c^3*d^3 - c*d*e^2)*Sqrt[1 + I*c
*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/
e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[
1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-7*c^2*d^2*e - 3*e^3)*Cos
h[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I
)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e
*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c
*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Elliptic
E[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*E
llipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)])
 + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[
(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin
[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I
+ c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]
*(2 + c^2*x^2))))/(15*c*d*e*(c^2*d^2 + e^2)^2*(d + e*x)^(7/2))))/c
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.51 (sec) , antiderivative size = 3782, normalized size of antiderivative = 5.84

method	result	size
derivativedivides	Expression too large to display	3782
default	Expression too large to display	3782
parts	Expression too large to display	3784

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsch(c*x)-2/15/c*(7*I*((
c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*e*(e*x+d)^3+3*EllipticPi((e*x+d)^(1
/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-
c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*e*(e*
x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*
(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c*d*e^4*(e*x+d)^(3/2)+3*(-(I*c*e*
(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2
```

```
*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+
I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*c*d*e^4*(e*x+d)^(3/2)-10*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(-(I*c*e*(e*x+d)+c
^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d
)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e^2*(e*x+d)^(3/2)+6*EllipticPi(
(e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)
/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*(-
(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x
+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e^2*(e*x+d)^(3/
2)+9*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c
*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d
)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^
2+e^2))^(1/2))*c^3*d^3*e^2*(e*x+d)^(3/2)-3*EllipticE((e*x+d)^(1/2)*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(
-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*
x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c*d*e^4*(e*x+d)^(3/2)+
I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*
(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(
1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e
^2))^(1/2))*c^4*d^4*e*(e*x+d)^(3/2)+I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),
(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^3*(e*x+d)^(3/2)-3
*I*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c
*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2))*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2
)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^4*d^4*e
*(e*x+d)^(3/2)-6*I*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/
2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^
2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2)*c^2*d^2*e^3*(e*x+d)^(3/2)-13*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4
*d^4*e*(e*x+d)^2+5*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^5*e*(e*x+d)+3*
I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e^3*(e*x+d)^3-5*I*((c*d+I*e)*c/(c
^2*d^2+e^2))^(1/2)*c^2*d^2*e^3*(e*x+d)^2+8*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2)*c^2*d^3*e^3*(e*x+d)-7*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(-(I*c*e*(e*x+d)+c
^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d
)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^5*d^5*(e*x+d)^(3/2)+3*EllipticPi((e*x
+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(
-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c
*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^5*d^5*(e*x+d)^(3/2)+6*(-(
I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+
```

```
d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^
(1/2))*c^5*d^5*(e*x+d)^(3/2)-3*I*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2
*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))
^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^
2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2)*e^5*(e*x+d)^(3/2)-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^
5*d^7-7*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^5*d^4*(e*x+d)^3+13*((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c^5*d^5*(e*x+d)^2-5*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)
*c^5*d^6*(e*x+d)-2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^5*e^2+I*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2)*d^2*e^5-((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d^3*e
^4-3*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*e^2*(e*x+d)^3+I*((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c^4*d^6*e+5*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3
*e^2*(e*x+d)^2-8*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4*e^2*(e*x+d)-3*((
c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^4*(e*x+d)+2*I*((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2)*c^2*d^4*e^3+3*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*d*e^5*(e*x+d
))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/d^3/(c
^2*d^2+e^2)^2/(e*x+d)^(3/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(10*c^2*integrate(1/5*x/((c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x +
d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x
^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(e*x
+ d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((e^3*x^2 + 2*d*e^2*x + d^2*e)*sqr
t(e*x + d)) + 5*integrate(1/5*((5*e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + 5*e
*log(c) + 5*(c^2*e*x^2 + e)*log(x))/((c^2*e^4*x^5 + 3*c^2*d*e^3*x^4 + 3*d^2
*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)*x^3 + (c^2*d^3*e + 3*d*e^3)*x^2)*sqr
t(e*x + d)), x))*b - 2/5*a/((e*x + d)^(5/2)*e)
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{(d+e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.76 $\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

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Giac [F] . . . 560
Mupad [F(-1)] . . . 560

Optimal result

Integrand size = 19, antiderivative size = 214

$$\begin{aligned}\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = &-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}\\ &+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}\\ &+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)\\ &-\frac{b(42c^2d-25e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{560c^6\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] 1/5*d*x^5*(a+b*arccsch(c*x))+1/7*e*x^7*(a+b*arccsch(c*x))-1/560*b*(42*c^2*d
-25*e)*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^6/(-c^2*x^2)^(1/2)-1/560*b*(42*c^
2*d-25*e)*x^2*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)+1/840*b*(42*c^2*d-25*
e)*x^4*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/42*b*e*x^6*(-c^2*x^2-1)^(1
/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

$= \{14, 6437, 12, 470, 327, 223, 209\}$

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)$$
$$-\frac{bx\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(42c^2d-25e)}{560c^6\sqrt{-c^2x^2}}$$
$$+\frac{bex^6\sqrt{-c^2x^2-1}}{42c\sqrt{-c^2x^2}}-\frac{bx^2\sqrt{-c^2x^2-1}(42c^2d-25e)}{560c^5\sqrt{-c^2x^2}}$$
$$+\frac{bx^4\sqrt{-c^2x^2-1}(42c^2d-25e)}{840c^3\sqrt{-c^2x^2}}$$

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/560*(b*(42*c^2*d - 25*e)*x^2*Sqrt[-1 - c^2*x^2])/(c^5*Sqrt[-(c^2*x^2)])
+ (b*(42*c^2*d - 25*e)*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*Sqrt[-(c^2*x^2)]) +
 (b*e*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)]) + (d*x^5*(a + b*ArcCs
ch[c*x]))/5 + (e*x^7*(a + b*ArcCsch[c*x]))/7 - (b*(42*c^2*d - 25*e)*x*ArcTa
n[(c*x)/Sqrt[-1 - c^2*x^2]])/(560*c^6*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
 + 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
  + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^4(7d+5ex^2)}{35\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\\
&=\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^4(7d+5ex^2)}{\sqrt{-1-c^2x^2}}\,dx}{35\sqrt{-c^2x^2}}\\
&=\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(bc\left(42d-\frac{25e}{c^2}\right)x\right)\int\frac{x^4}{\sqrt{-1-c^2x^2}}\,dx}{210\sqrt{-c^2x^2}}\\
&=\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{\left(b\left(42d-\frac{25e}{c^2}\right)x\right)\int\frac{x^2}{\sqrt{-1-c^2x^2}}\,dx}{280c\sqrt{-c^2x^2}}\\
&=-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}\\
&\quad+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(b\left(42d-\frac{25e}{c^2}\right)x\right)\int\frac{1}{\sqrt{-1-c^2x^2}}\,dx}{560c^3\sqrt{-c^2x^2}}
\end{aligned}$$

$$= -\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}} + \frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}$$
$$+ \frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)$$
$$- \frac{\left(b\left(42d-\frac{25e}{c^2}\right)x\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{x}{\sqrt{-1-c^2x^2}}\right)}{560c^3\sqrt{-c^2x^2}}$$
$$= -\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}} + \frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}} + \frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}$$
$$+ \frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right) - \frac{b(42c^2d-25e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{560c^6\sqrt{-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{48ac^7x^5(7d+5ex^2) + bc^2\sqrt{1+\frac{1}{c^2x^2}}x^2(75e-2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4)) + 48bc^7x^5(7d+5ex}{1680c^7}$$

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2*(75*e - 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
*x^2)*ArcCsch[c*x] + 3*b*(42*c^2*d - 25*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*
x])/(1680*c^7)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
parts	$a\left(\frac{1}{7}e\,x^7+\frac{1}{5}d\,x^5\right)+\frac{b\left(\frac{c^5\,\mathrm{arccsch}(cx)e\,x^7}{7}+\frac{\mathrm{arccsch}(cx)x^5c^5d}{5}+\frac{\sqrt{c^2x^2+1}\left(84d\,c^5x^3\sqrt{c^2x^2+1}+40e\,c^5x^5\sqrt{c^2x^2+1}-126d\,c^3\right.\right.}{c^5}$
derivativedivides	$\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsch}(cx)d\,c^7x^5}{5}+\frac{\mathrm{arccsch}(cx)e\,c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\left(84d\,c^5x^3\sqrt{c^2x^2+1}+40e\,c^5x^5\sqrt{c^2x^2+1}-126d\,c^3x\sqrt{c^2x^2+1}\right.\right.}{c^5}$
default	$\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsch}(cx)d\,c^7x^5}{5}+\frac{\mathrm{arccsch}(cx)e\,c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\left(84d\,c^5x^3\sqrt{c^2x^2+1}+40e\,c^5x^5\sqrt{c^2x^2+1}-126d\,c^3x\sqrt{c^2x^2+1}\right.\right.}{c^5}$

```
[In] int(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arccsch(c*x)*e*x^7+1/5*arccsch(c*x)*
x^5*c^5*d+1/1680/c^3*(c^2*x^2+1)^(1/2)*(84*d*c^5*x^3*(c^2*x^2+1)^(1/2)+40*e
*c^5*x^5*(c^2*x^2+1)^(1/2)-126*d*c^3*x*(c^2*x^2+1)^(1/2)-50*e*c^3*x^3*(c^2*
x^2+1)^(1/2)+126*d*c^2*arcsinh(c*x)+75*e*c*x*(c^2*x^2+1)^(1/2)-75*e*arcsinh
(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int x^4\left(d+ex^2\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{240\,ac^7ex^7+336\,ac^7dx^5+48\left(7\,bc^7d+5\,bc^7e\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-3\left(42\,bc^2d-25\,be\right)\log\left(cx\sqrt{\frac{c^2}{}}\right.}{}$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(7*b*c^7*d + 5*b*c^7*e)*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 3*(42*b*c^2*d - 25*b*e)*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 48*(7*b*c^7*d + 5*b*c^7*e)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d
*x^5 - 7*b*c^7*d - 5*b*c^7*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + (40*b*c^6*e*x^6 + 2*(42*b*c^6*d - 25*b*c^4*e)*x^4 - 3*(42*b*c^4*d -
 25*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^4(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)\,dx$$

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral(x**4*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.35

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{7}aex^7+\frac{1}{5}adx^5$$
$$+\frac{1}{80}\left(16\,x^5\,\text{arcsch}\,(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd$$
$$+\frac{1}{672}\left(96\,x^7\,\text{arcsch}\,(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3\,c^6\left(\frac{1}{c^2x^2}+1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right.}{c^6}}{c}\right.$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(
c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/
(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c
^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))
/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2)
+ 1) - c^6) - 15*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^
2) + 1) - 1)/c^6)/c)*b*e
```

Giac [F]

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (ex^2+d)(b\,\text{arcsch}\,(cx)+a)x^4\,dx$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^4\,(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^4*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.77 $\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 561
Rubi [A] (verified) . . . 561
Mathematica [A] (verified) . . . 564
Maple [A] (verified) . . . 564
Fricas [A] (verification not implemented) . . . 565
Sympy [F] . . . 565
Maxima [A] (verification not implemented) . . . 565
Giac [F] . . . 566
Mupad [F(-1)] . . . 566

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{b(20c^2d-9e)\,x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + \frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right) + \frac{b(20c^2d-9e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{120c^4\sqrt{-c^2x^2}}$$

```
[Out] 1/3*d*x^3*(a+b*arccsch(c*x))+1/5*e*x^5*(a+b*arccsch(c*x))+1/120*b*(20*c^2*d
-9*e)*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^4/(-c^2*x^2)^(1/2)+1/120*b*(20*c^2
*d-9*e)*x^2*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/20*b*e*x^4*(-c^2*x^2-
1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {14, 6437, 12, 470, 327, 223, 209}

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right) + \frac{bx\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(20c^2d-9e)}{120c^4\sqrt{-c^2x^2}} + \frac{bex^4\sqrt{-c^2x^2-1}}{20c\sqrt{-c^2x^2}} + \frac{bx^2\sqrt{-c^2x^2-1}(20c^2d-9e)}{120c^3\sqrt{-c^2x^2}}$$

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(20*c^2*d - 9*e)*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) + (b
*e*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + (d*x^3*(a + b*ArcCsch[
c*x]))/3 + (e*x^5*(a + b*ArcCsch[c*x]))/5 + (b*(20*c^2*d - 9*e)*x*ArcTan[(c
*x)/Sqrt[-1 - c^2*x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}dx^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{15\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\\
&=\frac{1}{3}dx^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{15\sqrt{-c^2x^2}}\\
&=\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(bc\left(20d-\frac{9e}{c^2}\right)x\right)\int\frac{x^2}{\sqrt{-1-c^2x^2}}\,dx}{60\sqrt{-c^2x^2}}\\
&=\frac{b(20c^2d-9e)\,x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{\left(b\left(20d-\frac{9e}{c^2}\right)x\right)\int\frac{1}{\sqrt{-1-c^2x^2}}\,dx}{120c\sqrt{-c^2x^2}}\\
&=\frac{b(20c^2d-9e)\,x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{\left(b\left(20d-\frac{9e}{c^2}\right)x\right)\operatorname{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,\frac{x}{\sqrt{-1-c^2x^2}}\right)}{120c\sqrt{-c^2x^2}}\\
&=\frac{b(20c^2d-9e)\,x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b(20c^2d-9e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{120c^4\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.71

$$\int x^2\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{c^2x^2\left(8ac^3x(5d+3ex^2)+b\sqrt{1+\frac{1}{c^2x^2}}(-9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\,\text{csch}^{-1}(cx)+b(-20c^2d}{120c^5}$$

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-9*e + c^2*(
20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsch[c*x] + b*(-20*c^2*d
 + 9*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\left(\frac{c^3\,\text{arccsch}(cx)e\,x^5}{5}+\frac{\text{arccsch}(cx)x^3c^3d}{3}-\frac{\sqrt{c^2x^2+1}\left(-6e\,c^3x^3\sqrt{c^2x^2+1}-20d\,c^3x\sqrt{c^2x^2+1}+20d\,c^2\,\text{ar}\right.}{120c^3\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x}\right.}{c^3}$
derivativedivides	$\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\text{arccsch}(cx)d\,c^5x^3}{3}+\frac{\text{arccsch}(cx)e\,c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\left(20d\,c^3x\sqrt{c^2x^2+1}+6e\,c^3x^3\sqrt{c^2x^2+1}-20d\,c^2\,\text{arcsinh}(cx\right.}{120\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^3}$
default	$\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\text{arccsch}(cx)d\,c^5x^3}{3}+\frac{\text{arccsch}(cx)e\,c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\left(20d\,c^3x\sqrt{c^2x^2+1}+6e\,c^3x^3\sqrt{c^2x^2+1}-20d\,c^2\,\text{arcsinh}(cx\right.}{120\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^3}$

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arccsch(c*x)*e*x^5+1/3*arccsch(c*x)*
x^3*c^3*d-1/120/c^3*(c^2*x^2+1)^(1/2)*(-6*e*c^3*x^3*(c^2*x^2+1)^(1/2)-20*d*
c^3*x*(c^2*x^2+1)^(1/2)+20*d*c^2*arcsinh(c*x)+9*e*c*x*(c^2*x^2+1)^(1/2)-9*e
*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.63

$$\int x^2\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{24\,ac^5ex^5+40\,ac^5dx^3+8\left(5\,bc^5d+3\,bc^5e\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)+\left(20\,bc^2d-9\,be\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right.}{}$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*
sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (20*b*c^2*d - 9*b*e)*log(c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 - 5*
b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (
6*b*c^4*e*x^4 + (20*b*c^4*d - 9*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
)/c^5
```

Sympy [F]

$$\int x^2\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\operatorname{acsch}(cx))\left(d+ex^2\right)\,dx$$

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.36

$$\int x^2\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3$$
$$+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd$$
$$+\frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + l
og(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsch(c*x) - (2*
(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1
)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4
 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

Giac [F]

$$\int x^2\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)\left(b\operatorname{arcsch}(cx)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^2\left(e\,x^2+d\right)\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.78 $\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 567
Rubi [A] (verified) . . . 567
Mathematica [A] (verified) . . . 569
Maple [A] (verified) . . . 569
Fricas [B] (verification not implemented) . . . 570
Sympy [F] . . . 571
Maxima [A] (verification not implemented) . . . 571
Giac [F] . . . 571
Mupad [F(-1)] . . . 572

Optimal result

Integrand size = 16, antiderivative size = 115

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{b(6c^2d-e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6c^2\sqrt{-c^2x^2}}$$

```
[Out] d*x*(a+b*arccsch(c*x))+1/3*e*x^3*(a+b*arccsch(c*x))-1/6*b*(6*c^2*d-e)*x*arc
tan(c*x/(-c^2*x^2-1)^(1/2))/c^2/(-c^2*x^2)^(1/2)+1/6*b*e*x^2*(-c^2*x^2-1)^(
1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6427, 12, 396, 223, 209}

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = dx\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{bx\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(6c^2d-e)}{6c^2\sqrt{-c^2x^2}} + \frac{bex^2\sqrt{-c^2x^2-1}}{6c\sqrt{-c^2x^2}}$$

```
[In] Int[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*x^2*Sqrt[-1 - c^2*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + d*x*(a + b*ArcCsch[c*
x]) + (e*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(6*c^2*d - e)*x*ArcTan[(c*x)/Sqrt
[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx\big(a + b\operatorname{csch}^{-1}(cx)\big) + \frac{1}{3}ex^3\big(a + b\operatorname{csch}^{-1}(cx)\big) - \frac{(bcx)\int \frac{3d+ex^2}{3\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= dx\big(a + b\operatorname{csch}^{-1}(cx)\big) + \frac{1}{3}ex^3\big(a + b\operatorname{csch}^{-1}(cx)\big) - \frac{(bcx)\int \frac{3d+ex^2}{\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx\big(a + b\operatorname{csch}^{-1}(cx)\big) \\
&\quad + \frac{1}{3}ex^3\big(a + b\operatorname{csch}^{-1}(cx)\big) - \frac{\big(bc\big(6d - \frac{e}{c^2}\big)x\big)\int \frac{1}{\sqrt{-1-c^2x^2}}\,dx}{6\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{\left(bc\left(6d-\frac{e}{c^2}\right)x\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{x}{\sqrt{-1-c^2x^2}}\right)}{6\sqrt{-c^2x^2}}$$

$$= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{b\left(6d-\frac{e}{c^2}\right)x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6\sqrt{-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = adx + \frac{1}{3}aex^3 + \frac{bex^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{6c} + bdx\text{csch}^{-1}(cx) + \frac{1}{3}bex^3\text{csch}^{-1}(cx) + \frac{2bd\sqrt{1+\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}} - \frac{be\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*x
*ArcCsch[c*x] + (b*e*x^3*ArcCsch[c*x])/3 + (2*b*d*Sqrt[1 + 1/(c^2*x^2)]*x*A
rcTanh[(-1 + Sqrt[1 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2] - (b*e*Log[x*(1 +
 Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

method	result
parts	$a\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b\left(\frac{c\,\operatorname{arccsch}(cx)e\,x^3}{3}+\operatorname{arccsch}(cx)dxc+\frac{\sqrt{c^2x^2+1}\left(6d\,c^2\operatorname{arcsinh}(cx)+ecx\sqrt{c^2x^2+1}-e\,\operatorname{arcsinh}(cx)\right)}{6c^3x\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c}$
derivativedivides	$\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^3x+\frac{\operatorname{arccsch}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(6d\,c^2\operatorname{arcsinh}(cx)+ecx\sqrt{c^2x^2+1}-e\,\operatorname{arcsinh}(cx)\right)}{6cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}}{c}$
default	$\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^3x+\frac{\operatorname{arccsch}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(6d\,c^2\operatorname{arcsinh}(cx)+ecx\sqrt{c^2x^2+1}-e\,\operatorname{arcsinh}(cx)\right)}{6cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}}{c}$

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arccsch(c*x)*e*x^3+arccsch(c*x)*d*x*c+1/6/c^3*
(c^2*x^2+1)^(1/2)*(6*d*c^2*arcsinh(c*x)+e*c*x*(c^2*x^2+1)^(1/2)-e*arcsinh(c
*x))/x/((c^2*x^2+1)/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(101)=202$.

Time $=0.28$ (sec) , antiderivative size $=245$, normalized size of antiderivative $=2.13$

$$\int \left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{2\,ac^3ex^3+bc^2ex^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}+6\,ac^3dx+2\left(3\,bc^3d+bc^3e\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-\left(6\,bc^2d-be\right)\log\left(cx\right.}{}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + b*c^2*e*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 6*a*c^3*d*
x + 2*(3*b*c^3*d + b*c^3*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - (6*b*c^2*d - b*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(3*b
*c^3*d + b*c^3*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c
^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)))/c^3
```

Sympy [F]

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\begin{aligned}&\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\ &= \frac{1}{3}\,aex^3+\frac{1}{12}\left(4\,x^3\,\text{arcsch}\left(cx\right)+\frac{\frac{2\,\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be \\ &\quad +adx+\frac{\left(2\,cx\,\text{arcsch}\left(cx\right)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd}{2\,c}\end{aligned}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/3*a*e*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2
*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/
(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d/c
```

Giac [F]

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)\left(b\,\text{arcsch}\left(cx\right)+a\right)\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(e\,x^2+d\right)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int((d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x} + ex\left(a+b\text{csch}^{-1}(cx)\right) - \frac{bex\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

```
[Out] -d*(a+b*arccsch(c*x))/x+e*x*(a+b*arccsch(c*x))-b*e*x*arctan(c*x/(-c^2*x^2-1
)^(1/2))/(-c^2*x^2)^(1/2)+b*c*d*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6437, 462, 223, 209}

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x} + ex\left(a+b\text{csch}^{-1}(cx)\right) - \frac{bex\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (b*c*d*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] - (d*(a + b*ArcCsch[c*x]))/x +
e*x*(a + b*ArcCsch[c*x]) - (b*e*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(
c^2*x^2)]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + ex\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{-d+ex^2}{x^2\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + ex\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcex)\int \frac{1}{\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + ex\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcex)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{x}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + ex\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{bex\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{bd\operatorname{csch}^{-1}(cx)}{x} + bex\operatorname{csch}^{-1}(cx) + \frac{2be\sqrt{1+\frac{1}{c^2x^2}}x\operatorname{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x + b*c*d*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsch[c*x
])/x + b*e*x*ArcCsch[c*x] + (2*b*e*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(-1 + Sq
rt[1 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result	size
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(-\frac{\operatorname{arccsch}(cx)d}{xc}+\frac{\operatorname{arccsch}(cx)ex}{c}+\frac{\sqrt{c^2x^2+1}\left(d\,c^2\sqrt{c^2x^2+1}+e\,\operatorname{arcsinh}(cx)cx\right)}{c^4x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)$	102
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\,\operatorname{arccsch}(cx)ex-\frac{\operatorname{arccsch}(cx)dc}{x}+\frac{\sqrt{c^2x^2+1}\left(d\,c^2\sqrt{c^2x^2+1}+e\,\operatorname{arcsinh}(cx)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}\right)$	107
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\,\operatorname{arccsch}(cx)ex-\frac{\operatorname{arccsch}(cx)dc}{x}+\frac{\sqrt{c^2x^2+1}\left(d\,c^2\sqrt{c^2x^2+1}+e\,\operatorname{arcsinh}(cx)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}\right)$	107

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e*x-d/x)+b*c*(-arccsch(c*x)*d/x/c+1/c*arccsch(c*x)*e*x+1/c^4*(c^2*x^2+1)
^(1/2)*(d*c^2*(c^2*x^2+1)^(1/2)+e*arcsinh(c*x)*c*x)/x^2/((c^2*x^2+1)/c^2/x^
2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.44

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

$$= \frac{bc^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + bc^2dx + acex^2 - bex\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - acd - (bcd-bce)x\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + \right.}{cx}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
[Out] (b*c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + b*c^2*d*x + a*c*e*x^2 - b*e*x*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - a*c*d - (b*c*d - b*c*e)*x*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c*d - b*c*e)*x*log(c*x*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*e*x^2 - b*c*d + (b*c*d - b*c*
e)*x)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\,(d+ex^2)}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**2,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

$$= \left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsch}(cx)}{x}\right)bd + aex$$

$$+ \frac{\left(2\,cx\operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)be}{2\,c} - \frac{ad}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arccsch
(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*
e/c - a*d/x
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.80 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result . . . 578
Rubi [A] (verified) . . . 578
Mathematica [A] (verified) . . . 580
Maple [A] (verified) . . . 580
Fricas [A] (verification not implemented) . . . 581
Sympy [F] . . . 581
Maxima [A] (verification not implemented) . . . 581
Giac [F] . . . 582
Mupad [F(-1)] . . . 582

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{bc(2c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

```
[Out] -1/3*d*(a+b*arccsch(c*x))/x^3-e*(a+b*arccsch(c*x))/x-1/9*b*c*(2*c^2*d-9*e)*
(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/9*b*c*d*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6437, 12, 464, 270}

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x} - \frac{bc\sqrt{-c^2x^2-1}(2c^2d-9e)}{9\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*(2*c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] + (b*c*d*Sqr
t[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(3*x^3
) - (e*(a + b*ArcCsch[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int \frac{-d-3ex^2}{3x^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int \frac{-d-3ex^2}{x^4\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} - \frac{(bc(2c^2d-9e)\,x)\int \frac{1}{x^2\sqrt{-1-c^2x^2}}\,dx}{9\sqrt{-c^2x^2}}
\end{aligned}$$

$$= -\frac{bc(2c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$
$$= \frac{-3a(d+3ex^2) + bc\sqrt{1+\frac{1}{c^2x^2}}\,x(d-2c^2dx^2+9ex^2) - 3b(d+3ex^2)\operatorname{csch}^{-1}(cx)}{9x^3}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 9*e*x^
2) - 3*b*(d + 3*e*x^2)*ArcCsch[c*x])/(9*x^3)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arccsch}(cx)e}{c^3x}-\frac{\operatorname{arccsch}(cx)d}{3x^3c^3}-\frac{\left(c^2x^2+1\right)\left(2c^4dx^2-9ec^2x^2-c^2d\right)}{9c^6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x^4}\right)$	109
derivativedivides	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3cx^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{cx}-\frac{\operatorname{arccsch}(cx)d}{3cx^3}-\frac{\left(c^2x^2+1\right)\left(2c^4dx^2-9ec^2x^2-c^2d\right)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	122
default	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3cx^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{cx}-\frac{\operatorname{arccsch}(cx)d}{3cx^3}-\frac{\left(c^2x^2+1\right)\left(2c^4dx^2-9ec^2x^2-c^2d\right)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	122

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arccsch(c*x)*e/x-1/3*arccsch(c*x)*d/x^3/c^
3-1/9/c^6*(c^2*x^2+1)*(2*c^4*d*x^2-9*c^2*e*x^2-c^2*d)/((c^2*x^2+1)/c^2/x^2)
^(1/2)/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

$$= -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(bcdx-(2\,bc^3d-9\,bce)x^3)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{9\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)) - (b*c*d*x - (2*b*c^3*d - 9*b*c*e)*x^3)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))/x^3
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(a+b\,\operatorname{acsch}(cx))\,(d+ex^2)}{x^4}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

$$= \left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)be + \frac{1}{9}\,bd\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arcsch}(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2)
 + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - a*e/x
- 1/3*a*d/x^3
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{cx}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.81 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result . . . 583
Rubi [A] (verified) . . . 583
Mathematica [A] (verified) . . . 585
Maple [A] (verified) . . . 586
Fricas [A] (verification not implemented) . . . 586
Sympy [F] . . . 587
Maxima [A] (verification not implemented) . . . 587
Giac [F] . . . 587
Mupad [F(-1)] . . . 588

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{2bc^3(12c^2d-25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

```
[Out] -1/5*d*(a+b*arccsch(c*x))/x^5-1/3*e*(a+b*arccsch(c*x))/x^3+2/225*b*c^3*(12*
c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/25*b*c*d*(-c^2*x^2-1)^(1/
2)/x^4/(-c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2/(-
c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6437, 12, 464, 277, 270}

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{bc\sqrt{-c^2x^2-1}(12c^2d-25e)}{225x^2\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}} + \frac{2bc^3\sqrt{-c^2x^2-1}(12c^2d-25e)}{225\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (2*b*c^3*(12*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c^2*x^2)]) + (b*
c*d*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (b*c*(12*c^2*d - 25*e)*
Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(
5*x^5) - (e*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
```

```
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{-3d-5ex^2}{15x^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{-3d-5ex^2}{x^6\sqrt{-1-c^2x^2}}\,dx}{15\sqrt{-c^2x^2}} \\
&= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{(bc(12c^2d-25e)\,x)\int \frac{1}{x^4\sqrt{-1-c^2x^2}}\,dx}{75\sqrt{-c^2x^2}} \\
&= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} \\
&\quad - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} + \frac{(2bc^3(12c^2d-25e)\,x)\int \frac{1}{x^2\sqrt{-1-c^2x^2}}\,dx}{225\sqrt{-c^2x^2}} \\
&= \frac{2bc^3(12c^2d-25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\begin{aligned}
&\int \frac{\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx \\
&= \frac{-15a(3d+5ex^2) + bc\sqrt{1+\frac{1}{c^2x^2}}x(25ex^2(1-2c^2x^2)+3d(3-4c^2x^2+8c^4x^4)) - 15b(3d+5ex^2)\operatorname{csch}^{-1}(c}{225x^5}
\end{aligned}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(25*e*x^2*(1 - 2*c^2*x
^2) + 3*d*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcCsch[c*x])
/(225*x^5)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

method	result
parts	$a\left(-\frac{e}{3x^3}-\frac{d}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arccsch}(cx)e}{3c^5x^3}-\frac{\operatorname{arccsch}(cx)d}{5x^5c^5}+\frac{\left(c^2x^2+1\right)\left(24c^6dx^4-50c^4ex^4-12c^4dx^2+25ec^2x^2+\right.}{225c^8\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x^6}$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsch}(cx)d}{5c^3x^5}+\frac{\left(c^2x^2+1\right)\left(24c^6dx^4-50c^4ex^4-12c^4dx^2+25ec^2x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$
default	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsch}(cx)d}{5c^3x^5}+\frac{\left(c^2x^2+1\right)\left(24c^6dx^4-50c^4ex^4-12c^4dx^2+25ec^2x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3*e/x^3-1/5*d/x^5)+b*c^5*(-1/3/c^5*arccsch(c*x)*e/x^3-1/5*arccsch(c*x
)*d/x^5/c^5+1/225/c^8*(c^2*x^2+1)*(24*c^6*d*x^4-50*c^4*e*x^4-12*c^4*d*x^2+2
5*c^2*e*x^2+9*c^2*d)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx =$$
$$-\frac{75\,aex^2+45\,ad+15\left(5\,bex^2+3\,bd\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(2\left(12\,bc^5d-25\,bc^3e\right)x^5+9\,bcdx-\left(12\,bc^3d-\right.\right.}{225\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) - (2*(12*b*c^5*d - 25*b*c^3*e)*x^5 + 9*b*c*d*x
 - (12*b*c^3*d - 25*b*c*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^5
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)}{x^6}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**6,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx \\ &= \frac{1}{75}\,bd\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{15\,\text{arcsch}\,(cx)}{x^5}\right) \\ &\quad + \frac{1}{9}\,be\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\text{arcsch}\,(cx)}{x^3}\right) - \frac{ae}{3\,x^3} - \frac{ad}{5\,x^5}\end{aligned}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
[Out] 1/75*b*d*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) +
 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) + 1/9*b*e*((c^4*(1/
(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3)
 - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.82 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal result . . . 589
Rubi [A] (verified) . . . 589
Mathematica [A] (verified) . . . 592
Maple [A] (verified) . . . 592
Fricas [A] (verification not implemented) . . . 593
Sympy [F] . . . 593
Maxima [A] (verification not implemented) . . . 593
Giac [F] . . . 594
Mupad [F(-1)] . . . 594

Optimal result

Integrand size = 19, antiderivative size = 205

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = &-\frac{8bc^5(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}}\\ &+\frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}\\ &+\frac{4bc^3(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675x^2\sqrt{-c^2x^2}}\\ &-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}\end{aligned}$$

```
[Out] -1/7*d*(a+b*arccsch(c*x))/x^7-1/5*e*(a+b*arccsch(c*x))/x^5-8/3675*b*c^5*(30
*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/49*b*c*d*(-c^2*x^2-1)^(1
/2)/x^6/(-c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^4/
(-c^2*x^2)^(1/2)+4/3675*b*c^3*(30*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{14, 6437, 12, 464, 277, 270\}$

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{bc\sqrt{-c^2x^2-1}(30c^2d-49e)}{1225x^4\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{49x^6\sqrt{-c^2x^2}} - \frac{8bc^5\sqrt{-c^2x^2-1}(30c^2d-49e)}{3675\sqrt{-c^2x^2}} + \frac{4bc^3\sqrt{-c^2x^2-1}(30c^2d-49e)}{3675x^2\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-8*b*c^5*(30*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(3675*Sqrt[-(c^2*x^2)]) + (
b*c*d*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]) - (b*c*(30*c^2*d - 49*e
)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2)]) + (4*b*c^3*(30*c^2*d - 49
*e)*Sqrt[-1 - c^2*x^2])/(3675*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x
]))/(7*x^7) - (e*(a + b*ArcCsch[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{(bcx)\int \frac{-5d-7ex^2}{35x^8\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{(bcx)\int \frac{-5d-7ex^2}{x^8\sqrt{-1-c^2x^2}}\,dx}{35\sqrt{-c^2x^2}} \\
&= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} \\
&\quad - \frac{(bc(30c^2d-49e)\,x)\int \frac{1}{x^6\sqrt{-1-c^2x^2}}\,dx}{245\sqrt{-c^2x^2}} \\
&= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} \\
&\quad - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} + \frac{(4bc^3(30c^2d-49e)\,x)\int \frac{1}{x^4\sqrt{-1-c^2x^2}}\,dx}{1225\sqrt{-c^2x^2}} \\
&= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{4bc^3(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675x^2\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{(8bc^5(30c^2d-49e)\,x)\int \frac{1}{x^2\sqrt{-1-c^2x^2}}\,dx}{3675\sqrt{-c^2x^2}} \\
&= -\frac{8bc^5(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} \\
&\quad + \frac{4bc^3(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$$

$$= \frac{-105a(5d+7ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}x(49ex^2(3-4c^2x^2+8c^4x^4)-15d(-5+6c^2x^2-8c^4x^4+16c^6x^6))-105}{3675x^7}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(49*e*x^2*(3 - 4*c^2*
x^2 + 8*c^4*x^4) - 15*d*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*
(5*d + 7*e*x^2)*ArcCsch[c*x])/(3675*x^7)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.71

method	result
parts	$a\left(-\frac{e}{5x^5}-\frac{d}{7x^7}\right)+b\,c^7\left(-\frac{\text{arccsch}(cx)e}{5c^7x^5}-\frac{\text{arccsch}(cx)d}{7x^7c^7}-\frac{(c^2x^2+1)(240c^8dx^6-392c^6ex^6-120c^6dx^4+196c^4e$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\text{arccsch}(cx)d}{7c^5x^7}-\frac{\text{arccsch}(cx)e}{5c^5x^5}-\frac{\left(c^2x^2+1\right)\left(240c^8dx^6-392c^6ex^6-120c^6dx^4+196c^4ex^4+90c^4dx\right.}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^8x^8}\right)}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\text{arccsch}(cx)d}{7c^5x^7}-\frac{\text{arccsch}(cx)e}{5c^5x^5}-\frac{\left(c^2x^2+1\right)\left(240c^8dx^6-392c^6ex^6-120c^6dx^4+196c^4ex^4+90c^4dx\right.}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^8x^8}\right)}{c^2}\right.$

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*e/x^5-1/7*d/x^7)+b*c^7*(-1/5/c^7*arccsch(c*x)*e/x^5-1/7*arccsch(c*x
)*d/x^7/c^7-1/3675/c^10*(c^2*x^2+1)*(240*c^8*d*x^6-392*c^6*e*x^6-120*c^6*d*
x^4+196*c^4*e*x^4+90*c^4*d*x^2-147*c^2*e*x^2-75*c^2*d)/((c^2*x^2+1)/c^2/x^2
)^(1/2)/x^8)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx =$$

$$-\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(8\,(30\,bc^7d-49\,bc^5e)x^7-4\,(30\,bc^5d-49$$
$$3675\,x^7$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*(30*b*c^7*d - 49*b*c^5*e)*x^7 - 4*(30
*b*c^5*d - 49*b*c^3*e)*x^5 - 75*b*c*d*x + 3*(30*b*c^3*d - 49*b*c*e)*x^3)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(a+b\,\operatorname{acsch}(cx))\,(d+ex^2)}{x^8}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**8,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx$$
$$= \frac{1}{245}\,bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\operatorname{arcsch}(cx)}{x^7}\right)$$
$$+\frac{1}{75}\,be\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsch}(cx)}{x^5}\right)$$
$$-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2)
+ 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*arc
csch(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*
x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) -
1/5*a*e/x^5 - 1/7*a*d/x^7
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcsch}(cx)+a)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.83 $\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 595
Rubi [A] (verified) . . . 595
Mathematica [A] (verified) . . . 597
Maple [A] (verified) . . . 598
Fricas [A] (verification not implemented) . . . 598
Sympy [F] . . . 599
Maxima [A] (verification not implemented) . . . 599
Giac [F] . . . 599
Mupad [F(-1)] . . . 600

Optimal result

Integrand size = 19, antiderivative size = 204

$$\begin{aligned}\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx &= \frac{b(4c^2d-3e)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}\\ &+\frac{b(8c^2d-9e)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}\\ &+\frac{b(4c^2d-9e)\,x(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}}-\frac{bex(-1-c^2x^2)^{7/2}}{56c^7\sqrt{-c^2x^2}}\\ &+\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)\end{aligned}$$

```
[Out] 1/6*d*x^6*(a+b*arccsch(c*x))+1/8*e*x^8*(a+b*arccsch(c*x))+1/72*b*(8*c^2*d-9
*e)*x*(-c^2*x^2-1)^(3/2)/c^7/(-c^2*x^2)^(1/2)+1/120*b*(4*c^2*d-9*e)*x*(-c^2
*x^2-1)^(5/2)/c^7/(-c^2*x^2)^(1/2)-1/56*b*e*x*(-c^2*x^2-1)^(7/2)/c^7/(-c^2*
x^2)^(1/2)+1/24*b*(4*c^2*d-3*e)*x*(-c^2*x^2-1)^(1/2)/c^7/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{14, 6437, 12, 457, 78\}$

$$\int x^5\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)$$
$$+\frac{bx(-c^2x^2-1)^{5/2}\left(4c^2d-9e\right)}{120c^7\sqrt{-c^2x^2}}$$
$$+\frac{bx(-c^2x^2-1)^{3/2}\left(8c^2d-9e\right)}{72c^7\sqrt{-c^2x^2}}$$
$$+\frac{bx\sqrt{-c^2x^2-1}(4c^2d-3e)}{24c^7\sqrt{-c^2x^2}}-\frac{bex(-c^2x^2-1)^{7/2}}{56c^7\sqrt{-c^2x^2}}$$

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(4*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2])/(24*c^7*Sqrt[-(c^2*x^2)]) + (b*(8*
c^2*d - 9*e)*x*(-1 - c^2*x^2)^(3/2))/(72*c^7*Sqrt[-(c^2*x^2)]) + (b*(4*c^2*
d - 9*e)*x*(-1 - c^2*x^2)^(5/2))/(120*c^7*Sqrt[-(c^2*x^2)]) - (b*e*x*(-1 -
c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*x^2)]) + (d*x^6*(a + b*ArcCsch[c*x]))/6
+ (e*x^8*(a + b*ArcCsch[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}dx^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5\left(4d+3ex^2\right)}{24\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5\left(4d+3ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{24\sqrt{-c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\operatorname{Subst}\left(\int\frac{x^2(4d+3ex)}{\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{48\sqrt{-c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad -\frac{(bcx)\operatorname{Subst}\left(\int\left(\frac{4c^2d-3e}{c^6\sqrt{-1-c^2x}}+\frac{\left(8c^2d-9e\right)\sqrt{-1-c^2x}}{c^6}+\frac{\left(4c^2d-9e\right)\left(-1-c^2x\right)^{3/2}}{c^6}-\frac{3e\left(-1-c^2x\right)^{5/2}}{c^6}\right)dx,x,x^2\right)}{48\sqrt{-c^2x^2}}\\
&= \frac{b(4c^2d-3e)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}+\frac{b(8c^2d-9e)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}\\
&\quad +\frac{b(4c^2d-9e)\,x(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}}-\frac{bex(-1-c^2x^2)^{7/2}}{56c^7\sqrt{-c^2x^2}}\\
&\quad +\frac{1}{6}dx^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\operatorname{csch}^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\begin{aligned}
&\int x^5\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx\\
&=\frac{x\left(105ax^5(4d+3ex^2)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(-144e+8c^2\left(28d+9ex^2\right)-2c^4\left(56dx^2+27ex^4\right)+c^6\left(84dx^4+45ex^6\right)\right)}{c^7}+105bx^5(4d+3ex^2)\operatorname{c}\right.}{2520}
\end{aligned}$$

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-144*e + 8*c^2*(2
8*d + 9*e*x^2) - 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/
c^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsch[c*x]))/2520
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

method	result
parts	$a\left(\frac{1}{8}e\,x^8+\frac{1}{6}d\,x^6\right)+\frac{b\left(\frac{c^6\,\mathrm{arccsch}(cx)e\,x^8}{8}+\frac{\mathrm{arccsch}(cx)d\,x^6c^6}{6}+\frac{\left(c^2x^2+1\right)\left(45c^6e\,x^6+84c^6d\,x^4-54c^4e\,x^4-112c^4d\,x^2+72e\,c\right.}{2520c^3\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x}\right.}{c^6}$
derivativedivides	$\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsch}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arccsch}(cx)e\,c^8x^8}{8}+\frac{\left(c^2x^2+1\right)\left(45c^6e\,x^6+84c^6d\,x^4-54c^4e\,x^4-112c^4d\,x^2+72e\,c^2x^2+2\right.}{2520\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^6}$
default	$\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsch}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arccsch}(cx)e\,c^8x^8}{8}+\frac{\left(c^2x^2+1\right)\left(45c^6e\,x^6+84c^6d\,x^4-54c^4e\,x^4-112c^4d\,x^2+72e\,c^2x^2+2\right.}{2520\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^6}$

```
[In] int(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e*x^8+1/6*d*x^6)+b/c^6*(1/8*c^6*arccsch(c*x)*e*x^8+1/6*arccsch(c*x)*
d*x^6*c^6+1/2520/c^3*(c^2*x^2+1)*(45*c^6*e*x^6+84*c^6*d*x^4-54*c^4*e*x^4-11
2*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d-144*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.81

$$\int x^5\left(d+ex^2\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{315\,ac^7ex^8+420\,ac^7dx^6+105\left(3\,bc^7ex^8+4\,bc^7dx^6\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(45\,bc^6ex^7+6\left(14\,bc^6d-9\,bc^4e\right)x\right.}{2520\,c^7}$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d*
x^6)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (45*b*c^6*e*x^7 +
 6*(14*b*c^6*d - 9*b*c^4*e)*x^5 - 8*(14*b*c^4*d - 9*b*c^2*e)*x^3 + 16*(14*b
*c^2*d - 9*b*e)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^5 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int x^5 (a+b\,\text{acsch}\,(cx)) \left(d+ex^2\right) \, dx$$

```
[In] integrate(x**5*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral(x**5*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

$$\int x^5 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \frac{1}{8}\,aex^8 + \frac{1}{6}\,adx^6$$
$$+ \frac{1}{90}\left(15\,x^6\,\text{arcsch}\,(cx) + \frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)bd$$
$$+ \frac{1}{280}\left(35\,x^8\,\text{arcsch}\,(cx) + \frac{5\,c^6x^7\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^7}\right)$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*
x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2
) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1
)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)
^(3/2) - 35*x*sqrt(1/(c^2*x^2) + 1))/c^7)*b*e
```

Giac [F]

$$\int x^5 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b\,\text{arcsch}\,(cx)+a)x^5 \, dx$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int x^5 \left(e\,x^2+d\right) \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^5*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.84 $\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 601
Rubi [A] (verified) . . . 601
Mathematica [A] (verified) . . . 603
Maple [A] (verified) . . . 604
Fricas [A] (verification not implemented) . . . 604
Sympy [F] . . . 605
Maxima [A] (verification not implemented) . . . 605
Giac [F] . . . 605
Mupad [F(-1)] . . . 606

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = -\frac{b(3c^2d-2e)\,x\sqrt{-1-c^2x^2}}{12c^5\sqrt{-c^2x^2}} - \frac{b(3c^2d-4e)\,x(-1-c^2x^2)^{3/2}}{36c^5\sqrt{-c^2x^2}} + \frac{bex(-1-c^2x^2)^{5/2}}{30c^5\sqrt{-c^2x^2}} + \frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/4*d*x^4*(a+b*arccsch(c*x))+1/6*e*x^6*(a+b*arccsch(c*x))-1/36*b*(3*c^2*d-4
*e)*x*(-c^2*x^2-1)^(3/2)/c^5/(-c^2*x^2)^(1/2)+1/30*b*e*x*(-c^2*x^2-1)^(5/2)
/c^5/(-c^2*x^2)^(1/2)-1/12*b*(3*c^2*d-2*e)*x*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6437, 12, 457, 78}

$$\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right) - \frac{bx(-c^2x^2-1)^{3/2}(3c^2d-4e)}{36c^5\sqrt{-c^2x^2}} - \frac{bx\sqrt{-c^2x^2-1}(3c^2d-2e)}{12c^5\sqrt{-c^2x^2}} + \frac{bex(-c^2x^2-1)^{5/2}}{30c^5\sqrt{-c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/12*(b*(3*c^2*d - 2*e)*x*Sqrt[-1 - c^2*x^2])/(c^5*Sqrt[-(c^2*x^2)]) - (b*
(3*c^2*d - 4*e)*x*(-1 - c^2*x^2)^(3/2))/(36*c^5*Sqrt[-(c^2*x^2)]) + (b*e*x*
(-1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + (d*x^4*(a + b*ArcCsch[c*x
]))/4 + (e*x^6*(a + b*ArcCsch[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{4}dx^4\left(a + b\text{csch}^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a + b\text{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{x^3(3d+2ex^2)}{12\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int \frac{x^3\left(3d+2ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{12\sqrt{-c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int \frac{x\left(3d+2ex\right)}{\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{24\sqrt{-c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)$$
$$-\frac{(bcx)\text{Subst}\left(\int \left(\frac{-3c^2d+2e}{c^4\sqrt{-1-c^2x}}+\frac{(-3c^2d+4e)\sqrt{-1-c^2x}}{c^4}+\frac{2e\left(-1-c^2x\right)^{3/2}}{c^4}\right)\,dx,x,x^2\right)}{24\sqrt{-c^2x^2}}$$

$$= -\frac{b(3c^2d-2e)\,x\sqrt{-1-c^2x^2}}{12c^5\sqrt{-c^2x^2}}-\frac{b(3c^2d-4e)\,x\left(-1-c^2x^2\right)^{3/2}}{36c^5\sqrt{-c^2x^2}}$$
$$+\frac{bex\left(-1-c^2x^2\right)^{5/2}}{30c^5\sqrt{-c^2x^2}}+\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int x^3\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}(16e-2c^2(15d+4ex^2)+3c^4(5dx^2+2ex^4))}{c^5}\right.$$
$$\left.+15bx^3\left(3d+2ex^2\right)\text{csch}^{-1}(cx)\right)$$

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(16*e - 2*c^2*(15*d
 + 4*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*Ar
cCsch[c*x]))/180
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\,\mathrm{arccsch}(cx)e\,x^6}{6}+\frac{\mathrm{arccsch}(cx)x^4c^4d}{4}+\frac{\left(c^2x^2+1\right)\left(6c^4e\,x^4+15c^4d\,x^2-8e\,c^2x^2-30c^2d+16e\right)}{180c^3\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x}\right)}{c^4}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^2}{2}-\frac{\left(e\,c^2x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}+b\left(-\frac{\mathrm{arccsch}(cx)c^6d^3}{12e^2}+\frac{\mathrm{arccsch}(cx)c^6d\,x^4}{4}+\frac{e\,\mathrm{arccsch}(cx)c^6x^6}{6}+\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\,\mathrm{arc}\right.}{c^4}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^2}{2}-\frac{\left(e\,c^2x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}+b\left(-\frac{\mathrm{arccsch}(cx)c^6d^3}{12e^2}+\frac{\mathrm{arccsch}(cx)c^6d\,x^4}{4}+\frac{e\,\mathrm{arccsch}(cx)c^6x^6}{6}+\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\,\mathrm{arc}\right.}{c^4}$

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arccsch(c*x)*e*x^6+1/4*arccsch(c*x)*
x^4*c^4*d+1/180/c^3*(c^2*x^2+1)*(6*c^4*e*x^4+15*c^4*d*x^2-8*c^2*e*x^2-30*c^
2*d+16*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

$$\int x^3\left(d+ex^2\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{30\,ac^5ex^6+45\,ac^5dx^4+15\left(2\,bc^5ex^6+3\,bc^5dx^4\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(6\,bc^4ex^5+\left(15\,bc^4d-8\,bc^2e\right)x^3-2\right.}{180\,c^5}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4)
*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e*x^5 + (15*
b*c^4*d - 8*b*c^2*e)*x^3 - 2*(15*b*c^2*d - 8*b*e)*x)*sqrt((c^2*x^2 + 1)/(c^
2*x^2)))/c^5
```

Sympy [F]

$$\int x^3 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int x^3 (a+b\,\text{acsch}\,(cx)) \left(d+ex^2\right) \, dx$$

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int x^3 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx \\
&= \frac{1}{6}\, aex^6 + \frac{1}{4}\, adx^4 + \frac{1}{12}\left(3\, x^4 \,\text{arcsch}\,(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\, x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right) bd \\
&\quad + \frac{1}{90}\left(15\, x^6 \,\text{arcsch}\,(cx) + \frac{3\, c^4 x^5 \left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\, c^2 x^3 \left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\, x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right) be
\end{aligned}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2
) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsch(c
*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/
2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e
```

Giac [F]

$$\int x^3 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b\,\text{arcsch}\,(cx)+a)x^3 \, dx$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d + e x^2\right) \left(a + b \operatorname{csch}^{-1}(c x)\right) \, dx = \int x^3 \left(e\, x^2 + d\right) \left(a + b \operatorname{asinh}\left(\frac{1}{c\, x}\right)\right) \, dx$$

```
[In] int(x^3*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.85 $\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 607
Rubi [A] (verified) . . . 607
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Optimal result

Integrand size = 17, antiderivative size = 146

$$\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bex(-1-c^2x^2)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e} - \frac{bcd^2x\arctan\left(\sqrt{-1-c^2x^2}\right)}{4e\sqrt{-c^2x^2}}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arccsch(c*x))/e-1/12*b*e*x*(-c^2*x^2-1)^(3/2)/c^3/(-c^
2*x^2)^(1/2)-1/4*b*c*d^2*x*arctan((-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/
4*b*(2*c^2*d-e)*x*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6435, 457, 90, 65, 211}

$$\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e} - \frac{bcd^2x\arctan\left(\sqrt{-c^2x^2-1}\right)}{4e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}(2c^2d-e)}{4c^3\sqrt{-c^2x^2}} - \frac{bex(-c^2x^2-1)^{3/2}}{12c^3\sqrt{-c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(2*c^2*d - e)*x*Sqrt[-1 - c^2*x^2])/(4*c^3*Sqrt[-(c^2*x^2)]) - (b*e*x*(-
1 - c^2*x^2)^(3/2))/(12*c^3*Sqrt[-(c^2*x^2)]) + ((d + e*x^2)^2*(a + b*ArcCs
ch[c*x]))/(4*e) - (b*c*d^2*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(4*e*Sqrt[-(c^2*x^
2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{(bcx)\int \frac{(d+ex^2)^2}{x\sqrt{-1-c^2x^2}}\,dx}{4e\sqrt{-c^2x^2}}$$

$$= \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{(d+ex)^2}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{8e\sqrt{-c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e} - \frac{(bcx)\text{Subst}\left(\int \left(-\frac{e\left(-2c^2d+e\right)}{c^2\sqrt{-1-c^2x}} + \frac{d^2}{x\sqrt{-1-c^2x}} - \frac{e^2\sqrt{-1-c^2x}}{c^2}\right)\,dx, x, x^2\right)}{8e\sqrt{-c^2x^2}}$$

$$= \frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e} - \frac{(bcd^2x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{8e\sqrt{-c^2x^2}}$$

$$= \frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e} + \frac{(bd^2x)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{4ce\sqrt{-c^2x^2}}$$

$$= \frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e} - \frac{bcd^2x\arctan\left(\sqrt{-1-c^2x^2}\right)}{4e\sqrt{-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.53

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{x\left(3ac^3x(2d+ex^2) + b\sqrt{1+\frac{1}{c^2x^2}}(-2e+c^2(6d+ex^2)) + 3bc^3x(2d+ex^2)\,\text{csch}^{-1}(cx)\right)}{12c^3}$$

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-2*e + c^2*(6*d + e*
x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsch[c*x]))/(12*c^3)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

method	result
parts	$\frac{a\left(e x^2+d\right)^2}{4e}+\frac{b\left(\frac{c^2 e \operatorname{arccsch}(cx) x^4}{4}+\frac{\operatorname{arccsch}(cx) x^2 c^2 d}{2}+\frac{c^2 \operatorname{arccsch}(cx) d^2}{4e}-\frac{\sqrt{c^2x^2+1}\left(3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-e^2c^2x^2\sqrt{c^2}\right.\right.}{12c^3e\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}$
derivativedivides	$\frac{a\left(e c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^4d^2}{4e}+\frac{\operatorname{arccsch}(cx)c^4dx^2}{2}+\frac{e\operatorname{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+1}\left(-3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)+6c^2de\sqrt{c^2}\right.\right.}{12e\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}$
default	$\frac{a\left(e c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^4d^2}{4e}+\frac{\operatorname{arccsch}(cx)c^4dx^2}{2}+\frac{e\operatorname{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+1}\left(-3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)+6c^2de\sqrt{c^2}\right.\right.}{12e\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}$

```
[In] int(x*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x^2+d)^2/e+b/c^2*(1/4*c^2*e*arccsch(c*x)*x^4+1/2*arccsch(c*x)*x^2*
c^2*d+1/4*c^2/e*arccsch(c*x)*d^2-1/12/c^3/e*(c^2*x^2+1)^(1/2)*(3*c^4*d^2*ar
ctanh(1/(c^2*x^2+1)^(1/2))-e^2*c^2*x^2*(c^2*x^2+1)^(1/2)-6*c^2*d*e*(c^2*x^2
+1)^(1/2)+2*e^2*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int x\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{3\,ac^3ex^4+6\,ac^3dx^2+3\left(bc^3ex^4+2\,bc^3dx^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(bc^2ex^3+2\left(3\,bc^2d-be\right)x\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e*x^3 + 2*(3*b*c^2*d
 - b*e)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]

$$\int x\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\operatorname{acsch}(cx))\left(d+ex^2\right)\,dx$$

```
[In] integrate(x*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\begin{aligned}&\int x\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx\\&=\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd\\&\quad+\frac{1}{12}\left(3\,x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be\end{aligned}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x
*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e
```

Giac [F]

$$\int x\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)(b\,\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x\left(e\,x^2+d\right)\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.86 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal result . . . 613
Rubi [A] (verified) . . . 613
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Maple [F] . . . 617
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Giac [F] . . . 618
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Optimal result

Integrand size = 19, antiderivative size = 115

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = {} & \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 \\ & +\frac{1}{2}ex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\ & -bd\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\ & +bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right)-d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ & -\frac{1}{2}bd\,\mathrm{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/2*b*d*arccsch(c*x)^2+1/2*e*x^2*(a+b*arccsch(c*x))-b*d*arccsch(c*x)*ln(1-(
1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*d*arccsch(c*x)*ln(1/x)-d*(a+b*arccsch(c*x))
*ln(1/x)-1/2*b*d*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1+1/c^
2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules

used = {6439, 14, 5822, 6874, 270, 2362, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = -d\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right)$$
$$+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}-\frac{1}{2}bd\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)$$
$$+\frac{1}{2}bd\text{csch}^{-1}(cx)^2-bd\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)$$
$$+bd\log\left(\frac{1}{x}\right)\text{csch}^{-1}(cx)$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (b*d*ArcCsch[c*x]^2)/2 + (e*x^2*(a +
b*ArcCsch[c*x]))/2 - b*d*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + b*d*Arc
Csch[c*x]*Log[x^(-1)] - d*(a + b*ArcCsch[c*x])*Log[x^(-1)] - (b*d*PolyLog[2
, E^(2*ArcCsch[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e}{2x^2}+d\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{e}{2x^2\sqrt{1+\frac{x^2}{c^2}}} + \frac{d\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\right)\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(be)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - (bd)\text{Subst}\left(\int \frac{\text{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - (bd)\text{Subst}\left(\int x\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (2bd)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (bd)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$
\begin{aligned}
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - bd\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}(bd)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - bd\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{1}{2}bd\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = &\frac{1}{2}aex^2 + \frac{bex\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bex^2\operatorname{csch}^{-1}(cx) \\
&+ \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 - bd\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&+ ad\log(x) - \frac{1}{2}bd\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (a*e*x^2)/2 + (b*e*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcCsc
h[c*x])/2 + (b*d*ArcCsch[c*x]^2)/2 - b*d*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[
c*x])] + a*d*Log[x] - (b*d*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Maple [F]

$$
\int \frac{\left(e\,x^2+d\right)\left(a+b\,\operatorname{arccsch}\left(cx\right)\right)}{x}dx
$$

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)}{x}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
[Out] 2*b*c^2*d*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqr
t(c^2*x^2 + 1) + 1), x) - 1/2*b*e*x^2*log(c) - 1/2*b*e*x^2*log(x) + 1/2*a*e
*x^2 - b*d*log(c)*log(x) - 1/2*b*d*log(x)^2 - 1/4*(2*log(c^2*x^2 + 1)*log(x
) + dilog(-c^2*x^2))*b*d + a*d*log(x) + 1/2*(b*e*x^2 + 2*b*d*log(x))*log(sq
rt(c^2*x^2 + 1) + 1) + 1/4*b*e*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2
 + 1/4*b*e*log(c^2*x^2 + 1)/c^2
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.87 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result . . . 620
Rubi [A] (verified) . . . 620
Mathematica [A] (verified) . . . 625
Maple [F] . . . 625
Fricas [F] . . . 625
Sympy [F] . . . 626
Maxima [F] . . . 626
Giac [F] . . . 626
Mupad [F(-1)] . . . 626

Optimal result

Integrand size = 19, antiderivative size = 128

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = {}& \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) \\ & + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} \\ & - be\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) \\ & + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ & - \frac{1}{2}be\,\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)\end{aligned}$$

```
[Out] -1/4*b*c^2*d*arccsch(c*x)+1/2*b*e*arccsch(c*x)^2-1/2*d*(a+b*arccsch(c*x))/x
^2-b*e*arccsch(c*x)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*e*arccsch(c*x)*ln
(1/x)-e*(a+b*arccsch(c*x))*ln(1/x)-1/2*b*e*polylog(2,(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)+1/4*b*c*d*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules

used = {6439, 14, 5822, 12, 6874, 327, 221, 2362, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} - e\log\left(\frac{1}{x}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) - \frac{1}{2}be\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - be\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + be\log\left(\frac{1}{x}\right)\operatorname{csch}^{-1}(cx)$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*d*ArcCsch[c*x])/4 + (b*e*ArcCs
ch[c*x]^2)/2 - (d*(a + b*ArcCsch[c*x]))/(2*x^2) - b*e*ArcCsch[c*x]*Log[1 -
E^(2*ArcCsch[c*x])] + b*e*ArcCsch[c*x]*Log[x^(-1)] - e*(a + b*ArcCsch[c*x])
*Log[x^(-1)] - (b*e*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+2e\log(x)}{2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+2e\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{1+\frac{x^2}{c^2}}} + \frac{2e\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\right)\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} + \frac{(be)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + be\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - e\Big(a + b\text{csch}^{-1}(cx)\Big)\log\left(\frac{1}{x}\right) - \frac{1}{4}(bcd)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) - (be)\text{Subst}\left(\int \frac{\text{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - (be)\text{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (2be)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} \\
&\quad - be\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (be)\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} \\
&\quad - be\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}(be)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\operatorname{csch}^{-1}(cx) + \frac{1}{2}be\operatorname{csch}^{-1}(cx)^2 - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} \\
&\quad - be\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + be\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{1}{2}be\,\text{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = &-\frac{ad}{2x^2}+\frac{bcd\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{4x}-\frac{bd\operatorname{csch}^{-1}(cx)}{2x^2}\\ &+\frac{1}{2}be\operatorname{csch}^{-1}(cx)^2-\frac{1}{4}bc^2d\operatorname{arcsinh}\left(\frac{1}{cx}\right)\\ &-be\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\\ &+ae\log(x)-\frac{1}{2}be\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)\end{aligned}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 + (b*c*d*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcCsch
[c*x])/(2*x^2) + (b*e*ArcCsch[c*x]^2)/2 - (b*c^2*d*ArcSinh[1/(c*x)])/4 - b*
e*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + a*e*Log[x] - (b*e*PolyLog[2, E
^(2*ArcCsch[c*x])])/2
```

Maple [F]

$$\int \frac{(e\,x^2+d)\left(a+b\ \operatorname{arccsch}\left(cx\right)\right)}{x^3}dx$$

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcsch}\left(cx\right)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**3,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
[Out] -1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log(
x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x))*
log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) + 2*
integrate(log(x)/(c^2*x^3 + x), x))*b*e + 1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2
) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
+ 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2) + a*
e*log(x) - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{cx}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.88 $\int x^2(d+ex^2)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 627
Rubi [A] (verified) . . . 628
Mathematica [A] (verified) . . . 631
Maple [A] (verified) . . . 631
Fricas [A] (verification not implemented) . . . 632
Sympy [F] . . . 632
Maxima [A] (verification not implemented) . . . 632
Giac [F] . . . 633
Mupad [F(-1)] . . . 634

Optimal result

Integrand size = 21, antiderivative size = 260

$$\begin{aligned}\int x^2(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx &= \frac{b(280c^4d^2-252c^2de+75e^2)\,x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}\\ &+\frac{b(84c^2d-25e)\,ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\\ &+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &+\frac{b(280c^4d^2-252c^2de+75e^2)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{1680c^6\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] 1/3*d^2*x^3*(a+b*arccsch(c*x))+2/5*d*e*x^5*(a+b*arccsch(c*x))+1/7*e^2*x^7*(
a+b*arccsch(c*x))+1/1680*b*(280*c^4*d^2-252*c^2*d*e+75*e^2)*x*arctan(c*x/(-
c^2*x^2-1)^(1/2))/c^6/(-c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2-252*c^2*d*e+75
*e^2)*x^2*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)+1/840*b*(84*c^2*d-25*e)*e
*x^4*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(-c^2*x^2-1)^(1
/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {276, 6437, 12, 1281, 470, 327, 223, 209}

$$\begin{aligned}\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx &= \frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &\quad+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &\quad+\frac{bx\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)\left(280c^4d^2-252c^2de+75e^2\right)}{1680c^6\sqrt{-c^2x^2}}\\ &\quad+\frac{be^2x^6\sqrt{-c^2x^2-1}}{42c\sqrt{-c^2x^2}}\\ &\quad+\frac{bex^4\sqrt{-c^2x^2-1}(84c^2d-25e)}{840c^3\sqrt{-c^2x^2}}\\ &\quad+\frac{bx^2\sqrt{-c^2x^2-1}(280c^4d^2-252c^2de+75e^2)}{1680c^5\sqrt{-c^2x^2}}\end{aligned}$$

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 - c^2*x^2])/(1680*c^5*S
qrt[-(c^2*x^2)]) + (b*(84*c^2*d - 25*e)*e*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*
Sqrt[-(c^2*x^2)]) + (b*e^2*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)])
 + (d^2*x^3*(a + b*ArcCsch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsch[c*x]))/5 + (
e^2*x^7*(a + b*ArcCsch[c*x]))/7 + (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x
*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(1680*c^6*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)\\ &\quad+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(35d^2+42dex^2+15e^2x^4\right)}{105\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(35d^2+42dex^2+15e^2x^4\right)}{\sqrt{-1-c^2x^2}}\,dx}{105\sqrt{-c^2x^2}}\\
&= \frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(bx)\int\frac{x^2\left(-210c^2d^2-3\left(84c^2d-25e\right)ex^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{630c\sqrt{-c^2x^2}}\\
&= \frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{\left(b\left(-840c^4d^2+9\left(84c^2d-25e\right)e\right)x\right)\int\frac{x^2}{\sqrt{-1-c^2x^2}}\,dx}{2520c^3\sqrt{-c^2x^2}}\\
&= \frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\\
&\quad+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(b\left(-840c^4d^2+9\left(84c^2d-25e\right)e\right)x\right)\int\frac{1}{\sqrt{-1-c^2x^2}}\,dx}{5040c^5\sqrt{-c^2x^2}}\\
&= \frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\\
&\quad+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad-\frac{\left(b\left(-840c^4d^2+9\left(84c^2d-25e\right)e\right)x\right)\operatorname{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,\frac{x}{\sqrt{-1-c^2x^2}}\right)}{5040c^5\sqrt{-c^2x^2}}\\
&= \frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b\left(84c^2d-25e\right)ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\\
&\quad+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\left(280c^4d^2-252c^2de+75e^2\right)x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{1680c^6\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.70

$$\int x^2\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{c^2x^2\Big(16ac^5x(35d^2+42dex^2+15e^2x^4)+b\sqrt{1+\frac{1}{c^2x^2}}(75e^2-2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+$$

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 + 1/(c^2
*x^2)]*(75*e^2 - 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x]
 + b*(-280*c^4*d^2 + 252*c^2*d*e - 75*e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*
x])/(1680*c^7)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.04

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\left(\frac{c^3\,\text{arccsch}(cx)e^2x^7}{7}+\frac{2c^3\,\text{arccsch}(cx)de\,x^5}{5}+\frac{\text{arccsch}(cx)d^2x^3c^3}{3}-\frac{\sqrt{c^2x^2+1}\left(-40e\right.}{}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\text{arccsch}(cx)d^2c^7x^3}{3}+\frac{2\,\text{arccsch}(cx)d\,c^7e\,x^5}{5}+\frac{\text{arccsch}(cx)e^2c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\left(280d^2c^5x\sqrt{c^2}\right.}{}\right.}{}$
default	$\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\text{arccsch}(cx)d^2c^7x^3}{3}+\frac{2\,\text{arccsch}(cx)d\,c^7e\,x^5}{5}+\frac{\text{arccsch}(cx)e^2c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\left(280d^2c^5x\sqrt{c^2}\right.}{}\right.}{}$

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+b/c^3*(1/7*c^3*arccsch(c*x)*e^2*x^7
+2/5*c^3*arccsch(c*x)*d*e*x^5+1/3*arccsch(c*x)*d^2*x^3*c^3-1/1680/c^5*(c^2*
x^2+1)^(1/2)*(-40*e^2*c^5*x^5*(c^2*x^2+1)^(1/2)-168*d*c^5*e*x^3*(c^2*x^2+1)
^(1/2)-280*d^2*c^5*x*(c^2*x^2+1)^(1/2)+280*d^2*c^4*arcsinh(c*x)+50*e^2*c^3*
x^3*(c^2*x^2+1)^(1/2)+252*d*c^3*e*x*(c^2*x^2+1)^(1/2)-252*d*c^2*e*arcsinh(c
*x)-75*e^2*c*x*(c^2*x^2+1)^(1/2)+75*e^2*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)
^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.49

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\left(35\,bc^7d^2+42\,bc^7de+15\,bc^7e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-}{}$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(35*
b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) - c*x + 1) + (280*b*c^4*d^2 - 252*b*c^2*d*e + 75*b*e^2)*log(c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) - c*x) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*
e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 16*(15*b*c^7*e^2*x^
7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e - 15*
b*c^7*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (40*b*c^6*e
^2*x^6 + 2*(84*b*c^6*d*e - 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 - 252*b*c^4*d
*e + 75*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^2(a+b\,\operatorname{acsch}(cx))\left(d+ex^2\right)^2\,dx$$

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.52

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3$$
$$+\frac{1}{12}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2$$
$$+\frac{1}{40}\left(16x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde$$
$$+\frac{1}{672}\left(96x^7\operatorname{arcsch}(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3c^6\left(\frac{1}{c^2x^2}+1\right)^2+3c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsch(c*x) +
(2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^
2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(16*
x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))
/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(
c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*e + 1
/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2
) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(
1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(sqrt(1/(c^2*x^
2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

Giac [F]

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2(b\operatorname{arcsch}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^2 \left(e\,x^2+d\right)^2 \left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.89 $\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx$

Optimal result . . . 635
Rubi [A] (verified) . . . 635
Mathematica [A] (verified) . . . 638
Maple [A] (verified) . . . 638
Fricas [B] (verification not implemented) . . . 639
Sympy [F] . . . 640
Maxima [A] (verification not implemented) . . . 640
Giac [F] . . . 641
Mupad [F(-1)] . . . 641

Optimal result

Integrand size = 18, antiderivative size = 197

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = &\frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}\\ &+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)\\ &+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)\\ &-\frac{b(120c^4d^2-40c^2de+9e^2)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{120c^4\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] d^2*x*(a+b*arccsch(c*x))+2/3*d*e*x^3*(a+b*arccsch(c*x))+1/5*e^2*x^5*(a+b*ar
ccsch(c*x))-1/120*b*(120*c^4*d^2-40*c^2*d*e+9*e^2)*x*arctan(c*x/(-c^2*x^2-1
)^(1/2))/c^4/(-c^2*x^2)^(1/2)+1/120*b*(40*c^2*d-9*e)*e*x^2*(-c^2*x^2-1)^(1/
2)/c^3/(-c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(-c^2*x^2-1)^(1/2)/c/(-c^2*x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used

$= \{200, 6427, 12, 1173, 396, 223, 209\}$

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = d^2x\left(a+b\text{csch}^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right) - \frac{bx\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)\left(120c^4d^2-40c^2de+9e^2\right)}{120c^4\sqrt{-c^2x^2}} + \frac{be^2x^4\sqrt{-c^2x^2-1}}{20c\sqrt{-c^2x^2}} + \frac{bex^2\sqrt{-c^2x^2-1}(40c^2d-9e)}{120c^3\sqrt{-c^2x^2}}$$

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(40*c^2*d - 9*e)*e*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) +
(b*e^2*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + d^2*x*(a + b*ArcCs
ch[c*x]) + (2*d*e*x^3*(a + b*ArcCsch[c*x]))/3 + (e^2*x^5*(a + b*ArcCsch[c*x
]))/5 - (b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*x*ArcTan[(c*x)/Sqrt[-1 - c^2*
x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{15d^2+10dex^2+3e^2x^4}{15\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\\
&= d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{15d^2+10dex^2+3e^2x^4}{\sqrt{-1-c^2x^2}}\,dx}{15\sqrt{-c^2x^2}}\\
&= \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(bx)\int\frac{-60c^2d^2-(40c^2d-9e)ex^2}{\sqrt{-1-c^2x^2}}\,dx}{60c\sqrt{-c^2x^2}}\\
&= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)\\
&\quad+\frac{\left(b(-120c^4d^2+(40c^2d-9e)\,e\right)x)\int\frac{1}{\sqrt{-1-c^2x^2}}\,dx}{120c^3\sqrt{-c^2x^2}}
\end{aligned}
$$

$$= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$+ \frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$+ \frac{\left(b(-120c^4d^2+(40c^2d-9e)\,e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{x}{\sqrt{-1-c^2x^2}}\right)}{120c^3\sqrt{-c^2x^2}}$$

$$= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$+ \frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$- \frac{b(120c^4d^2-40c^2de+9e^2)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{120c^4\sqrt{-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int \left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)+be\sqrt{1+\frac{1}{c^2x^2}}x(-9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2x}{120c^5}$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/(c^2*x^2
)]*x*(-9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^
2*x^4)*ArcCsch[c*x] + b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1
+ 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+x\,d^2\right)+\frac{b\left(\frac{c\ \mathrm{arccsch}(cx)e^2x^5}{5}+\frac{2c\ \mathrm{arccsch}(cx)de\,x^3}{3}+\mathrm{arccsch}(cx)xc\,d^2+\frac{\sqrt{c^2x^2+1}\left(120d^2c^4\ \mathrm{ar}\right.\right.}{}$
derivativedivides	$\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\mathrm{arccsch}(cx)d^2c^5x+\frac{2\ \mathrm{arccsch}(cx)d\,c^5e\,x^3}{3}+\frac{\mathrm{arccsch}(cx)e^2c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\left(120d^2c^4\ \mathrm{arcsinh}(cx\right.\right.}{c}$
default	$\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\mathrm{arccsch}(cx)d^2c^5x+\frac{2\ \mathrm{arccsch}(cx)d\,c^5e\,x^3}{3}+\frac{\mathrm{arccsch}(cx)e^2c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\left(120d^2c^4\ \mathrm{arcsinh}(cx\right.\right.}{c}$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e^2*x^5+2/3*d*e*x^3+x*d^2)+b/c*(1/5*c*arccsch(c*x)*e^2*x^5+2/3*c*arc
csch(c*x)*d*e*x^3+arccsch(c*x)*x*c*d^2+1/120/c^5*(c^2*x^2+1)^(1/2)*(120*d^2
*c^4*arcsinh(c*x)+40*d*c^3*e*x*(c^2*x^2+1)^(1/2)+6*e^2*c^3*x^3*(c^2*x^2+1)^
(1/2)-40*d*c^2*e*arcsinh(c*x)-9*e^2*c*x*(c^2*x^2+1)^(1/2)+9*e^2*arcsinh(c*x
))/x/((c^2*x^2+1)/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(175)=350$.

Time $=0.33$ (sec) , antiderivative size $=353$, normalized size of antiderivative $=1.79$

$$\int \left(d+ex^2\right)^2\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\left(15\,bc^5d^2+10\,bc^5de+3\,bc^5e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-(12}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(15*b*c^5*
d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x + 1) - (120*b*c^4*d^2 - 40*b*c^2*d*e + 9*b*e^2)*log(c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - c*x) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*
d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e^2*x^4 + (40*b*c^
4*d*e - 9*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(a+b\,\text{acsch}\left(cx\right)\right) \left(d+ex^2\right)^2\, dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 \\
&+\frac{1}{6}\left(4\,x^3\,\text{arcsch}\left(cx\right) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right) bde \\
&+\frac{1}{80}\left(16\,x^5\,\text{arcsch}\left(cx\right) - \frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2 - 2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4} - \frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right) be^2 \\
&+ad^2x + \frac{\left(2\,cx\,\text{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^2}{2\,c}
\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^
2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2
+ log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsch(c*x)
- (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2
) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1
)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/2*(2*c
*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1
) - 1))*b*d^2/c
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsch}\,(cx)+a)\, dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(e\,x^2+d\right)^2 \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int((d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int((d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.90 $\int \frac{\left(d+ex^2\right)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result . . . 642
Rubi [A] (verified) . . . 642
Mathematica [A] (verified) . . . 645
Maple [A] (verified) . . . 646
Fricas [B] (verification not implemented) . . . 646
Sympy [F] . . . 647
Maxima [A] (verification not implemented) . . . 647
Giac [F] . . . 648
Mupad [F(-1)] . . . 648

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{b(12c^2d-e)\,ex\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6c^2\sqrt{-c^2x^2}}$$

```
[Out] -d^2*(a+b*arccsch(c*x))/x+2*d*e*x*(a+b*arccsch(c*x))+1/3*e^2*x^3*(a+b*arccs
ch(c*x))-1/6*b*(12*c^2*d-e)*e*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^2/(-c^2*x^
2)^(1/2)+b*c*d^2*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(-c^2*x^
2-1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{276, 6437, 12, 1279, 396, 223, 209\}$

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{bex\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)\left(12c^2d-e\right)}{6c^2\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-c^2x^2-1}}{6c\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] + (b*e^2*x^2*Sqrt[-1 - c^2*x^
2])/(6*c*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/x + 2*d*e*x*(a + b*
ArcCsch[c*x]) + (e^2*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(12*c^2*d - e)*e*x*Ar
cTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{-3d^2+6dex^2+e^2x^4}{3x^2\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{-3d^2+6dex^2+e^2x^4}{x^2\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{6de+e^2x^2}{\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} \\
&\quad - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{\left(b\left(12c^2de-e^2\right)x\right)\int \frac{1}{\sqrt{-1-c^2x^2}}\,dx}{6c\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$+ \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{\left(b(12c^2de-e^2)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{x}{\sqrt{-1-c^2x^2}}\right)}{6c\sqrt{-c^2x^2}}$$
$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$
$$+ 2dex\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$- \frac{b(12c^2d-e)\,ex\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6c^2\sqrt{-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.79

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$
$$= \frac{c^2\left(b\sqrt{1+\frac{1}{c^2x^2}}x(6c^2d^2+e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\operatorname{csch}^{-1}(cx)+}{6c^3x}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (c^2*(b*Sqrt[1 + 1/(c^2*x^2)]*x*(6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*d
*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsch[c*x] +
b*(12*c^2*d - e)*e*x*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3*x)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\left(\frac{\operatorname{arccsch}(cx)x^3e^2}{3c}+\frac{2\operatorname{arccsch}(cx)dex}{c}-\frac{\operatorname{arccsch}(cx)d^2}{xc}+\frac{\sqrt{c^2x^2+1}\left(6c^4d^2\sqrt{c^2}\right.\right.$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arccsch}(cx)c^3dex+\frac{\operatorname{arccsch}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arccsch}(cx)c^3d^2}{x}+\frac{\sqrt{c^2x^2+1}\left(6c^4d^2\sqrt{c^2x^2+1}\right.\right.}{c^4}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arccsch}(cx)c^3dex+\frac{\operatorname{arccsch}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arccsch}(cx)c^3d^2}{x}+\frac{\sqrt{c^2x^2+1}\left(6c^4d^2\sqrt{c^2x^2+1}\right.\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arccsch(c*x)*x^3*e^2+2/c*arccsch(c
*x)*d*e*x-arccsch(c*x)*d^2/x/c+1/6/c^6*(c^2*x^2+1)^(1/2)*(6*c^4*d^2*(c^2*x^
2+1)^(1/2)+12*c^3*d*e*arcsinh(c*x)*x+e^2*c^2*x^2*(c^2*x^2+1)^(1/2)-arcsinh(
c*x)*e^2*c*x)/x^2/((c^2*x^2+1)/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(152) = 304.

Time = 0.32 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.04

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

$$=\frac{2\,ac^3e^2x^4+6\,bc^4d^2x+12\,ac^3dex^2-6\,ac^3d^2-2\left(3\,bc^3d^2-6\,bc^3de-bc^3e^2\right)x\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*(
3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) - c*x + 1) - (12*b*c^2*d*e - b*e^2)*x*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) - c*x) + 2*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log(c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*
c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*d^2*x + b*c^2*e^2*x^3)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/(c^3*x)
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^2}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**2,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx \\
&= \frac{1}{3}\,ae^2x^3 + \left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\text{arcsch}\left(cx\right)}{x}\right)bd^2 \\
&\quad + \frac{1}{12}\left(4\,x^3\,\text{arcsch}\left(cx\right) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 \\
&\quad + 2\,adex + \frac{\left(2\,cx\,\text{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bde}{c} - \frac{ad^2}{x}
\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
[Out] 1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d^2 + 1/12*(4*
x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) -
 log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c
)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1)
- log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d*e/c - a*d^2/x
```

Giac [F]

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(e\,x^2+d)^2\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.91 $\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result . . . 649
Rubi [A] (verified) . . . 649
Mathematica [A] (verified) . . . 652
Maple [A] (verified) . . . 652
Fricas [B] (verification not implemented) . . . 653
Sympy [F] . . . 653
Maxima [A] (verification not implemented) . . . 653
Giac [F] . . . 654
Mupad [F(-1)] . . . 654

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{be^2x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

```
[Out] -1/3*d^2*(a+b*arccsch(c*x))/x^3-2*d*e*(a+b*arccsch(c*x))/x+e^2*x*(a+b*arccs
ch(c*x))-b*e^2*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/(-c^2*x^2)^(1/2)-2/9*b*c*d*
(c^2*d-9*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/9*b*c*d^2*(-c^2*x^2-1)^(1
/2)/x^2/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 6437, 12, 1279, 462, 223, 209}

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{be^2x\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}} - \frac{2bcd\sqrt{-c^2x^2-1}(c^2d-9e)}{9\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*d*(c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/(9*Sqrt[-(c^2*x^2)]) + (b*c*d^2
*Sqrt[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/
(3*x^3) - (2*d*e*(a + b*ArcCsch[c*x]))/x + e^2*x*(a + b*ArcCsch[c*x]) - (b*
e^2*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(c^2*x^2)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} \\
&\quad + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{-d^2-6dex^2+3e^2x^4}{3x^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} \\
&\quad + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{-d^2-6dex^2+3e^2x^4}{x^4\sqrt{-1-c^2x^2}}\,dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} \\
&\quad + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bcx)\int \frac{2d(c^2d-9e)+9e^2x^2}{x^2\sqrt{-1-c^2x^2}}\,dx}{9\sqrt{-c^2x^2}} \\
&= -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bce^2x)\int \frac{1}{\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} \\
&\quad - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} \\
&\quad + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{(bce^2x)\operatorname{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{x}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}} \\
&= -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{be^2x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$
$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}x(d-2c^2dx^2+18ex^2)-3a(d^2+6dex^2-3e^2x^4)}{9x^3}$$
$$-\frac{b(d^2+6dex^2-3e^2x^4)\operatorname{csch}^{-1}(cx)}{3x^3}+\frac{be^2\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{c}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2 + 6*
d*e*x^2 - 3*e^2*x^4))/(9*x^3) - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsch[c*
x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/c
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.14

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,c^3\left(\frac{\operatorname{arccsch}(cx)e^2x}{c^3}-\frac{2\operatorname{arccsch}(cx)de}{c^3x}-\frac{\operatorname{arccsch}(cx)d^2}{3x^3c^3}-\frac{\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2+1}\,c$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arccsch}(cx)e^2cx-\frac{\operatorname{arccsch}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arccsch}(cx)cde}{x}-\frac{\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2+1}\,c^6d^2x^2-c^4d^2\sqrt{}\right.\right.\right.}{9}}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arccsch}(cx)e^2cx-\frac{\operatorname{arccsch}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arccsch}(cx)cde}{x}-\frac{\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2+1}\,c^6d^2x^2-c^4d^2\sqrt{}\right.\right.\right.}{9}}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*c^3*(1/c^3*arccsch(c*x)*e^2*x-2/c^3*arccsch
(c*x)*d*e/x-1/3*arccsch(c*x)*d^2/x^3/c^3-1/9/c^8*(c^2*x^2+1)^(1/2)*(2*(c^2*
x^2+1)^(1/2)*c^6*d^2*x^2-c^4*d^2*(c^2*x^2+1)^(1/2)-18*c^4*d*e*(c^2*x^2+1)^(
1/2)*x^2-9*e^2*arcsinh(c*x)*c^3*x^3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(146) = 292$.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.04

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

$$= \frac{9\,ace^2x^4 - 9\,be^2x^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - 18\,acdex^2 - 3\left(bcd^2+6\,bcde-3\,bce^2\right)x^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - c\right.}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 - 9*b*e^2*x^3*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*
x) - 18*a*c*d*e*x^2 - 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x
^3*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*a*c*d^2 - 2*(b*c^4*
d^2 - 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2 + (b*c*
d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c*x)) + (b*c^2*d^2*x - 2*(b*c^4*d^2 - 9*b*c^2*d*e)*x^3)*sqrt((c^2*x^2 +
 1)/(c^2*x^2)))/(c*x^3)
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(a+b\operatorname{acsch}(cx)\right)\left(d+ex^2\right)^2}{x^4}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$
$$= 2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)bde+ae^2x$$
$$+\frac{1}{9}bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arcsch}(cx)}{x^3}\right)$$
$$+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)be^2}{2\,c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*((
c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*
x)/x^3) + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sq
rt(1/(c^2*x^2) + 1) - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Giac [F]

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(ex^2+d)^2(b\operatorname{arcsch}(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(e\,x^2+d)^2\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.92 $\int \frac{(d+ex^2)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result . . . 655
Rubi [A] (verified) . . . 655
Mathematica [A] (verified) . . . 658
Maple [A] (verified) . . . 658
Fricas [A] (verification not implemented) . . . 659
Sympy [F] . . . 659
Maxima [A] (verification not implemented) . . . 659
Giac [F] . . . 660
Mupad [F(-1)] . . . 660

Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{bc(24c^4d^2-100c^2de+225e^2)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd(6c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$

```
[Out] -1/5*d^2*(a+b*arccsch(c*x))/x^5-2/3*d*e*(a+b*arccsch(c*x))/x^3-e^2*(a+b*arc
csch(c*x))/x+1/225*b*c*(24*c^4*d^2-100*c^2*d*e+225*e^2)*(-c^2*x^2-1)^(1/2)/
(-c^2*x^2)^(1/2)+1/25*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)-2/225
*b*c*d*(6*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {276, 6437, 12, 1279, 464, 270}

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + \frac{bcd^2\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd\sqrt{-c^2x^2-1}(6c^2d-25e)}{225x^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-c^2x^2-1}(24c^4d^2-100c^2de+225e^2)}{225\sqrt{-c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c*(24*c^4*d^2 - 100*c^2*d*e + 225*e^2)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c
^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (2*b*c
*d*(6*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(
a + b*ArcCsch[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsch[c*x]))/(3*x^3) - (e^2*
(a + b*ArcCsch[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int \frac{-3d^2-10dex^2-15e^2x^4}{15x^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int \frac{-3d^2-10dex^2-15e^2x^4}{x^6\sqrt{-1-c^2x^2}}\,dx}{15\sqrt{-c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} \\
&\quad - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} - \frac{(bcx)\int \frac{2d(6c^2d-25e)-75e^2x^2}{x^4\sqrt{-1-c^2x^2}}\,dx}{75\sqrt{-c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd(6c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} \\
&\quad - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x} \\
&\quad - \frac{\left(bc(-24c^4d^2+100c^2de-225e^2)\,x\right)\int \frac{1}{x^2\sqrt{-1-c^2x^2}}\,dx}{225\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{bc(24c^4d^2 - 100c^2de + 225e^2)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd(6c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx$$

$$= \frac{-15a(3d^2+10dex^2+15e^2x^4) + bc\sqrt{1+\frac{1}{c^2x^2}}x(225e^2x^4 - 50dex^2(-1+2c^2x^2) + 3d^2(3-4c^2x^2+8c^4x^4))}{225x^5}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(225
*e^2*x^4 - 50*d*e*x^2*(-1 + 2*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4))
 - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x])/(225*x^5)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{2de}{3x^3}-\frac{d^2}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arccsch}(cx)e^2}{c^5x}-\frac{2\operatorname{arccsch}(cx)de}{3c^5x^3}-\frac{\operatorname{arccsch}(cx)d^2}{5x^5c^5}+\frac{(c^2x^2+1)(24c^8d^2x^4-$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5cx^5}-\frac{2de}{3cx^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{cx}-\frac{\operatorname{arccsch}(cx)d^2}{5cx^5}-\frac{2\operatorname{arccsch}(cx)de}{3cx^3}+\frac{\left(c^2x^2+1\right)\left(24c^8d^2x^4-100c^6dex^4-12c^6\right.}{225\sqrt{\frac{c^2x^2}{c^2x^2}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5cx^5}-\frac{2de}{3cx^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{cx}-\frac{\operatorname{arccsch}(cx)d^2}{5cx^5}-\frac{2\operatorname{arccsch}(cx)de}{3cx^3}+\frac{\left(c^2x^2+1\right)\left(24c^8d^2x^4-100c^6dex^4-12c^6\right.}{225\sqrt{\frac{c^2x^2}{c^2x^2}}}\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e^2/x-2/3*d*e/x^3-1/5*d^2/x^5)+b*c^5*(-1/c^5*arccsch(c*x)*e^2/x-2/3/c^5
*arccsch(c*x)*d*e/x^3-1/5*arccsch(c*x)*d^2/x^5/c^5+1/225/c^10*(c^2*x^2+1)*(
```

```
24*c^8*d^2*x^4-100*c^6*d*e*x^4-12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^
2+9*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx =$$

$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\left(15\,be^2x^4+10\,bdex^2+3\,bd^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-((24\,bc^5d^2-10}{225\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*
d*e*x^2 + 3*b*d^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - ((2
4*b*c^5*d^2 - 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x - 2*(6*b*c^3*d
^2 - 25*b*c*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^5
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\left(a+b\operatorname{acsch}(cx)\right)\left(d+ex^2\right)^2}{x^6}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx \\
&= \left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)be^2 \\
&\quad+\frac{1}{75}bd^2\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsch}(cx)}{x^5}\right) \\
&\quad+\frac{2}{9}bde\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arcsch}(cx)}{x^3}\right)-\frac{ae^2}{x}-\frac{2\,ade}{3\,x^3}-\frac{ad^2}{5\,x^5}
\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(c
^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^
2) + 1))/c - 15*arccsch(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) + 1)^(3/2)
 - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - a*e^2/x - 2/3*a*d
*e/x^3 - 1/5*a*d^2/x^5
```

Giac [F]

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(ex^2+d)^2(b\operatorname{arcsch}(cx)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(e\,x^2+d)^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.93 $\int \frac{(d+ex^2)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal result . . . 661
Rubi [A] (verified) . . . 662
Mathematica [A] (verified) . . . 664
Maple [A] (verified) . . . 665
Fricas [A] (verification not implemented) . . . 665
Sympy [F] . . . 666
Maxima [A] (verification not implemented) . . . 666
Giac [F] . . . 667
Mupad [F(-1)] . . . 667

Optimal result

Integrand size = 21, antiderivative size = 249

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = &-\frac{2bc^3(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025\sqrt{-c^2x^2}}\\ &+\frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}\\ &+\frac{bc(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025x^2\sqrt{-c^2x^2}}\\ &-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}\\ &-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}\end{aligned}$$

```
[Out] -1/7*d^2*(a+b*arccsch(c*x))/x^7-2/5*d*e*(a+b*arccsch(c*x))/x^5-1/3*e^2*(a+b
*arccsch(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*
x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/49*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^6/(-c^2*x^2
)^(1/2)-2/1225*b*c*d*(15*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2
)+1/11025*b*c*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*x^2-1)^(1/2)/x^2/(-
c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 6437, 12, 1279, 464, 277, 270}

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = &-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} \\ &- \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} + \frac{bcd^2\sqrt{-c^2x^2-1}}{49x^6\sqrt{-c^2x^2}} \\ &- \frac{2bcd\sqrt{-c^2x^2-1}(15c^2d-49e)}{1225x^4\sqrt{-c^2x^2}} \\ &+ \frac{bc\sqrt{-c^2x^2-1}(360c^4d^2-1176c^2de+1225e^2)}{11025x^2\sqrt{-c^2x^2}} \\ &- \frac{2bc^3\sqrt{-c^2x^2-1}(360c^4d^2-1176c^2de+1225e^2)}{11025\sqrt{-c^2x^2}}\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-2*b*c^3*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(1102
5*Sqrt[-(c^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]
) - (2*b*c*d*(15*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2
)]) + (b*c*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(110
25*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/(7*x^7) - (2*d*e*(a +
 b*ArcCsch[c*x]))/(5*x^5) - (e^2*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} \\ &\quad - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{-15d^2-42dex^2-35e^2x^4}{105x^8\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\ &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5} \\ &\quad - \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{-15d^2-42dex^2-35e^2x^4}{x^8\sqrt{-1-c^2x^2}}\,dx}{105\sqrt{-c^2x^2}}\end{aligned}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bcx)\int \frac{6d\left(15c^2d-49e\right)-245e^2x^2}{x^6\sqrt{-1-c^2x^2}}\,dx}{735\sqrt{-c^2x^2}}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{(bc(-360c^4d^2+1176c^2de-1225e^2)\,x)\int \frac{1}{x^4\sqrt{-1-c^2x^2}}\,dx}{3675\sqrt{-c^2x^2}}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{bc(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} + \frac{(2bc^3(-360c^4d^2+1176c^2de-1225e^2)\,x)\int \frac{1}{x^2\sqrt{-1-c^2x^2}}\,dx}{11025\sqrt{-c^2x^2}}$$

$$= -\frac{2bc^3(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{bc(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.61

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$$

$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}x(1225e^2x^4(1-2c^2x^2)+294dex^2(3-4c^2x^2+8c^4x^4)-45}{11025x^7}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]

[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1
225*e^2*x^4*(1 - 2*c^2*x^2) + 294*d*e*x^2*(3 - 4*c^2*x^2 + 8*c^4*x^4) - 45*
d^2*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2
 + 35*e^2*x^4)*ArcCsch[c*x])/(11025*x^7)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83

method	result
parts	$a\left(-\frac{e^2}{3x^3}-\frac{2de}{5x^5}-\frac{d^2}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arccsch}(cx)e^2}{3c^7x^3}-\frac{2\operatorname{arccsch}(cx)de}{5c^7x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7x^7c^7}-\frac{(c^2x^2+1)(720c^{10}$
derivativedivides	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arccsch}(cx)de}{5c^3x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arccsch}(cx)e^2}{3c^3x^3}-\frac{\left(c^2x^2+1\right)\left(720c^{10}d^2x^6-2352c^8d\right.}\right.}\right.$
default	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arccsch}(cx)de}{5c^3x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arccsch}(cx)e^2}{3c^3x^3}-\frac{\left(c^2x^2+1\right)\left(720c^{10}d^2x^6-2352c^8d\right.}\right.}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3/x^3*e^2-2/5*d*e/x^5-1/7*d^2/x^7)+b*c^7*(-1/3/c^7*arccsch(c*x)/x^3*e
^2-2/5/c^7*arccsch(c*x)*d*e/x^5-1/7*arccsch(c*x)*d^2/x^7/c^7-1/11025/c^12*(
c^2*x^2+1)*(720*c^10*d^2*x^6-2352*c^8*d*e*x^6-360*c^8*d^2*x^4+2450*c^6*e^2*
x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2-1225*c^4*e^2*x^4-882*c^4*d*e*x^2-225*c
^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^8)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx =$$

$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\left(35\,be^2x^4+42\,bdex^2+15\,bd^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(2\,(360$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + (2*(360*b*c^7*d^2 - 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 - (360*b*c^5*
d^2 - 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 - 225*b*c*d^2*x + 18*(15*b*c^3*d^2
 - 49*b*c*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^2}{x^8}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**8,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx \\
&= \frac{1}{245}\,bd^2\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35\,c^8\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{35\ \text{arcsch}\left(cx\right)}{x^7}\right) \\
&\quad + \frac{2}{75}\,bde\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{15\ \text{arcsch}\left(cx\right)}{x^5}\right) \\
&\quad + \frac{1}{9}\,be^2\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\ \text{arcsch}\left(cx\right)}{x^3}\right) - \frac{ae^2}{3\,x^3} - \frac{2\,ade}{5\,x^5} - \frac{ad^2}{7\,x^7}
\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
[Out] 1/245*b*d^2*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2
) + 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*a
rccsch(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(
c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5
) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/
c - 3*arccsch(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{\left(ex^2+d\right)^2(b\,\text{arcsch}\,(cx)+a)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{\left(e\,x^2+d\right)^2\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.94 $\int x^3(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 668
Rubi [A] (verified) . . . 668
Mathematica [A] (verified) . . . 671
Maple [A] (verified) . . . 671
Fricas [A] (verification not implemented) . . . 672
Sympy [F] . . . 672
Maxima [A] (verification not implemented) . . . 672
Giac [F] . . . 673
Mupad [F(-1)] . . . 673

Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}&\int x^3(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx\\&=-\frac{b(6c^4d^2-8c^2de+3e^2)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}-\frac{b(6c^4d^2-16c^2de+9e^2)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}\\&\quad+\frac{b(8c^2d-9e)\,ex(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}}-\frac{be^2x(-1-c^2x^2)^{7/2}}{56c^7\sqrt{-c^2x^2}}\\&\quad+\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)\end{aligned}$$

```
[Out] 1/4*d^2*x^4*(a+b*arccsch(c*x))+1/3*d*e*x^6*(a+b*arccsch(c*x))+1/8*e^2*x^8*(
a+b*arccsch(c*x))-1/72*b*(6*c^4*d^2-16*c^2*d*e+9*e^2)*x*(-c^2*x^2-1)^(3/2)/
c^7/(-c^2*x^2)^(1/2)+1/120*b*(8*c^2*d-9*e)*e*x*(-c^2*x^2-1)^(5/2)/c^7/(-c^2
*x^2)^(1/2)-1/56*b*e^2*x*(-c^2*x^2-1)^(7/2)/c^7/(-c^2*x^2)^(1/2)-1/24*b*(6*
c^4*d^2-8*c^2*d*e+3*e^2)*x*(-c^2*x^2-1)^(1/2)/c^7/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{272, 45, 6437, 12, 1265, 785\}$

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx &= \frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)\\ &+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)\\ &+\frac{bex(-c^2x^2-1)^{5/2}\left(8c^2d-9e\right)}{120c^7\sqrt{-c^2x^2}}\\ &-\frac{be^2x(-c^2x^2-1)^{7/2}}{56c^7\sqrt{-c^2x^2}}\\ &-\frac{bx(-c^2x^2-1)^{3/2}\left(6c^4d^2-16c^2de+9e^2\right)}{72c^7\sqrt{-c^2x^2}}\\ &-\frac{bx\sqrt{-c^2x^2-1}(6c^4d^2-8c^2de+3e^2)}{24c^7\sqrt{-c^2x^2}}\end{aligned}$$

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/24*(b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 - c^2*x^2])/(c^7*Sqrt[-(
c^2*x^2)]) - (b*(6*c^4*d^2 - 16*c^2*d*e + 9*e^2)*x*(-1 - c^2*x^2)^(3/2))/(7
2*c^7*Sqrt[-(c^2*x^2)]) + (b*(8*c^2*d - 9*e)*e*x*(-1 - c^2*x^2)^(5/2))/(120
*c^7*Sqrt[-(c^2*x^2)]) - (b*e^2*x*(-1 - c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*
x^2)]) + (d^2*x^4*(a + b*ArcCsch[c*x]))/4 + (d*e*x^6*(a + b*ArcCsch[c*x]))/
3 + (e^2*x^8*(a + b*ArcCsch[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
```

```
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad +\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int \frac{x^3\left(6d^2+8dex^2+3e^2x^4\right)}{24\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= \frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad +\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\int \frac{x^3\left(6d^2+8dex^2+3e^2x^4\right)}{\sqrt{-1-c^2x^2}}\,dx}{24\sqrt{-c^2x^2}} \\
&= \frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad +\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(bcx)\operatorname{Subst}\left(\int \frac{x\left(6d^2+8dex+3e^2x^2\right)}{\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{48\sqrt{-c^2x^2}} \\
&= \frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad -\frac{(bcx)\operatorname{Subst}\left(\int \left(\frac{-6c^4d^2+8c^2de-3e^2}{c^6\sqrt{-1-c^2x}}+\frac{\left(-6c^4d^2+16c^2de-9e^2\right)\sqrt{-1-c^2x}}{c^6}+\frac{\left(8c^2d-9e\right)e\left(-1-c^2x\right)^{3/2}}{c^6}-\frac{3e^2\left(-1-c^2x\right)^{5/}}{c^6}\right.\right.}{48\sqrt{-c^2x^2}}
\end{aligned}$$

$$= -\frac{b(6c^4d^2 - 8c^2de + 3e^2)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}} - \frac{b(6c^4d^2 - 16c^2de + 9e^2)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}$$
$$+ \frac{b(8c^2d - 9e)\,ex(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}} - \frac{be^2x(-1-c^2x^2)^{7/2}}{56c^7\sqrt{-c^2x^2}}$$
$$+ \frac{1}{4}d^2x^4\left(a + b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{3}dex^6\left(a + b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{8}e^2x^8\left(a + b\operatorname{csch}^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.64

$$\int x^3\left(d + ex^2\right)^2\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{x\left(105ax^3(6d^2 + 8dex^2 + 3e^2x^4) + \frac{b\sqrt{1+\frac{1}{c^2x^2}}(-144e^2+8c^2e(56d+9ex^2)-2c^4(210d^2+112dex^2+27e^2x^4)+3c^6(70d^2x^2+56dex^4+}{c^7}\right.}{2520}$$

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-
144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) - 2*c^4*(210*d^2 + 112*d*e*x^2 + 27*e^2*
x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^
2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCsch[c*x]))/2520
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{8}e^2x^8 + \frac{1}{3}de\,x^6 + \frac{1}{4}x^4d^2\right) + \frac{b\left(\frac{c^4\operatorname{arccsch}(cx)e^2x^8}{8} + \frac{c^4\operatorname{arccsch}(cx)de\,x^6}{3} + \frac{\operatorname{arccsch}(cx)d^2x^4c^4}{4} + \frac{\left(c^2x^2+1\right)\left(45c^6e^}{}\right.}{}$
derivativedivides	$-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3} - \frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2} + \frac{b\left(-\frac{\operatorname{arccsch}(cx)c^8d^4}{24e^2} + \frac{\operatorname{arccsch}(cx)c^8d^2x^4}{4} + \frac{e\,\operatorname{arccsch}(cx)c^8d\,x^6}{3} + \frac{e^2\,\operatorname{arccsch}(cx)c^8x^}{8}\right.}{}$
default	$-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3} - \frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2} + \frac{b\left(-\frac{\operatorname{arccsch}(cx)c^8d^4}{24e^2} + \frac{\operatorname{arccsch}(cx)c^8d^2x^4}{4} + \frac{e\,\operatorname{arccsch}(cx)c^8d\,x^6}{3} + \frac{e^2\,\operatorname{arccsch}(cx)c^8x^}{8}\right.}{}$

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arccsch(c*x)*e^2*x^8
+1/3*c^4*arccsch(c*x)*d*e*x^6+1/4*arccsch(c*x)*d^2*x^4*c^4+1/2520/c^5*(c^2*
```

```
x^2+1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2-54*c^4*e^2*x^4-224*c
^4*d*e*x^2-420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e-144*e^2)/((c^2*x^2+1)/c^2
/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.90

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{315\,ac^7e^2x^8+840\,ac^7dex^6+630\,ac^7d^2x^4+105\left(3\,bc^7e^2x^8+8\,bc^7dex^6+6\,bc^7d^2x^4\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(4}{}$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3*
b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) + (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e - 9*b*c^4*
e^2)*x^5 + 2*(105*b*c^6*d^2 - 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 - 4*(105*b*
c^4*d^2 - 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^3(a+b\operatorname{acsch}(cx))\left(d+ex^2\right)^2\,dx$$

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{8}ae^2x^8+\frac{1}{3}adex^6+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{12}\left(3x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2$$
$$+\frac{1}{45}\left(15x^6\operatorname{arcsch}(cx)+\frac{3c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)bde$$
$$+\frac{1}{280}\left(35x^8\operatorname{arcsch}(cx)+\frac{5c^6x^7\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35x\sqrt{\frac{1}{c^2x^2}+1}}{c^7}\right.$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsch(c*x) +
(c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d^2 +
1/45*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3
*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/280*(
35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1)^(7/2) - 21*c^4*x^5*(1/(c
^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 35*x*sqrt(1/(c^2*
x^2) + 1))/c^7)*b*e^2
```

Giac [F]

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2(b\operatorname{arcsch}(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x^3\left(e\,x^2+d\right)^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.95 $\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 674
Rubi [A] (verified) . . . 674
Mathematica [A] (verified) . . . 677
Maple [A] (verified) . . . 677
Fricas [A] (verification not implemented) . . . 678
Sympy [F] . . . 678
Maxima [A] (verification not implemented) . . . 678
Giac [F] . . . 679
Mupad [F(-1)] . . . 679

Optimal result

Integrand size = 19, antiderivative size = 203

$$\begin{aligned}\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx = &\frac{b(3c^4d^2-3c^2de+e^2)\,x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}}\\ &-\frac{b(3c^2d-2e)\,ex(-1-c^2x^2)^{3/2}}{18c^5\sqrt{-c^2x^2}}\\ &+\frac{be^2x(-1-c^2x^2)^{5/2}}{30c^5\sqrt{-c^2x^2}}\\ &+\frac{(d+ex^2)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}\\ &-\frac{bcd^3x\arctan\left(\sqrt{-1-c^2x^2}\right)}{6e\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccsch(c*x))/e-1/18*b*(3*c^2*d-2*e)*e*x*(-c^2*x^2-1)^
(3/2)/c^5/(-c^2*x^2)^(1/2)+1/30*b*e^2*x*(-c^2*x^2-1)^(5/2)/c^5/(-c^2*x^2)^(
1/2)-1/6*b*c*d^3*x*arctan((-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/6*b*(3*c
^4*d^2-3*c^2*d*e+e^2)*x*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{6435, 457, 90, 65, 211\}$

$$\int x\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e} - \frac{bcd^3x\arctan\left(\sqrt{-c^2x^2-1}\right)}{6e\sqrt{-c^2x^2}} - \frac{bex(-c^2x^2-1)^{3/2}\left(3c^2d-2e\right)}{18c^5\sqrt{-c^2x^2}} + \frac{be^2x(-c^2x^2-1)^{5/2}}{30c^5\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}(3c^4d^2-3c^2de+e^2)}{6c^5\sqrt{-c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x*Sqrt[-1 - c^2*x^2])/(6*c^5*Sqrt[-(c^2*x^
2)]) - (b*(3*c^2*d - 2*e)*e*x*(-1 - c^2*x^2)^(3/2))/(18*c^5*Sqrt[-(c^2*x^2)
]) + (b*e^2*x*(-1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + ((d + e*x^2
)^3*(a + b*ArcCsch[c*x]))/(6*e) - (b*c*d^3*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(6
*e*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex^2)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\int \frac{(d+ex^2)^3}{x\sqrt{-1-c^2x^2}}\,dx}{6e\sqrt{-c^2x^2}} \\
&= \frac{(d+ex^2)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{(d+ex)^3}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{12e\sqrt{-c^2x^2}} \\
&= \frac{(d+ex^2)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \left(\frac{e\left(3c^4d^2-3c^2de+e^2\right)}{c^4\sqrt{-1-c^2x}} + \frac{d^3}{x\sqrt{-1-c^2x}} - \frac{\left(3c^2d-2e\right)e^2\sqrt{-1-c^2x}}{c^4} + \frac{e^3\left(-1-c^2x\right)^{3/2}}{c^4}\right)\,dx, x, x^2\right)}{12e\sqrt{-c^2x^2}} \\
&= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/2}}{18c^5\sqrt{-c^2x^2}} \\
&\quad + \frac{be^2x\left(-1-c^2x^2\right)^{5/2}}{30c^5\sqrt{-c^2x^2}} + \frac{(d+ex^2)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} \\
&\quad - \frac{(bcd^3x)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{12e\sqrt{-c^2x^2}} \\
&= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/2}}{18c^5\sqrt{-c^2x^2}} + \frac{be^2x\left(-1-c^2x^2\right)^{5/2}}{30c^5\sqrt{-c^2x^2}} \\
&\quad + \frac{(d+ex^2)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} + \frac{(bd^3x)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{6ce\sqrt{-c^2x^2}} \\
&= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} \\
&\quad - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/2}}{18c^5\sqrt{-c^2x^2}} + \frac{be^2x\left(-1-c^2x^2\right)^{5/2}}{30c^5\sqrt{-c^2x^2}} \\
&\quad + \frac{(d+ex^2)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} - \frac{bcd^3x\arctan\left(\sqrt{-1-c^2x^2}\right)}{6e\sqrt{-c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.61

$$\int x\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)\right.$$
$$+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(8e^2-2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4)\right)}{c^5}$$
$$\left.+15bx\left(3d^2+3dex^2+e^2x^4\right)\operatorname{csch}^{-1}(cx)\right)$$

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(8*e^2
- 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsch[c*x]))/90
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31

method	result
parts	$\frac{a\left(e x^2+d\right)^3}{6e}+\frac{b\left(\frac{c^2e^2\operatorname{arccsch}(cx)x^6}{6}+\frac{c^2e\operatorname{arccsch}(cx)x^4d}{2}+\frac{\operatorname{arccsch}(cx)x^2c^2d^2}{2}+\frac{c^2\operatorname{arccsch}(cx)d^3}{6e}-\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\operatorname{arctanh}\right.\right.}{}$
derivativedivides	$\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^6d^3}{6e}+\frac{\operatorname{arccsch}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arccsch}(cx)c^6dx^4}{2}+\frac{e^2\operatorname{arccsch}(cx)c^6x^6}{6}-\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\operatorname{arctanh}\right.\right.}{}$
default	$\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^6d^3}{6e}+\frac{\operatorname{arccsch}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arccsch}(cx)c^6dx^4}{2}+\frac{e^2\operatorname{arccsch}(cx)c^6x^6}{6}-\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\operatorname{arctanh}\right.\right.}{}$

```
[In] int(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*(e*x^2+d)^3/e+b/c^2*(1/6*c^2*e^2*arccsch(c*x)*x^6+1/2*c^2*e*arccsch(c
*x)*x^4*d+1/2*arccsch(c*x)*x^2*c^2*d^2+1/6*c^2/e*arccsch(c*x)*d^3-1/90/c^5/
e*(c^2*x^2+1)^(1/2)*(15*c^6*d^3*arctanh(1/(c^2*x^2+1)^(1/2))-45*c^4*d^2*e*(
c^2*x^2+1)^(1/2)-15*c^4*d*e^2*x^2*(c^2*x^2+1)^(1/2)-3*e^3*c^4*x^4*(c^2*x^2+
```

```
1)^(1/2)+30*c^2*d*e^2*(c^2*x^2+1)^(1/2)+4*e^3*c^2*x^2*(c^2*x^2+1)^(1/2)-8*e
^3*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int x\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{15\,ac^5e^2x^6+45\,ac^5dex^4+45\,ac^5d^2x^2+15\left(bc^5e^2x^6+3\,bc^5dex^4+3\,bc^5d^2x^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(3\,bc^4e^2x}{90\,c^5}$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e^
2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) + 1)/(c*x)) + (3*b*c^4*e^2*x^5 + (15*b*c^4*d*e - 4*b*c^2*e^2)*x^3 +
(45*b*c^4*d^2 - 30*b*c^2*d*e + 8*b*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c
^5
```

Sympy [F]

$$\int x\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)^2\,dx$$

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90

$$\int x\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2\operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2$$
$$+ \frac{1}{6}\left(3\,x^4\operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bde$$
$$+ \frac{1}{90}\left(15\,x^6\operatorname{arcsch}(cx) + \frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsch(c*x) + x*s
qrt(1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*
x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arcc
sch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1
)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e^2
```

Giac [F]

$$\int x\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2(b\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x\left(e\,x^2+d\right)^2\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.96 $\int \frac{\left(d+ex^2\right)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal result . . . 680
Rubi [A] (verified) . . . 681
Mathematica [A] (verified) . . . 685
Maple [F] . . . 685
Fricas [F] . . . 686
Sympy [F] . . . 686
Maxima [F] . . . 686
Giac [F] . . . 687
Mupad [F(-1)] . . . 687

Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = {} & \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3}+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c}\\ & +\frac{1}{2}bd^2\operatorname{csch}^{-1}(cx)^2+dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ & +\frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ & -bd^2\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\\ & +bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right)\\ & -d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)\\ & -\frac{1}{2}bd^2\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/2*b*d^2*arccsch(c*x)^2+d*e*x^2*(a+b*arccsch(c*x))+1/4*e^2*x^4*(a+b*arccsc
h(c*x))-b*d^2*arccsch(c*x)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*d^2*arccsc
h(c*x)*ln(1/x)-d^2*(a+b*arccsch(c*x))*ln(1/x)-1/2*b*d^2*polylog(2,(1/c/x+(1
+1/c^2/x^2)^(1/2))^2)+1/6*b*(6*c^2*d-e)*e*x*(1+1/c^2/x^2)^(1/2)/c^3+1/12*b*
e^2*x^3*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6439, 272, 45, 5822, 6874, 464, 270, 2362, 5775, 3797, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx &= -d^2\log\left(\frac{1}{x}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &\quad + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &\quad + \frac{be^2x^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}(6c^2d-e)}{6c^3}\\ &\quad - \frac{1}{2}bd^2\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)+\frac{1}{2}bd^2\operatorname{csch}^{-1}(cx)^2\\ &\quad - bd^2\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\\ &\quad + bd^2\log\left(\frac{1}{x}\right)\operatorname{csch}^{-1}(cx)\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*(6*c^2*d - e)*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 + 1/(c^
2*x^2)]*x^3)/(12*c) + (b*d^2*ArcCsch[c*x]^2)/2 + d*e*x^2*(a + b*ArcCsch[c*x
]) + (e^2*x^4*(a + b*ArcCsch[c*x]))/4 - b*d^2*ArcCsch[c*x]*Log[1 - E^(2*Arc
Csch[c*x])] + b*d^2*ArcCsch[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsch[c*x])*Log
[x^(-1)] - (b*d^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
```

```
b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right)$$

$$- d^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e\left(e+4dx^2\right)}{4x^4}+d^2\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right)$$

$$- d^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{e(e+4dx^2)}{4x^4\sqrt{1+\frac{x^2}{c^2}}} + \frac{d^2\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\right)\,dx, x, \frac{1}{x}\right)}{c}$$

$$= dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right) - d^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$

$$+ \frac{(bd^2)\,\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(be)\text{Subst}\left(\int \frac{e+4dx^2}{x^4\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$
\begin{aligned}
&= \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad - \left(bd^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right) \\
&\quad - \frac{\left(b(6c^2d-e)\,e\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c^3} \\
&= \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) + bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \left(bd^2\right)\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\operatorname{csch}^{-1}(cx)^2 \\
&\quad + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) + bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \left(2bd^2\right)\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\operatorname{csch}^{-1}(cx)^2 + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) - bd^2\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \left(bd^2\right)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\operatorname{csch}^{-1}(cx)^2 + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + \frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) - bd^2\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}\left(bd^2\right)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2\operatorname{csch}^{-1}(cx)^2 \\
&\quad + dex^2\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - bd^2\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + bd^2\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{1}{2}bd^2\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.84

$$
\begin{aligned}
\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx &= adex^2 + \frac{1}{4}ae^2x^4 + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x(-2+c^2x^2)}{12c^3} \\
&\quad + \frac{1}{4}be^2x^4\operatorname{csch}^{-1}(cx) \\
&\quad + \frac{bdex\left(\sqrt{1+\frac{1}{c^2x^2}} + cx\operatorname{csch}^{-1}(cx)\right)}{c} \\
&\quad + ad^2\log(x) + \frac{1}{2}bd^2\Big(\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx)\right. \\
&\quad \left. - 2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\right) - \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)\Big)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + c^2*x^2))/
(12*c^3) + (b*e^2*x^4*ArcCsch[c*x])/4 + (b*d*e*x*(Sqrt[1 + 1/(c^2*x^2)] + c
*x*ArcCsch[c*x]))/c + a*d^2*Log[x] + (b*d^2*(ArcCsch[c*x]*(ArcCsch[c*x] - 2
*Log[1 - E^(2*ArcCsch[c*x])]) - PolyLog[2, E^(2*ArcCsch[c*x])]))/2
```

Maple [F]

$$
\int \frac{(e\,x^2+d)^2\left(a+b\,\operatorname{arccsch}(cx)\right)}{x}dx
$$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

Fricas [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsch(c*x))/x, x)
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\,\text{acsch}\,(cx)\right)\left(d+ex^2\right)^2}{x}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + 4*b*c^2*d^2*integrate(1/4*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + a*d*e*x^2 - b*d^2*log(c)*log(x)
 - 1/4*(2*log(c^2*x^2 + 1)*log(x) + dilog(-c^2*x^2))*b*d^2 + a*d^2*log(x) +
 1/2*b*d*e*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2 - 1/24*(3*c^2*x^2 -
 2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*b*e^
2/c^4 - 1/8*(2*b*c^2*e^2*x^4*log(c) + 4*b*c^2*d^2*log(x)^2 + (8*c^2*d*e*log
(c) - e^2)*b*x^2 + 2*(b*c^2*e^2*x^4 + 4*b*c^2*d*e*x^2)*log(x) - 2*(b*c^2*e^
2*x^4 + 4*b*c^2*d*e*x^2 + 4*b*c^2*d^2*log(x))*log(sqrt(c^2*x^2 + 1) + 1))/c
^2 + 1/8*(4*c^2*d*e - e^2)*b*log(c^2*x^2 + 1)/c^4
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(ex^2+d\right)^2(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(e\,x^2+d\right)^2\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x, x)
```

3.97 $\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result . . . 688
Rubi [A] (verified) . . . 689
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Mupad [F(-1)] . . . 695

Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = {} & \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c}\\ & -\frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx)+bde\operatorname{csch}^{-1}(cx)^2\\ & -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ & -2bde\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\\ & +2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right)\\ & -2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)\\ & -bde\,\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)\end{aligned}$$

```
[Out] -1/4*b*c^2*d^2*arccsch(c*x)+b*d*e*arccsch(c*x)^2-1/2*d^2*(a+b*arccsch(c*x))
/x^2+1/2*e^2*x^2*(a+b*arccsch(c*x))-2*b*d*e*arccsch(c*x)*ln(1-(1/c/x+(1+1/c
^2/x^2)^(1/2))^2)+2*b*d*e*arccsch(c*x)*ln(1/x)-2*d*e*(a+b*arccsch(c*x))*ln(
1/x)-b*d*e*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/4*b*c*d^2*(1+1/c^2/x^
2)^(1/2)/x+1/2*b*e^2*x*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6439, 272, 45, 5822, 12, 6874, 270, 327, 221, 2362, 5775, 3797, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = &-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2}\\ &-2de\log\left(\frac{1}{x}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\\ &+\frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{bcd^2\sqrt{\frac{1}{c^2x^2}+1}}{4x}\\ &-\frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx)+\frac{be^2x\sqrt{\frac{1}{c^2x^2}+1}}{2c}\\ &-bde\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)+bde\operatorname{csch}^{-1}(cx)^2\\ &-2bde\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\\ &+2bde\log\left(\frac{1}{x}\right)\operatorname{csch}^{-1}(cx)\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[1 + 1/(c^2*x^2)])/(4*x) + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*
c) - (b*c^2*d^2*ArcCsch[c*x])/4 + b*d*e*ArcCsch[c*x]^2 - (d^2*(a + b*ArcCsc
h[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsch[c*x]))/2 - 2*b*d*e*ArcCsch[c*x]*
Log[1 - E^(2*ArcCsch[c*x])] + 2*b*d*e*ArcCsch[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsch[c*x])*Log[x^(-1)] - b*d*e*PolyLog[2, E^(2*ArcCsch[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right) \\ &= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) \\ &\quad - 2de\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\operatorname{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad + \frac{b\operatorname{Subst}\left(\int \left(-\frac{e^2}{x^2\sqrt{1+\frac{x^2}{c^2}}} + \frac{d^2x^2}{\sqrt{1+\frac{x^2}{c^2}}} + \frac{4de\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\right)\,dx, x, \frac{1}{x}\right)}{2c} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd^2)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} \\
&\quad + \frac{(2bde)\operatorname{Subst}\left(\int \frac{\log(x)}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(be^2)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} \\
&= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad + 2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad - \frac{1}{4}(bcd^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) - (2bde)\operatorname{Subst}\left(\int \frac{\operatorname{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right) \\
&= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx) - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} \\
&\quad + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) + 2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - (2bde)\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx) + bde\operatorname{csch}^{-1}(cx)^2 \\
&\quad - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) + 2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (4bde)\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx) \\
&\quad + bde\operatorname{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - 2bde\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + 2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (2bde)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx) \\
&\quad + bde\operatorname{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - 2bde\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + 2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (bde)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{csch}^{-1}(cx)}\right) \\
&= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx) \\
&\quad + bde\operatorname{csch}^{-1}(cx)^2 - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right) \\
&\quad - 2bde\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + 2bde\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - bde\,\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx \\
&= \frac{1}{4}\Bigg(-\frac{2ad^2}{x^2} + 2ae^2x^2 - \frac{2bd^2\operatorname{csch}^{-1}(cx)}{x^2} + \frac{2be^2x\left(\sqrt{1+\frac{1}{c^2x^2}} + cx\operatorname{csch}^{-1}(cx)\right)}{c} \\
&\qquad\qquad - \frac{bd^2\left(-1-c^2x^2+c^2x^2\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)\right)}{c\sqrt{1+\frac{1}{c^2x^2}}x^3} + 8ade\log(x) \\
&\quad + 4bde\left(\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx) - 2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\right) - \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsch[c*x])/x^2 + (2*b*e^2*x*(Sq
rt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c*x]))/c - (b*d^2*(-1 - c^2*x^2 + c^2*x^2
*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]]))/(c*Sqrt[1 + 1/(c^2*x^2)]*x^
3) + 8*a*d*e*Log[x] + 4*b*d*e*(ArcCsch[c*x]*(ArcCsch[c*x] - 2*Log[1 - E^(2*
ArcCsch[c*x])]) - PolyLog[2, E^(2*ArcCsch[c*x])]))/4
```

Maple [F]

$$\int \frac{(e\,x^2+d)^2\,(a+b\,\operatorname{arccsch}(cx))}{x^3}dx$$

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^2\,(a+b\mathrm{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arcsch}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsch(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2\,(a+b\mathrm{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))\,(d+ex^2)^2}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^2\left(b\operatorname{arcsch}(cx)+a\right)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 4*b*c^2*d*e*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + s
qrt(c^2*x^2 + 1) + 1), x) - 1/2*b*e^2*x^2*log(c) - 1/2*b*e^2*x^2*log(x) + 1
/2*a*e^2*x^2 - 2*b*d*e*log(c)*log(x) - b*d*e*log(x)^2 - 1/2*(2*log(c^2*x^2
+ 1)*log(x) + dilog(-c^2*x^2))*b*d*e + 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2)
 + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) +
 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2) + 2*a
*d*e*log(x) + 1/4*b*e^2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2 + 1/4*
b*e^2*log(c^2*x^2 + 1)/c^2 + 1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*log(sqrt(c^2*
x^2 + 1) + 1) - 1/2*a*d^2/x^2
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^2\left(b\operatorname{arcsch}(cx)+a\right)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(e\,x^2+d\right)^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.98 $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result . . . 696
Rubi [A] (verified) . . . 697
Mathematica [C] (verified) . . . 703
Maple [F] . . . 704
Fricas [F] . . . 704
Sympy [F] . . . 705
Maxima [F(-2)] . . . 705
Giac [F] . . . 705
Mupad [F(-1)] . . . 705

Optimal result

Integrand size = 21, antiderivative size = 512

$$\begin{aligned}\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce}\\ &+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\\ &+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}\end{aligned}$$

```
[Out] x*(a+b*arccsch(c*x))/e+b*arctanh((1+1/c^2/x^2)^(1/2))/c/e+1/2*(a+b*arccsch(
c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/
```

```
2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*a
rccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d
+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2
*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^
(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,-c*
(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/
2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6439, 5823, 5776, 272, 65, 214, 5793, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx &= \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}\\ &- \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^{3/2}}\\ &+ \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^{3/2}}\\ &- \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^{3/2}}\\ &+ \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{b\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{ce}\\ &- \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}\\ &+ \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}\\ &- \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^{3/2}}\\ &+ \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^{3/2}}\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/e + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e) + (Sq
rt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] -
Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 +
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) +
(Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^(3/2))
 - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c
*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^(3/2))
 + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e])])/(2*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e} \\
&= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce} \\
&\quad + \frac{d\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e} \\
&= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ce} \\
&= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2e^{3/2}} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2e^{3/2}} \\
&\quad - \frac{(bc)\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)}{e}
\end{aligned}$$

$$= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce}$$
$$+ \frac{d\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{d\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{d\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{d\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce}$$
$$+ \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}$$
$$+ \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$
$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{3/2}}$$

$$
\begin{aligned}
&= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{3/2}} \\
&\quad + \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{3/2}} \\
&\quad - \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{3/2}} \\
&\quad + \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&\quad - \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 1239, normalized size of antiderivative = 2.42

$$
\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx
$$

$$
= \frac{4ac\sqrt{e}x + 4bc\sqrt{e}x\operatorname{csch}^{-1}(cx) - 4ac\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 8ibc\sqrt{d}\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\right.\right.\right.}{\sqrt{-}}
$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]

[Out] (4*a*c*Sqrt[e]*x + 4*b*c*Sqrt[e]*x*ArcCsch[c*x] - 4*a*c*Sqrt[d]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] + b*c*Sqrt[d]*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*
(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d
]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - b*c*Sqrt[d]*Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqrt
```

```
[d]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
b*c*Sqrt[d]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqr
t[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*c*Sq
rt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[Sqrt[e
] - (I*Sqrt[d])/x] - b*c*Sqrt[d]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*b*Sqrt
[e]*Log[Cosh[ArcCsch[c*x]/2]] - 4*b*Sqrt[e]*Log[Sinh[ArcCsch[c*x]/2]] + (2*
I)*b*c*Sqrt[d]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*PolyLog[2, ((-I)*
(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqr
t[d]*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])])/(4*c*e^(3/2))
```

Maple [F]

$$\int \frac{x^2(a + b \operatorname{arccsch}(cx))}{e\,x^2 + d}\,dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{x^2\left(a + b\operatorname{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{ex^2 + d}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{x^2(a+b\,\operatorname{acsch}(cx))}{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d),x)
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^2}{ex^2+d}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{x^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.99 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result . . . 706
Rubi [A] (verified) . . . 707
Mathematica [C] (verified) . . . 713
Maple [F] . . . 714
Fricas [F] . . . 714
Sympy [F] . . . 715
Maxima [F] . . . 715
Giac [F] . . . 715
Mupad [F(-1)] . . . 715

Optimal result

Integrand size = 19, antiderivative size = 467

$$\begin{aligned}\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e}\\&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\\&+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\end{aligned}$$

```
[Out] -(a+b*arccsch(c*x))^2/b/e-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1
/2))^2)/e+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
```

```
/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsch(
c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/
2)))/e+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2)
)^2)/e+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6439, 5823, 5775, 3797, 2221, 2317, 2438, 5827, 5680}

$$\begin{aligned}\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = {}& \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e} \\ &+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e} \\ &+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e} \\ &+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e} \\ &- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be} \\ &- \frac{\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} \\ &+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e} \\ &+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e} \\ &+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e} \\ &+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e}\end{aligned}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a + b*ArcCsch[c*x])^2/(b*e)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCs
ch[c*x])])/e + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(S
qrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcC
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] + Sqrt[-(c^2*d) + e])])/(2*e) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*
e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) +
 e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x]
)/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e} \\
&= \frac{\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{csch}^{-1}(cx)\right)}{be} \\
&\quad + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be} - \frac{2\operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{be} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e} + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e} \\
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e} \\
&\quad + \frac{\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{e} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} \\
&\quad + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 1103, normalized size of antiderivative = 2.36

$$
\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx
$$

$$
= \frac{b\pi^2 - 4ib\pi\operatorname{csch}^{-1}(cx) - 8b\operatorname{csch}^{-1}(cx)^2 + 16b\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right)}{\sqrt{-c^2d+e}}\right)}{}
$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (b*Pi^2 - (4*I)*b*Pi*ArcCsch[c*x] - 8*b*ArcCsch[c*x]^2 + 16*b*ArcSin[Sqrt[1
 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2
*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]/(c
*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c
*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])
] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
```

```
])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*
Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (2*I)*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqrt
[e] + (I*Sqrt[d])/x] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*
x])] + 4*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(8*e)
```

Maple [F]

$$\int \frac{x(a+b\ \mathrm{arccsch}\,(cx))}{e\,x^2+d}dx$$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{x\big(a+b\mathrm{csch}^{-1}(cx)\big)}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{x(a+b\,\operatorname{acsch}(cx))}{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d),x)
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d), x) + 1/2*a*
log(e*x^2 + d)/e
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{x\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{e\,x^2+d}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.100 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx$

Optimal result . . . 716
Rubi [A] (verified) . . . 717
Mathematica [C] (verified) . . . 722
Maple [F] . . . 723
Fricas [F] . . . 723
Sympy [F] . . . 723
Maxima [F(-2)] . . . 724
Giac [F] . . . 724
Mupad [F(-1)] . . . 724

Optimal result

Integrand size = 18, antiderivative size = 477

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx = {}& \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccsch(c*x))*ln(1
+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c
```

```
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/
2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6429, 5793, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex^2}\,dx = &\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}\\ &+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6429

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} \\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} \\
&= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 1055, normalized size of antiderivative = 2.21

$$
\begin{aligned}
&\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx \\
&= \frac{4a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + 8ib\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right)}{\sqrt{-c^2d+e}}\right) + 8ib\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2),x]

[Out] (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4
])/Sqrt[-(c^2*d) + e]] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x]
)/(c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
```

```
qrt[d])] + b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] - b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
 b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - (2
*I)*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + (2*I)*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + (2*I)*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(4*Sqrt[d]*Sqrt[e])
```

Maple [F]

$$\int \frac{a + b\ \mathrm{arccsch}\left(cx\right)}{e\,x^2 + d}dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{d + ex^2}\,dx = \int \frac{b\,\mathrm{arcsch}\left(cx\right) + a}{ex^2 + d}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{d + ex^2}\,dx = \int \frac{a + b\,\mathrm{acsch}\left(cx\right)}{d + ex^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex^2}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{ex^2+d}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex^2}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{e\,x^2+d}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2), x)
```

3.101 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx$

Optimal result . . . 725
Rubi [A] (verified) . . . 726
Mathematica [C] (verified) . . . 731
Maple [F] . . . 732
Fricas [F] . . . 732
Sympy [F] . . . 732
Maxima [F] . . . 732
Giac [F] . . . 733
Mupad [F(-1)] . . . 733

Optimal result

Integrand size = 21, antiderivative size = 425

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d}$$

```
[Out] 1/2*(a+b*arccsch(c*x))^2/b/d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))
*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/
d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)+(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x
+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*b*polylo
g(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-
1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+
```

```
e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(-c^2*d+e)^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6439, 5823, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a + b*ArcCsch[c*x])^2/(2*b*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -((c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d) - (b*PolyLog
[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])
/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
 + e])])/(2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}(a+b\text{arcsinh}\left(\frac{x}{c}\right))}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}(a+b\text{arcsinh}\left(\frac{x}{c}\right))}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\, dx, x, \frac{1}{x}\right)}{2\sqrt{-d}} + \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\, dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\, dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\, dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$+ \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$+ \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$
\begin{aligned}
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d} \\
&= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 1141, normalized size of antiderivative = 2.68

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx =$$

$$-\frac{b\pi^2-4ib\pi\operatorname{csch}^{-1}(cx)-12b\operatorname{csch}^{-1}(cx)^2+16b\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right)}{\sqrt{-c^2d+e}}\right.}{}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] -1/8*(b*Pi^2 - (4*I)*b*Pi*ArcCsch[c*x] - 12*b*ArcCsch[c*x]^2 + 16*b*ArcSin[
Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(P
i + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*Arc
Csch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch
[c*x])] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c
^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] + 8*b*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] - (2*I)*b*
Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] -
 8*a*Log[x] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*
b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, E^(2*ArcCsch[c*x])])/d
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x\,(e\,x^2 + d)}\,dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{a + b \operatorname{acsch}(cx)}{x\,(d + ex^2)}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d),x)
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="maxima")
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c^2*x^2) +
 1) + 1/(c*x))/(e*x^3 + d*x), x)
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.102 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx$

Optimal result . . . 734
Rubi [A] (verified) . . . 735
Mathematica [C] (verified) . . . 741
Maple [F] . . . 742
Fricas [F] . . . 743
Sympy [F] . . . 743
Maxima [F(-2)] . . . 743
Giac [F] . . . 743
Mupad [F(-1)] . . . 744

Optimal result

Integrand size = 21, antiderivative size = 518

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = {} & \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\operatorname{csch}^{-1}(cx)}{dx} \\ & +\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}}\end{aligned}$$

```
[Out] -a/d/x-b*arccsch(c*x)/d/x+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b
*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1
/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b
*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/
2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylog(2,c*(1/
c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d
)^(3/2)+b*c*(1+1/c^2/x^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6439, 5823, 5772, 267, 5793, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx &= \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}\\ &\quad-\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2(-d)^{3/2}}\\ &\quad+\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2(-d)^{3/2}}\\ &\quad-\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2(-d)^{3/2}}\\ &\quad-\frac{a}{dx}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}\\ &\quad+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}\\ &\quad-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}\\ &\quad+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{d}-\frac{b\operatorname{csch}^{-1}(cx)}{dx}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcCsch[c*x])/(d*x) + (Sqrt[e]
*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) + (
Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/
2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyL
og[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*(-
d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sq
rt[-(c^2*d) + e])])/(2*(-d)^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}
\end{aligned}$$

$$= -\frac{a}{dx} - \frac{b\text{Subst}\left(\int \text{arcsinh}\left(\frac{x}{c}\right)\, dx, x, \frac{1}{x}\right)}{d}$$
$$+ \frac{e\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e}-\sqrt{-d}x)} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)\, dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x^2}{c^2}}}\, dx, x, \frac{1}{x}\right)}{cd}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\, dx, x, \frac{1}{x}\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\, dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\, dx, x, \text{csch}^{-1}(cx)\right)}{2d}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\, dx, x, \text{csch}^{-1}(cx)\right)}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\text{csch}^{-1}(cx)}{dx}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2d}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2d}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2d}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\, dx, x, \text{csch}^{-1}(cx)\right)}{2d}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\
&\quad - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.34

$$
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = -\frac{a}{dx} - \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + b\left(\frac{c\sqrt{1+\frac{1}{c^2x^2}} - \frac{\operatorname{csch}^{-1}(cx)}{x}}{d}\right.
$$

$$
\left. - \frac{i\sqrt{e}\left(\pi^2 - 4i\pi\operatorname{csch}^{-1}(cx) - 8\operatorname{csch}^{-1}(cx)^2 + 32\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right.\right.\right.\right.}{\sqrt{-c^2d+e}}\right.
$$

$$
\left. + \frac{i\sqrt{e}\left(\pi^2 - 4i\pi\operatorname{csch}^{-1}(cx) - 8\operatorname{csch}^{-1}(cx)^2 - 32\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right.\right.\right.\right.}{\sqrt{-c^2d+e}}\right.
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[1
 + 1/(c^2*x^2)] - ArcCsch[c*x]/x)/d - ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcC
```

```
sch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1
 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsc
h[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e
] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/d^(3/2)
+ ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*Arc
Sin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Co
t[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1
- E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^
(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])]))/d^(3/2))
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 (e x^2 + d)} dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x^2\left(d+ex^2\right)}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2\,(d + ex^2)}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2\,(e\,x^2 + d)}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.103 $\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result 746
Rubi [A] (verified) 747
Mathematica [C] (warning: unable to verify) 756
Maple [F] 757
Fricas [F] 758
Sympy [F] 758
Maxima [F] 758
Giac [F] 758
Mupad [F(-1)] 759

Optimal result

Integrand size = 21, antiderivative size = 591

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2}$$
$$+ \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} - \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}}$$
$$+ \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}$$

```
[Out] 1/2*d*(a+b*arccsch(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsch(c*x))/e^2+2*d*(
a+b*arccsch(c*x))^2/b/e^3+2*d*(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2
)^(1/2))^2)/e^3-d*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+
(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-d*(a+b*arcc
sch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/e^3-d*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(
```

```
1/2))^2)/e^3-b*d*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-1/2*b*d*arct
an((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d-e)^(1/2)
+1/2*b*x*(1+1/c^2/x^2)^(1/2)/c/e^2
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {6439, 5823, 5776, 270, 5775, 3797, 2221, 2317, 2438, 5821, 385, 211, 5827, 5680}

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{e^3} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{e^3} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{e^3} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{2d\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(\frac{d}{x^2}+e\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{5/2}\sqrt{c^2d-e}} - \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{e^3} + \frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2ce^2} - \frac{bd\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}$$

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsch[c*x]))/(2*e^2*(e
+ d/x^2)) + (x^2*(a + b*ArcCsch[c*x]))/(2*e^2) + (2*d*(a + b*ArcCsch[c*x])^
2)/(b*e^3) - (b*d*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x
)])/(2*Sqrt[c^2*d - e]*e^(5/2)) + (2*d*(a + b*ArcCsch[c*x])*Log[1 - E^(-2*A
rcCsch[c*x])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
```

```
*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x
])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, E^(-2*ArcCsch[c*x
])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(
c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} \right.\right. \\
&\qquad \left.\left. + \frac{2d^2x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= \frac{(2d)\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x^3}\,dx, x, \frac{1}{x}\right)}{e^2} - \frac{d^2\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2} \\
&= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} \\
&\quad - \frac{(2d)\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{csch}^{-1}(cx)\right)}{be^3} \\
&\quad - \frac{\left(2d^2\right)\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e^3} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce^2} - \frac{(bd)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} \\
&\quad + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} - \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{e^3} \\
&\quad + \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{e^3} \\
&\quad + \frac{(4d)\operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{be^3} \\
&\quad - \frac{(bd)\operatorname{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2ce^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} \\
&\quad - \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad - \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{(2d)\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} \\
&\quad - \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad - \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(-d)^{3/2}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} \\
&\quad - \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{bd\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} \\
&\quad - \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3} \\
&\quad - \frac{bd\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{e^3}
\end{aligned}
$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3}$$
$$- \frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{e^3}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 1447, normalized size of antiderivative = 2.45

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx =$$
$$-2aex^2 + \frac{2ad^2}{d+ex^2} + 4ad\log\left(d+ex^2\right) + b\left(d\pi^2 - \frac{2e\sqrt{1+\frac{1}{c^2x^2}}x}{c} - 4id\pi\operatorname{csch}^{-1}(cx) - 2ex^2\operatorname{csch}^{-1}(cx) + \frac{d^{3/2}\text{cs}}{\sqrt{d}-}\right.$$
$$-$$

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]

[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(d*Pi^2
 - (2*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsch[c*x] - 2*e*x^2*ArcC
sch[c*x] + (d^(3/2)*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcCsc
h[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 8*d*ArcCsch[c*x]^2 - 2*d*ArcSinh[1/(c*x)]
 + 16*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] -
```

```
Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*d*ArcSi
n[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[
(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*d*ArcCsch[c*x]*Log[1
 - E^(-2*ArcCsch[c*x])] + (2*I)*d*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] +
 Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[1 +
 Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[
1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sqr
t[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c
^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 + (I*(Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*d*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]
 - (2*I)*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[
e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*
x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e] + (d*
Sqrt[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) +
 e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))
])/Sqrt[-(c^2*d) + e] + 4*d*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*d*PolyLog[2
, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*
PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
 + 4*d*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sq
rt[d])] + 4*d*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])]))/e^3
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^2}dx$$

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^5*arccsch(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^5(a+b\operatorname{acsch}(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**5*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2),
x)
```

Giac [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^5}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^5\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result 761
Rubi [A] (verified) 762
Mathematica [C] (warning: unable to verify) 769
Maple [F] 770
Fricas [F] 771
Sympy [F] 771
Maxima [F] 771
Giac [F] 771
Mupad [F(-1)] 772

Optimal result

Integrand size = 21, antiderivative size = 553

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^2} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2}$$

```
[Out] 1/2*(-a-b*arccsch(c*x))/e/(e+d/x^2)-(a+b*arccsch(c*x))^2/b/e^2-(a+b*arccsch
(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^2+1/2*(a+b*arccsch(c*x))*ln(
1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^2+
1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsch(c*x))*
ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
^2+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^2+1/2*b*polylog(2,-c*
(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^2+1/2*
b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1
/2)))/e^2+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
```

```
)+(-c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(
1/2)/(1+1/c^2/x^2)^(1/2))/e^(3/2)/(c^2*d-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6439, 5823, 5775, 3797, 2221, 2317, 2438, 5821, 385, 211, 5827, 5680}

$$\begin{aligned}
\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx &= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2} \\
&+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^2} \\
&+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^2} \\
&+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^2} \\
&- \frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(\frac{d}{x^2}+e\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^2} \\
&- \frac{\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&+ \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{3/2}\sqrt{c^2d-e}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2} \\
&+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2} \\
&+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^2} \\
&+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e^2}
\end{aligned}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsch[c*x])/(e*(e + d/x^2)) - (a + b*ArcCsch[c*x])^2/(b*e^2)
+ (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c
^2*d - e]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[c*x])])/e^
2 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sq
rt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
```

```
rcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x
])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*
e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*e^2) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*e^2
) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) +
e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqr
t[e] + Sqrt[-(c^2*d) + e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a + b\text{arcsinh}\left(\frac{x}{c}\right)}{x\left(e + dx^2\right)^2}\, dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e^2 x} - \frac{dx\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{dx\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x}\, dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\, dx, x, \frac{1}{x}\right)}{e^2}$$
$$+ \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\, dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\, dx, x, a+b\text{csch}^{-1}(cx)\right)}{be^2}$$
$$+ \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{e^2}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)}\, dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be^2}$$
$$- \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\, dx, x, a+b\text{csch}^{-1}(cx)\right)}{be^2}$$
$$- \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\, dx, x, \frac{1}{x}\right)}{2e^2} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\, dx, x, \frac{1}{x}\right)}{2e^2}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2}\, dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2ce}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}}$$
$$- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^2}$$
$$+ \frac{\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}$$
$$- \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}$$
$$+ \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}}$$
$$- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^2}$$
$$- \frac{b\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^2}$$
$$- \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}$$
$$- \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}$$
$$+ \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}$$
$$+ \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}} \\
&\quad - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)}{e^2} \\
&\quad + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} \\
&\quad + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2} + \frac{b\,\text{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\text{csch}^{-1}(cx)}{b}\right)}\right)}{2e^2}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 1410, normalized size of antiderivative = 2.55

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$
\begin{aligned}
&= \frac{b\pi^2 + \frac{4ad}{d+ex^2} - 4ib\pi\text{csch}^{-1}(cx) + \frac{2b\sqrt{d}\text{csch}^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{2b\sqrt{d}\text{csch}^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} - 8b\text{csch}^{-1}(cx)^2 - 4b\text{arcsinh}\left(\frac{1}{cx}\right) + 16b\,\text{ar}}{}
\end{aligned}
$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]

[Out] (b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsch[c*x] + (2*b*Sqrt[d]*ArcC
sch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] + I
*Sqrt[e]*x) - 8*b*ArcCsch[c*x]^2 - 4*b*ArcSinh[1/(c*x)] + 16*b*ArcSin[Sqrt[
1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (
2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[
```

```
c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x]
)] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]
/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] - (2*I)*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqr
t[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*
(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e] + (2*b*Sqrt[e]*Log[(-2*
Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(
c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
 + e] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*b*Poly
Log[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])])/(8*e^2)
```

Maple [F]

$$\int \frac{x^3(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^2}dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^3(a+b\operatorname{acsch}(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqrt
(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^3\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.105 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result . . . 773
Rubi [A] (verified) . . . 773
Mathematica [C] (verified) . . . 775
Maple [B] (verified) . . . 776
Fricas [B] (verification not implemented) . . . 777
Sympy [F] . . . 777
Maxima [F] . . . 778
Giac [F] . . . 778
Mupad [F(-1)] . . . 778

Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{bcx\arctan\left(\sqrt{-1-c^2x^2}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\sqrt{e}\sqrt{-c^2x^2}}$$

```
[Out] 1/2*(-a-b*arccsch(c*x))/e/(e*x^2+d)+1/2*b*c*x*arctan((-c^2*x^2-1)^(1/2))/d/
e/(-c^2*x^2)^(1/2)+1/2*b*c*x*arctanh(e^(1/2)*(-c^2*x^2-1)^(1/2)/(c^2*d-e)^(
1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6435, 457, 88, 65, 211, 214}

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{bcx\arctan\left(\sqrt{-c^2x^2-1}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{e}\sqrt{-c^2x^2}\sqrt{c^2d-e}}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsch[c*x])/(e*(d + e*x^2)) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2
]])/(2*d*e*Sqrt[-(c^2*x^2)]) + (b*c*x*ArcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/
Sqrt[c^2*d - e]])/(2*d*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{-c^2x^2}} \\ &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)}\,dx, x, x^2\right)}{4e\sqrt{-c^2x^2}}\end{aligned}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}(d+ex)}\,dx, x, x^2\right)}{4d\sqrt{-c^2x^2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{4de\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bx)\text{Subst}\left(\int \frac{1}{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{2cd\sqrt{-c^2x^2}} - \frac{(bx)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{2cde\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{bcx\arctan\left(\sqrt{-1-c^2x^2}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\sqrt{e}\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx =$$

$$-\frac{\frac{2a}{d+ex^2} + \frac{2b\text{csch}^{-1}(cx)}{d+ex^2} - \frac{2b\text{arcsinh}\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}\log\left(-\frac{4\left(ide+cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b\sqrt{-c^2d+e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d+e}} + \frac{b\sqrt{e}\log\left(\frac{4i\left(de+cd\sqrt{e}\left(ic\sqrt{d}+\right.\right.\right.}{b\sqrt{-c^2d-}}\right.}{d\sqrt{-c^2}}}{4e}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsch[c*x])/(d + e*x^2) - (2*b*ArcSinh[1/(
c*x)])/d + (b*Sqrt[e]*Log[(-4*(I*d*e + c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^
2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^2*d) + e]*(Sqrt[d] - I*Sqr
t[e]*x))])/(d*Sqrt[-(c^2*d) + e]) + (b*Sqrt[e]*Log[((4*I)*(d*e + c*d*Sqrt[e
]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^
2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) + e]))/e
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(120) = 240.

Time = 6.22 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

method	result
parts	$-\frac{a}{2e(ex^2+d)}+\frac{b\left(-\frac{c^4\,\mathrm{arccsch}(cx)}{2e\left(ec^2x^2+c^2d\right)}+\frac{c\sqrt{c^2x^2+1}\left(2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2+1}\sqrt{-\frac{c^2d-e}{e}}\,e+\sqrt{-c^2d}\right.\right.\right.\right.}{-cex+\sqrt{-c^2de}}\right.}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd\sqrt{-\frac{c^2d-}{e}}}\right)}{c^2}$
derivativedivides	$-\frac{a\,c^4}{2e\left(ec^2x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arccsch}(cx)}{2e\left(ec^2x^2+c^2d\right)}+\frac{\sqrt{c^2x^2+1}\left(2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2+1}\sqrt{-\frac{c^2d-e}{e}}\,e+\sqrt{-c}\right.\right.\right.\right.}{-cex+\sqrt{-c^2de}}\right.}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3xd\sqrt{-\frac{c^2}{}}}\right)}{c^2}$
default	$-\frac{a\,c^4}{2e\left(ec^2x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arccsch}(cx)}{2e\left(ec^2x^2+c^2d\right)}+\frac{\sqrt{c^2x^2+1}\left(2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2+1}\sqrt{-\frac{c^2d-e}{e}}\,e+\sqrt{-c}\right.\right.\right.\right.}{-cex+\sqrt{-c^2de}}\right.}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3xd\sqrt{-\frac{c^2}{}}}\right)}{c^2}$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arccsch(c*x)+1/4*c/e
*(c^2*x^2+1)^(1/2)*(2*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)-ln(
-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*
x+(-c^2*d*e)^(1/2)))-ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2
*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x
/d/(-(c^2*d-e)/e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.42

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2-2\,ade+\sqrt{-c^2de+e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d-2\sqrt{-c^2de+e^2}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,e}{ex^2+d}\right)-2\,(bc^2d^2-bde+}{} \right.$$

$$\left. -\frac{ac^2d^2-ade+\sqrt{c^2de-e^2}(bex^2+bd)\arctan\left(-\frac{\sqrt{c^2de-e^2}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{c^2d-e}\right)-(bc^2d^2-bde+(bc^2de-be^2)x^2)}{2\,(c^2d^3} \right]$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*a*d*e + sqrt(-c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^
2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 2*e)/(e*x^2 + d)) - 2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^2*d^2 - b*d*e + (b*c^2
*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*
c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d
^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*e + sqrt
(c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x
^2 + 1)/(c^2*x^2))/(c^2*d - e)) - (b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*
x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^2 - b*d*e
+ (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1)
 + (b*c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/
(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x(a+b\,\operatorname{acsch}(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^2*integrate(1/2*x/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e + (c^2
*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(c^2*x^2 + 1)), x) - (2*c^2*d^2*l
og(c) - 2*(c^2*d*e - e^2)*x^2*log(x) - 2*d*e*log(c) + (c^2*d*e*x^2 + c^2*d^
2)*log(c^2*x^2 + 1) - 2*(c^2*d^2 - d*e)*log(sqrt(c^2*x^2 + 1) + 1))/(c^2*d^
3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2) + log(e*x^2 + d)/(c^2*d^2 - d*e)
)*b - 1/2*a/(e^2*x^2 + d*e)
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.106 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal result . . . 779
Rubi [A] (verified) . . . 780
Mathematica [C] (warning: unable to verify) . . . 786
Maple [F] . . . 787
Fricas [F] . . . 788
Sympy [F] . . . 788
Maxima [F] . . . 788
Giac [F] . . . 788
Mupad [F(-1)] . . . 789

Optimal result

Integrand size = 21, antiderivative size = 515

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}$$

```
[Out] -1/2*e*(a+b*arccsch(c*x))/d^2/(e+d/x^2)+1/2*(a+b*arccsch(c*x))^2/b/d^2-1/2*
(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
-c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1
```

```
/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsch(c*x))*ln(1
-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^2-1
/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^2-1/2*b*
polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2
)))/d^2+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/
2)/d^2/(c^2*d-e)^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6439, 5823, 5821, 385, 211, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d^2}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^2}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d^2}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}\\ &+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2d^2\sqrt{c^2d-e}}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^2}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcCsch[c*x]))/(d^2*(e + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2
*b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)
]*x)])/(2*d^2*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
```

```
(2*d^2) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^2) - (b*PolyLog[2,
 -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^2) -
(b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*d^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2} + \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d} \\
&\quad + \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2cd^2}
\end{aligned}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}}$$
$$- \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} + \frac{(be)\operatorname{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2cd^2}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}$$
$$+ \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$- \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}$$
$$+ \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$+ \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$- \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$- \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$
\begin{aligned}
&= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^2}
\end{aligned}
$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}$$
$$- \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}$$
$$- \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}$$
$$- \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}$$
$$- \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}$$
$$- \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}$$
$$- \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 1428, normalized size of antiderivative = 2.77

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \frac{a}{2d^2+2dex^2} + \frac{a\log(x)}{d^2} - \frac{a\log\left(d+ex^2\right)}{2d^2}$$
$$- \frac{b\left(\pi^2 - 4i\pi\text{csch}^{-1}(cx) - \frac{2\sqrt{d}\text{csch}^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} - \frac{2\sqrt{d}\text{csch}^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} - 12\text{csch}^{-1}(cx)^2 + 4\text{arcsinh}\left(\frac{1}{cx}\right) + 16\arcsin\left(\frac{\sqrt{1}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]

[Out] a/(2*d^2 + 2*d*e*x^2) + (a*Log[x])/d^2 - (a*Log[d + e*x^2])/(2*d^2) - (b*(P
i^2 - (4*I)*Pi*ArcCsch[c*x] - (2*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[e]
*x) - (2*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 12*ArcCsch[c*x]^2
+ 4*ArcSinh[1/(c*x)] + 16*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Arc
Tan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d)
+ e]] - 16*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d]
 + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCs
ch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (2*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*Log[1 - (I*(-
```

```
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*ArcSin[S
qrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-
(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*Log[1 + (I*(-Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*ArcSin[Sqr
t[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*ArcSin[Sqrt[1
- Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*ArcSin[Sqrt[1 + Sqr
t[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] - (2*I)
*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] -
(2*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]
*x))])/Sqrt[-(c^2*d) + e] - (2*Sqrt[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c
*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d
) + e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) + e] + 4*PolyLog[2, E^(-2*A
rcCsch[c*x])] + 4*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + 4*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + 4*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*PolyLog[2, E^(2*ArcCsch[c*x])]))/(8*d
^2)
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex^2+d)^2x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\operatorname{acsch}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**2,x)
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex^2+d)^2x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex^2+d)^2x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{x\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^2),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.107 $\int \frac{x^4\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result 791
Rubi [A] (verified) 792
Mathematica [C] (warning: unable to verify) 803
Maple [F] 806
Fricas [F] 806
Sympy [F] 806
Maxima [F(-2)] 806
Giac [F] 807
Mupad [F(-1)] 807

Optimal result

Integrand size = 21, antiderivative size = 756

$$\begin{aligned}\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx &= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\ &+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\ &+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\ &+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\ &+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\ &+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\end{aligned}$$

```
[Out] x*(a+b*arccsch(c*x))/e^2+b*arctanh((1+1/c^2/x^2)^(1/2))/c/e^2+3/4*(a+b*arcc
sch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)
^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a
+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c
^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)
```

```
-3/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d
+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylog(2
,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)
^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+1/4*b*arctanh((c^2*d-(-d)^(1/2)
*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2
*d-e)^(1/2)+1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^
(1/2)/(1+1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d-e)^(1/2)-1/4*d*(a+b*arccsch(c
*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsch(c*x))/e^2/(d/x+(-d)^(
1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules

used = {6439, 5823, 5776, 272, 65, 214, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \frac{3\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}$$
$$-\frac{3\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4e^{5/2}}$$
$$+\frac{3\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4e^{5/2}}$$
$$-\frac{3\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4e^{5/2}}$$
$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4e^2\sqrt{c^2d-e}}$$
$$+\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4e^2\sqrt{c^2d-e}}+\frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{ce^2}$$
$$-\frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}$$
$$+\frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}$$
$$-\frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4e^{5/2}}$$
$$+\frac{3b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4e^{5/2}}$$

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(d*(a + b*ArcCsch[c*x]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*Ar
cCsch[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsch[c*x]))/e^
2 + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d
 - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])
/(4*Sqrt[c^2*d - e]*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x
)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[c^2*d - e]*e^
2) + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4
*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[
```

```
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*Arc
Csch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*
b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x
])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{e}$$

$$
\begin{aligned}
&= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{d\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} \\
&\quad + \frac{d\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ce^2} \\
&\quad - \frac{d^2\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} - \frac{d^2\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} \\
&\quad - \frac{d^2\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e^2} \\
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{d\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}} \\
&\quad + \frac{d\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}} \\
&\quad - \frac{(bc)\operatorname{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)}{e^2} \\
&\quad + \frac{(bd)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce^2} \\
&\quad - \frac{(bd)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce^2} \\
&\quad - \frac{d^2\operatorname{Subst}\left(\int \left(-\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{4e^{5/2}}+\frac{d\operatorname{Subst}\left(\int\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{4e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad-\frac{(bd)\operatorname{Subst}\left(\int\frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx,x,\frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4ce^2}\\
&\quad+\frac{(bd)\operatorname{Subst}\left(\int\frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx,x,\frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4ce^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\
&\quad+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\
&\quad+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\
&\quad+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\
&\quad+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad+\frac{d\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\
&\quad+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\
&\quad+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\
&\quad+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\
&\quad+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4e^{5/2}}\\
&\quad-\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4e^{5/2}}\\
&\quad+\frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}\\
&\quad+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}\\
&\quad+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad+\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}}\\
&\quad-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 1593, normalized size of antiderivative = 2.11

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \frac{ax}{e^2} + \frac{adx}{2e^2\left(d+ex^2\right)} - \frac{3a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}$$

$$+ b\left(-\frac{d\left(-\frac{\operatorname{csch}^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex} - \frac{i\left(\frac{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{\sqrt{e}} - \frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d+e}\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)}{4e^2}\right.$$

$$\left. - \frac{d\left(-\frac{\operatorname{csch}^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} + \frac{i\left(\frac{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{\sqrt{e}} - \frac{\log\left(-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d+e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)}{4e^2}\right)$$

$$- \frac{3i\sqrt{d}\left(\pi^2 - 4i\pi\operatorname{csch}^{-1}(cx) - 8\operatorname{csch}^{-1}(cx)^2 + 32\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right.}{\sqrt{-c^2d+e}}\right.\right.}{}$$

$$+ \frac{3i\sqrt{d}\left(\pi^2 - 4i\pi\operatorname{csch}^{-1}(cx) - 8\operatorname{csch}^{-1}(cx)^2 - 32\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right.}{\sqrt{-c^2d+e}}\right.\right.}{}$$

$$+ \frac{\frac{1}{2}\operatorname{csch}^{-1}(cx)\coth\left(\frac{1}{2}\operatorname{csch}^{-1}(cx)\right) + \log\left(\cosh\left(\frac{1}{2}\operatorname{csch}^{-1}(cx)\right)\right) - \log\left(\sinh\left(\frac{1}{2}\operatorname{csch}^{-1}(cx)\right)\right) - \frac{1}{2}\operatorname{csch}^{-1}(cx)\,\mathrm{ta}}{ce^2}$$

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)
) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c
*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) +
 e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/e^2 - (d*(-(Ar
cCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - L
og[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[
1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(
c^2*d) + e]))/Sqrt[d]))/(4*e^2) - (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I)*Pi*ArcC
sch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1
 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1
 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsc
h[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e
] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/e^(5/2)
+ (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Lo
g[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - S
qrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2
, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/e^(5/2) + ((ArcCsch[c*x]*Coth[ArcCsch[c*
x]/2])/2 + Log[Cosh[ArcCsch[c*x]/2]] - Log[Sinh[ArcCsch[c*x]/2]] - (ArcCsch
[c*x]*Tanh[ArcCsch[c*x]/2])/2)/(c*e^2))
```

Maple [F]

$$\int \frac{x^4(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^2}dx$$

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^4}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^4(a+b\,\mathrm{acsch}\,(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**4*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^4}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^4\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.108 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result 809
Rubi [A] (verified) 810
Mathematica [C] (warning: unable to verify) 817
Maple [F] 818
Fricas [F] 819
Sympy [F] 819
Maxima [F(-2)] 819
Giac [F] 819
Mupad [F(-1)] 820

Optimal result

Integrand size = 21, antiderivative size = 719

$$\begin{aligned}\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx &= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\ &-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e}\\ &-\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e}\\ &+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\ &+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\end{aligned}$$

```
[Out] 1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1+c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)
^(1/2)+1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1
+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/
2)/(-d)^(1/2)-1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(1+1/c
```

```
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/
4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e/d
^(1/2)/(c^2*d-e)^(1/2)-1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d-e)^(1/2)+1/4*(a+b*ar
ccsch(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsch(c*x))/e/(d/x+(-d)
^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {6439, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4\sqrt{-d}e^{3/2}} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4\sqrt{-d}e^{3/2}} + \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4\sqrt{d}e\sqrt{c^2d-e}} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4\sqrt{d}e\sqrt{c^2d-e}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCsch[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsch[c*x])/
(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/
(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d -
 e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d - e]*e) + ((a + b*ArcCsch
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])
/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcC
```

```
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*Sqr
t[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLo
g[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-
d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e}
\end{aligned}$$

$$= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce}$$
$$+ \frac{b\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce}$$
$$+ \frac{d\operatorname{Subst}\left(\int \left(-\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}-\sqrt{-d}x)} - \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)\,dx, x, \frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}}$$
$$- \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4ce}$$
$$- \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4ce}$$

$$= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e}$$
$$- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4e^{3/2}}$$
$$- \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4e^{3/2}}$$

$$
\begin{aligned}
&= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4e^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4e^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4e^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4e^{3/2}} \\
&= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 1442, normalized size of antiderivative = 2.01

$$
\begin{aligned}
&\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx \\
&= -\frac{4a\sqrt{e}x}{d+ex^2} + \frac{4a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + b\left(\frac{2\operatorname{csch}^{-1}(cx)}{i\sqrt{d}-\sqrt{e}x} - \frac{2\operatorname{csch}^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x} + \frac{8i\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right.}{\sqrt{-c^2d+e}}\right.}{\sqrt{d}}\right.
\end{aligned}
$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]

[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((2*ArcCsch[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsch[c*x])/(I*Sqrt[d]
+ Sqrt[e]*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[
((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]
])/Sqrt[d] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]])
```

```
/Sqrt[d] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])])/Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] - (4*ArcSin[Sqrt[1 + Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])])/Sqrt[d] + (Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] - ((2*I)*ArcCsch[c*x]*Log[1 + (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] + (4
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] + (Pi*Log[1 - (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] - ((2*I)*Ar
cCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])])/Sqrt[d] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 -
(I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] - (
Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/
Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] + (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])])/Sqrt[d] - (Pi*Log[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[d] + (Pi*Log[Sqrt[
e] + (I*Sqrt[d])/x])/Sqrt[d] + ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqr
t[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt
[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/(Sqrt[d]*Sqrt[-(c^2*d) + e]) - ((
2*I)*Sqrt[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e
]*x))])/(Sqrt[d]*Sqrt[-(c^2*d) + e]) - ((2*I)*PolyLog[2, ((-I)*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] + ((2*I)*PolyLog[
2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d]
 + ((2*I)*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])])/Sqrt[d] - ((2*I)*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d]))/(8*e^(3/2))
```

Maple [F]

$$\int \frac{x^2(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^2}dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^2}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^2(a+b\,\operatorname{acsch}(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^2}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{x^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.109 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal result . . . 821
Rubi [A] (verified) . . . 822
Mathematica [C] (warning: unable to verify) . . . 831
Maple [F] . . . 833
Fricas [F] . . . 833
Sympy [F] . . . 833
Maxima [F(-2)] . . . 834
Giac [F] . . . 834
Mupad [F(-1)] . . . 834

Optimal result

Integrand size = 18, antiderivative size = 713

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\ &+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\ &+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\ &+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\ &+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\ &+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

```
[Out] 1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c
^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(1/2)+1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1
```

```
/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(1/
2)-1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccsch(c*x))*ln(1+c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccsch(c*x))*l
n(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-
d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(1+
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
+1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d
+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-a-b*arccs
ch(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsch(c*x))/d/(d/x+(-d)^(1/
2)*e^(1/2))
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6429, 5823, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$
\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcCsch[c*x])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsch[c*x
])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/
x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*
d - e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
 e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[
c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsc
h[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*A
rcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) +
 e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E
^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])
/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[
-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]
) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6429

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d} \\
&\quad + \frac{e\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d\sqrt{e}} \\
&= -\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd} - \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd} \\
&\quad - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2d\sqrt{e}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2d\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&\quad-\frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx,x,\frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cd}+\frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx,x,\frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cd}\\
&\quad+\frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{4d\sqrt{e}}+\frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{4d\sqrt{e}}\\
&\quad-\frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2d\sqrt{e}}\\
&\quad-\frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2d\sqrt{e}}\\
&\quad-\frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2d\sqrt{e}}\\
&\quad-\frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2d\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&\quad+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\operatorname{Subst}\left(\int\frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4d\sqrt{e}}\\
&\quad+\frac{\operatorname{Subst}\left(\int\frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4d\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&-\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&-\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&+\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4d\sqrt{e}}\\
&+\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4d\sqrt{e}}\\
&+\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4d\sqrt{e}}\\
&+\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\operatorname{csch}^{-1}(cx)\right)}{4d\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&\quad+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&\quad+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&\quad+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&\quad+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.78 (sec) , antiderivative size = 1437, normalized size of antiderivative = 2.02

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}\right.$$

$$\left. +\, b\left(\frac{2\sqrt{d}\text{csch}^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}+\frac{2\sqrt{d}\text{csch}^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{8i\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\text{csch}^{-1}(cx)\right)\right)}{\sqrt{-c^2d+e}}\right)}{\sqrt{e}}+\frac{8i\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right.}{}\right.\right.$$

$$+\frac{}{}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]

[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((2*Sqrt[d]*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*Ar
cCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*S
qrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x
])/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])
/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcCsch[c*x]*Log[1 -
 (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] -
 (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + (Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - ((2*I
)*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])])/Sqrt[e] + (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[
e] + (Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])])/Sqrt[e] - ((2*I)*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])])/Sqrt[e] - (Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
```

```
Csch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + (4*ArcSin[Sqr
t[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - (Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]
)/Sqrt[e] + (Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/Sqrt[e] - ((2*I)*Log[(2*Sqrt[
d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2
*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) +
e] + ((2*I)*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]
*x))])/Sqrt[-(c^2*d) + e] - ((2*I)*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*PolyLog[2, (I*(-Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*P
olyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
)/Sqrt[e] - ((2*I)*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])])/Sqrt[e]))/(4*d^(3/2)))/2
```

Maple [F]

$$\int \frac{a + b\ \mathrm{arccsch}\left(cx\right)}{\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{\left(d + ex^2\right)^2}\,dx = \int \frac{b\,\mathrm{arcsch}\left(cx\right) + a}{\left(ex^2 + d\right)^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{\left(d + ex^2\right)^2}\,dx = \int \frac{a + b\,\mathrm{acsch}\left(cx\right)}{\left(d + ex^2\right)^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^2, x)
```

3.110 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal result 836
Rubi [A] (verified) 837
Mathematica [C] (warning: unable to verify) 847
Maple [F] 848
Fricas [F] 849
Sympy [F(-1)] 849
Maxima [F(-2)] 849
Giac [F] 849
Mupad [F(-1)] 850

Optimal result

Integrand size = 21, antiderivative size = 758

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx &= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\operatorname{csch}^{-1}(cx)}{d^2x}\\ &+\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\ &-\frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}}-\frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}}\\ &-\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &+\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &-\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &+\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\\ &-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}\end{aligned}$$

```
[Out] -a/d^2/x-b*arccsch(c*x)/d^2/x-1/4*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)-1/4*
b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2
/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)-3/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/
2)+3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1-c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(
-d)^(5/2)+3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
```

```
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/
x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^
(5/2)-3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*poly
log(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*
e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsch(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1
/4*e*(a+b*arccsch(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+b*c*(1+1/c^2/x^2)^(1/2
)/d^2
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6439, 5823, 5772, 267, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx &= -\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} \\
&\quad+\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{5/2}} \\
&\quad-\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&\quad+\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4(-d)^{5/2}} \\
&\quad+\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{a}{d^2x} \\
&\quad-\frac{b\operatorname{earctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4d^{5/2}\sqrt{c^2d-e}}-\frac{b\operatorname{earctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4d^{5/2}\sqrt{c^2d-e}} \\
&\quad+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{d^2}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} \\
&\quad-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} \\
&\quad+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} \\
&\quad-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}-\frac{b\operatorname{csch}^{-1}(cx)}{d^2x}
\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcCsch[c*x])/(d^2*x) + (e
*(a + b*ArcCsch[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsch
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*
Sqrt[c^2*d - e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d - e]) - (3*Sq
rt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/
2)) - (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/
(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqr
t[e] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (
3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*
d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{d^2}+\frac{e^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}-\frac{2e\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad + \frac{(2e)\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} \\
&= -\frac{a}{d^2x} - \frac{b\text{Subst}\left(\int \text{arcsinh}\left(\frac{x}{c}\right)\,dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad + \frac{(2e)\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad - \frac{e^2\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2}
\end{aligned}$$

$$= -\frac{a}{d^2 x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{b\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2}$$
$$+ \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2}$$
$$+ \frac{e\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d} + \frac{e\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d}$$
$$+ \frac{e\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{d^2}$$
$$- \frac{(be)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{(be)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{e\operatorname{Subst}\left(\int \left(-\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2} - \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2}$$
$$+ \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{d^2}$$
$$+ \frac{(be)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cd^2}$$
$$- \frac{(be)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cd^2}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&\quad - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} \\
&\quad - \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{(-d)^{5/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{(-d)^{5/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{(-d)^{5/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{(-d)^{5/2}} \\
&\quad - \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}dx, x, \operatorname{csch}^{-1}(cx)\right)}{4d^2} \\
&\quad - \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}dx, x, \operatorname{csch}^{-1}(cx)\right)}{4d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} \\
&- \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&+ \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&- \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&+ \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&+ \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{(-d)^{5/2}} \\
&- \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{(-d)^{5/2}} \\
&+ \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{(-d)^{5/2}} \\
&- \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{(-d)^{5/2}} \\
&- \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4d^2} \\
&- \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4d^2} \\
&- \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4d^2} \\
&- \frac{\sqrt{e}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} - \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} \\
&\quad - \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad - \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{5/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{5/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{5/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{4(-d)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} - \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} \\
&\quad - \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad - \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{5/2}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{4(-d)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} - \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}} \\
&\quad - \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad - \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 1487, normalized size of antiderivative = 1.96

$$
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx
$$

$$
= \frac{-\frac{8a\sqrt{d}}{x} - \frac{4a\sqrt{d}ex}{d+ex^2} - 12a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(8c\sqrt{d}\sqrt{1+\frac{1}{c^2x^2}} - \frac{8\sqrt{d}\operatorname{csch}^{-1}(cx)}{x} - \frac{2\sqrt{d}e\operatorname{csch}^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} - \frac{2\sqrt{d}e\operatorname{csch}}{i\sqrt{d}\sqrt{e}-}\right.}{}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(8*c*Sqrt[d]*Sqrt[1 + 1/(c^2*x^2)] - (8*Sqrt[d]*ArcCs
ch[c*x])/x - (2*Sqrt[d]*e*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2*S
qrt[d]*e*ArcCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (24*I)*Sqrt[e]*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
```

```
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - (24*I)*Sqrt[e]*ArcSin[Sqrt[
1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (
2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] + 3*Sqrt[e]*Pi*Log[1 - (I*(-Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 3*Sqrt[e]*P
i*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] - 3*Sqrt[e]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + 3*Sqrt[e]*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcS
in[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 3*Sqrt[e]*Pi*Log[Sqrt[e] - (I*Sqrt[
d])/x] - 3*Sqrt[e]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + ((2*I)*e*Log[(2*Sqrt[d
]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*
x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e
] - ((2*I)*e*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e
]*x))])/Sqrt[-(c^2*d) + e] + (6*I)*Sqrt[e]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*PolyLog[2, (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[
e]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + (6*I)*Sqrt[e]*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])]))/(8*d^(5/2))
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 \left(e\,x^2 + d\right)^2} dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

Fricas [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2\left(d + ex^2\right)^2}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.111 $\int \frac{x^5\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result 852
Rubi [A] (verified) 853
Mathematica [C] (warning: unable to verify) 862
Maple [F] 864
Fricas [F] 864
Sympy [F(-1)] 864
Maxima [F] 864
Giac [F] 865
Mupad [F(-1)] 865

Optimal result

Integrand size = 21, antiderivative size = 694

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8\left(c^2d-e\right)^{3/2}e^{5/2}}$$
$$+ \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}}$$
$$- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$

```
[Out] 1/4*(-a-b*arccsch(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsch(c*x))/e^2/(e+d/x^2)
-(a+b*arccsch(c*x))^2/b/e^3+1/8*b*(c^2*d-2*e)*arctan((c^2*d-e)^(1/2)/c/x/e^
(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e^(5/2)-(a+b*arccsch(c*x))*ln(1-
1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arc
csch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e
```

```
)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/
x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*pol
ylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,
c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/
2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^
2/x^2)^(1/2))/e^(5/2)/(c^2*d-e)^(1/2)+1/8*b*c*d*(1+1/c^2/x^2)^(1/2)/(c^2*d-
e)/e^2/(e+d/x^2)/x
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules

used = {6439, 5823, 5775, 3797, 2221, 2317, 2438, 5821, 390, 385, 211, 5827, 5680}

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}$$
$$+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^3}$$
$$+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^3}$$
$$+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^3}$$
$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\frac{d}{x^2}+e\right)^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3}$$
$$-\frac{\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+\frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{5/2}\sqrt{c^2d-e}}$$
$$+\frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{8e^{5/2}\left(c^2d-e\right)^{3/2}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}$$
$$+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^3}$$
$$+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^3}$$
$$+\frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{8e^2x\left(c^2d-e\right)\left(\frac{d}{x^2}+e\right)}+\frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e^3}$$

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(8*(c^2*d - e)*e^2*(e + d/x^2)*x) - (a + b*Ar
cCsch[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsch[c*x])/(2*e^2*(e + d/x^2))
- (a + b*ArcCsch[c*x])^2/(b*e^3) + (b*(c^2*d - 2*e)*ArcTan[Sqrt[c^2*d - e]/
(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*(c^2*d - e)^(3/2)*e^(5/2)) + (b*Ar
```

```
cTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d -
e]*e^(5/2)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[c*x])])/e^3 + ((a
 + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^
2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[
1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) +
((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-
(c^2*d) + e])])/(2*e^3) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*e^3) + (b*
PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/
(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d
) + e])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/
(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e^3 x} - \frac{dx\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} \right.\right. \\
&\qquad \left.\left. - \frac{dx\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{e} \\
&= -\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&\quad + \frac{\text{Subst}\left(\int x\coth\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{csch}^{-1}(cx)\right)}{be^3} \\
&\quad + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e^3} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{4ce}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be^3} - \frac{2\operatorname{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{be^3} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2ce^2} \\
&\quad + \frac{\left(b\left(c^2d-2e\right)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{8c\left(c^2d-e\right)e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2be^3} \\
&\quad + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{\operatorname{Subst}\left(\int \log\left(1-e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad + \frac{\left(b\left(c^2d-2e\right)\right)\operatorname{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8c\left(c^2d-e\right)e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8\left(c^2d-e\right)^{3/2}e^{5/2}} \\
&\quad + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^3} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad - \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad + \frac{\sqrt{-d}\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8\left(c^2d-e\right)^{3/2}e^{5/2}} \\
&\quad + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx,x,\operatorname{csch}^{-1}(cx)\right)}{2e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8\left(c^2d-e\right)^{3/2}e^{5/2}} \\
&\quad + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^3} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8\left(c^2d-e\right)^{3/2}e^{5/2}} \\
&\quad + \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{5/2}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} \\
&\quad + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{csch}^{-1}(cx)}{b}\right)}\right)}{2e^3}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.89 (sec) , antiderivative size = 2023, normalized size of antiderivative = 2.91

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]

[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*(-1/16*(d*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*
(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[
d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4
*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt
[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d
- e)^(3/2))))/e^(5/2) - (d*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[
```

```
d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d]
+ Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I
)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqr
t[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d
 - e)^(3/2))))/(16*e^(5/2)) - (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/(I*Sqrt[d
]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/
(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d])
)/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x
)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I
*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) +
 e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/e^(5/2) + (Pi^
2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[
c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])]
 + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
 + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4
*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8
*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
 + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])])/(16*e^3) + (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*Ar
cSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*C
ot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1
 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-
(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E
^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])])/(16*e^3))
```

Maple [F]

$$\int \frac{x^5(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^3}dx$$

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^5}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsch(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^5}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^3*x^6 +
 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^5}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{x^5\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.112 $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result . . . 866
Rubi [A] (verified) . . . 866
Mathematica [C] (verified) . . . 869
Maple [B] (verified) . . . 869
Fricas [B] (verification not implemented) . . . 871
Sympy [F(-1)] . . . 872
Maxima [F] . . . 872
Giac [F] . . . 872
Mupad [F(-1)] . . . 873

Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{bc(c^2d-2e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d\left(c^2d-e\right)^{3/2}e^{3/2}\sqrt{-c^2x^2}}$$

```
[Out] 1/4*x^4*(a+b*arccsch(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d-2*e)*x*arctanh(e^(1
/2)*(-c^2*x^2-1)^(1/2)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(3/2)/e^(3/2)/(-c^2*x^2
)^(1/2)-1/8*b*c*x*(-c^2*x^2-1)^(1/2)/(c^2*d-e)/e/(e*x^2+d)/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 6437, 12, 457, 79, 65, 214}

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} + \frac{bcx(c^2d-2e)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8de^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}} - \frac{bcx\sqrt{-c^2x^2-1}}{8e\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x*Sqrt[-1 - c^2*x^2])/((c^2*d - e)*e*Sqrt[-(c^2*x^2)]*(d + e*x^2)
) + (x^4*(a + b*ArcCsch[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d - 2*e)*x*A
```

```
rcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d*(c^2*d - e)^(3/2
)*e^(3/2)*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bcx)\int \frac{x^3}{4d\sqrt{-1-c^2x^2}(d+ex^2)^2}\,dx}{\sqrt{-c^2x^2}} \\
&= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bcx)\int \frac{x^3}{\sqrt{-1-c^2x^2}(d+ex^2)^2}\,dx}{4d\sqrt{-c^2x^2}} \\
&= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{x}{\sqrt{-1-c^2x}(d+ex)^2}\,dx, x, x^2\right)}{8d\sqrt{-c^2x^2}} \\
&= -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} \\
&\quad - \frac{\left(bc(c^2d-2e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}(d+ex)}\,dx, x, x^2\right)}{16d\left(c^2d-e\right)e\sqrt{-c^2x^2}} \\
&= -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} \\
&\quad + \frac{\left(b(c^2d-2e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{8cd\left(c^2d-e\right)e\sqrt{-c^2x^2}} \\
&= -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} \\
&\quad + \frac{bc(c^2d-2e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d\left(c^2d-e\right)^{3/2}e^{3/2}\sqrt{-c^2x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.25

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx =$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2bce\sqrt{1+\frac{1}{c^2x^2}}x}{(-c^2d+e)(d+ex^2)}+\frac{4b(d+2ex^2)\text{csch}^{-1}(cx)}{(d+ex^2)^2}-\frac{4b\text{arcsinh}\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}\left(-c^2d+2e\right)\log\left(\frac{16de^{3/2}\sqrt{-c^2d+e}}{b}\right.}{d(-c^2}}{16e^2}$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 + 1/(c^
2*x^2)]*x)/((-(c^2*d) + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcCsch[c*x])/
(d + e*x^2)^2 - (4*b*ArcSinh[1/(c*x)])/d + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[
(16*d*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(-(c^2*d) + 2*e)*(I*Sqrt[d] + Sqrt[e]
*x))])/(d*(-(c^2*d) + e)^(3/2)) + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[((-16*I)*
d*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]
*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(c^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*
(-(c^2*d) + e)^(3/2)))/e^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(145) = 290$.

Time = 7.63 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.79

method	result
parts	$a\left(\frac{d}{4e^2(e\,x^2+d)^2}-\frac{1}{2e^2(e\,x^2+d)}\right)+b\left(\frac{c^8\,\mathrm{arccsch}(cx)\,d}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{c^6\,\mathrm{arccsch}(cx)}{2e^2\left(e\,c^2x^2+c^2d\right)}-\frac{c^3\sqrt{c^2x^2+1}\left(4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-c^{\cdots}}\right.\right.$
derivativedivides	$a\,c^6\left(\frac{d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{1}{2e^2\left(e\,c^2x^2+c^2d\right)}\right)+b\,c^6\left(\frac{\mathrm{arccsch}(cx)\,d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{\mathrm{arccsch}(cx)}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\sqrt{c^2x^2+1}\left(-4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right.\right.\right.$
default	$a\,c^6\left(\frac{d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{1}{2e^2\left(e\,c^2x^2+c^2d\right)}\right)+b\,c^6\left(\frac{\mathrm{arccsch}(cx)\,d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{\mathrm{arccsch}(cx)}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\sqrt{c^2x^2+1}\left(-4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right.\right.\right.$

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(1/4*c^8*arccsch(c*x)*d/e
^2/(c^2*e*x^2+c^2*d)^2-1/2*c^6*arccsch(c*x)/e^2/(c^2*e*x^2+c^2*d)-1/16*c^3*
(c^2*x^2+1)^(1/2)/e*(4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*c^
4*d*e*x^2+4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*c^4*d^2-ln(-2
*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+
(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2
)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-ln(-2*(-(c^2
*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d
*e)^(1/2)))*x^2*c^4*d*e-ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-4*arctanh(1/(c^2*x^
2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*e^2*c^2*x^2-4*arctanh(1/(c^2*x^2+1)^(1/2))
*(-(c^2*d-e)/e)^(1/2)*c^2*d*e-2*(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*c^2*
d*e+2*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+
e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2+2*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2
*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e
+2*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)
/(c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^2*e^2+2*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d
-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e)/(
(c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(c*e*x+(-c^2*
d*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(145) = 290$.

Time $= 0.43$ (sec) , antiderivative size $= 1381$, normalized size of antiderivative $= 8.27$

$$\int \frac{x^3\left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e - 2*a*c^
2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d
^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e + e^2)*log((c^2*e*x^2
 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/
(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 -
2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^
2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 4*(b*c^4*d^4 - 2*b*c^
2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^
4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e
 - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + 2*((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^
2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4
 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e
^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*
c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*
b*e^3)*x^4 - 2*b*d^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(c^2*d*e - e^
2)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d - e
)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*
e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e +
b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e -
2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
 - 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e - 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + ((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*
e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5
)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)x^3}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/8*b*((2*c^4*d^4*log(c) - 2*(c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4*log(x) +
 2*d^2*e^2*log(c) + d^2*e^2 - (4*d^3*e*log(c) + d^3*e)*c^2 + (4*c^4*d^3*e*l
og(c) + 4*d*e^3*log(c) + d*e^3 - (8*d^2*e^2*log(c) + d^2*e^2)*c^2)*x^2 + (c
^4*d^4 - 2*c^2*d^3*e + (c^4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + 2*(c^4*d^3*e - 2*c
^2*d^2*e^2)*x^2)*log(c^2*x^2 + 1) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2 + 2*
(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*x^2)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d
^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^
4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2) + log(e*x^2 + d)/(c^4*d^
3 - 2*c^2*d^2*e + d*e^2) - 8*integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)/(c^2*e^4
*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3)*x^2 + (c
^2*e^4*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3)*x^
2)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d
^2*e^2)
```

Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)x^3}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{x^3\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.113 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result . . . 874
Rubi [A] (verified) . . . 874
Mathematica [C] (verified) . . . 877
Maple [B] (verified) . . . 877
Fricas [B] (verification not implemented) . . . 879
Sympy [F(-1)] . . . 880
Maxima [F] . . . 880
Giac [F] . . . 880
Mupad [F(-1)] . . . 881

Optimal result

Integrand size = 19, antiderivative size = 205

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{bcx\arctan\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}} + \frac{bc\left(3c^2d-2e\right)x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d^2\left(c^2d-e\right)^{3/2}\sqrt{e}\sqrt{-c^2x^2}}$$

```
[Out] 1/4*(-a-b*arccsch(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*arctan((-c^2*x^2-1)^(1/2))/
d^2/e/(-c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d-2*e)*x*arctanh(e^(1/2)*(-c^2*x^2-1)
^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(3/2)/e^(1/2)/(-c^2*x^2)^(1/2)+1/8*b*
c*x*(-c^2*x^2-1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6435, 457, 105, 162, 65, 211, 214}

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{bcx\arctan\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}} + \frac{bcx\left(3c^2d-2e\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\sqrt{e}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}} + \frac{bcx\sqrt{-c^2x^2-1}}{8d\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]

[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(8*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*(d + e*x^2)) -
 (a + b*ArcCsch[c*x])/(4*e*(d + e*x^2)^2) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2
]])/(4*d^2*e*Sqrt[-(c^2*x^2)]) + (b*c*(3*c^2*d - 2*e)*x*ArcTanh[(Sqrt[e]*Sq
rt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d^2*(c^2*d - e)^(3/2)*Sqrt[e]*Sqrt[-
(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\int\frac{1}{x\sqrt{-1-c^2x^2}(d+ex^2)^2}\,dx}{4e\sqrt{-c^2x^2}} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{-1-c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8e\sqrt{-c^2x^2}} \\
&= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\operatorname{Subst}\left(\int\frac{c^2d-e-\frac{1}{2}c^2ex}{x\sqrt{-1-c^2x}(d+ex)}\,dx,x,x^2\right)}{8d\left(c^2d-e\right)e\sqrt{-c^2x^2}} \\
&= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} \\
&\quad+\frac{(bcx)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{8d^2e\sqrt{-c^2x^2}} \\
&\quad-\frac{\left(bc\left(\frac{1}{2}c^2de+\left(c^2d-e\right)e\right)x\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1-c^2x}(d+ex)}\,dx,x,x^2\right)}{8d^2\left(c^2d-e\right)e\sqrt{-c^2x^2}} \\
&= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} \\
&\quad-\frac{(bx)\operatorname{Subst}\left(\int\frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{-1-c^2x^2}\right)}{4cd^2e\sqrt{-c^2x^2}} \\
&\quad+\frac{\left(b\left(\frac{1}{2}c^2de+\left(c^2d-e\right)e\right)x\right)\operatorname{Subst}\left(\int\frac{1}{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx,x,\sqrt{-1-c^2x^2}\right)}{4cd^2\left(c^2d-e\right)e\sqrt{-c^2x^2}} \\
&= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} \\
&\quad+\frac{bcx\arctan\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bc(3c^2d-2e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d^2\left(c^2d-e\right)^{3/2}\sqrt{e}\sqrt{-c^2x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.80

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

$$= \frac{1}{16}\left(-\frac{4a}{e\left(d+ex^2\right)^2}+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}x}{d\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{4b\operatorname{csch}^{-1}(cx)}{e\left(d+ex^2\right)^2}+\frac{4b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d^2e}\right.$$

$$+\frac{b(3c^2d-2e)\log\left(\frac{16d^2\sqrt{e}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b(-3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}\left(-c^2d+e\right)^{3/2}}$$

$$\left.+\frac{b(3c^2d-2e)\log\left(-\frac{16id^2\sqrt{e}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d-2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2\sqrt{e}\left(-c^2d+e\right)^{3/2}}\right)$$

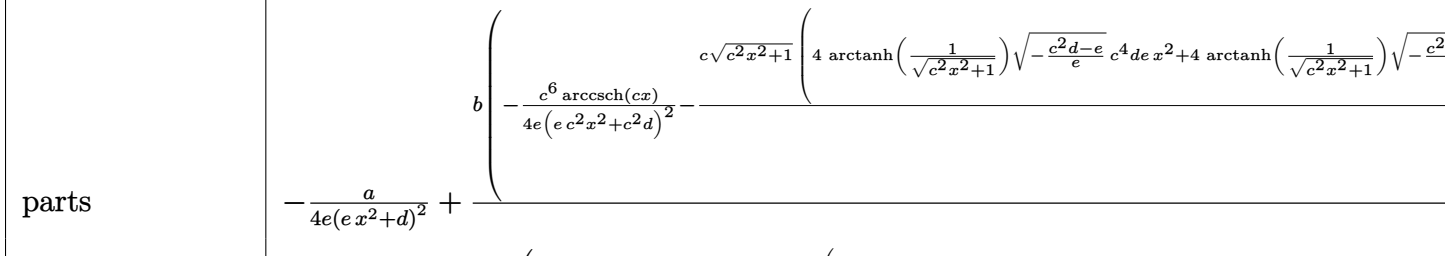
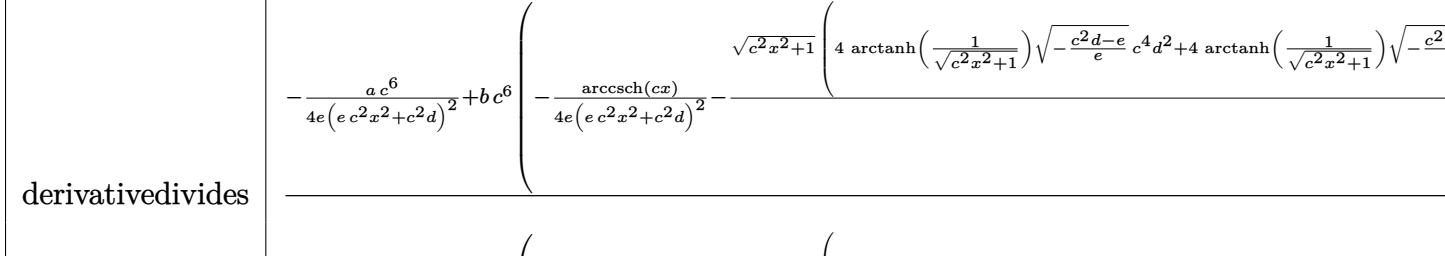
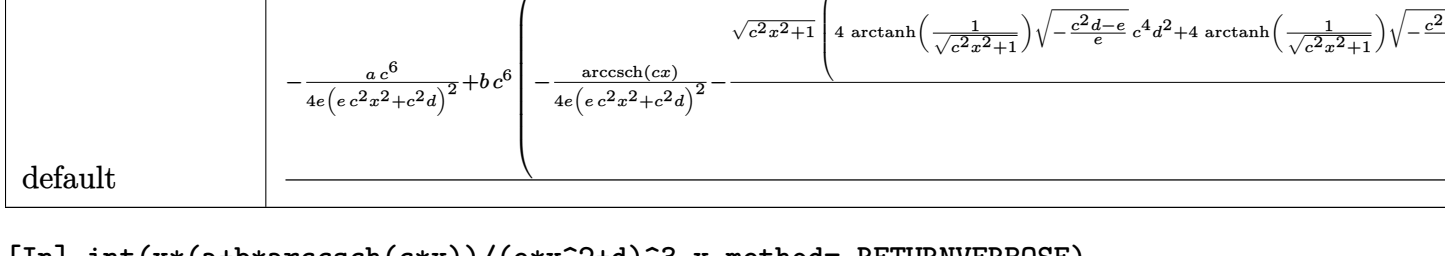
```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d - e)*
(d + e*x^2)) - (4*b*ArcCsch[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSinh[1/(c*x)]
)/(d^2*e) + (b*(3*c^2*d - 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(Sqrt
[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*
(-3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(-(c^2*d) + e)^(3/
2)) + (b*(3*c^2*d - 2*e)*Log[((-16*I)*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(Sqrt[
e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(3*c
^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*Sqrt[e]*(-(c^2*d) + e)^(3/2)))/
16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(180) = 360$.

Time = 6.26 (sec) , antiderivative size = 916, normalized size of antiderivative = 4.47

method	result
parts	
derivativedivides	
default	

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arccsch(c*x)-1/1
6*c*(c^2*x^2+1)^(1/2)*(4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*
c^4*d*e*x^2+4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*c^4*d^2-3*l
n(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*
e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e
)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*ln(-
2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x
+(-c^2*d*e)^(1/2)))*x^2*c^4*d*e-3*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^
(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+2*(c^2*x^
2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*c^2*d*e-4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c
^2*d-e)/e)^(1/2)*e^2*c^2*x^2-4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^
(1/2)*c^2*d*e+2*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^
(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2+2*ln(-2*((c^2*x^2+1)^(1
/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)
))*c^2*d*e+2*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1
/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^2*e^2+2*ln(-2*(-(c^2*x^2+1)^(1/2
)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*
c^2*d*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(
c*e*x+(-c^2*d*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(177) = 354$.

Time = 0.41 (sec) , antiderivative size = 1256, normalized size of antiderivative = 6.13

$$\int \frac{x\left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2
*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt
(-c^2*d*e + e^2)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^3*e
+ b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e
- 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x + 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c
^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3
*e + b*d^2*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*((b*
c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c
^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2),
 -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2*
d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(
c^2*d*e - e^2)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
)/(c^2*d - e)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2
- 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*
x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^4*d^4 - 2*b*
c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*
c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - c*x - 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - ((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 +
(b*c^3*d^3*e - b*c*d^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^6*e -
2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(
c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] -1/8*(8*c^2*integrate(1/4*x/(c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*e
+ (c^2*d^2*e + 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*
e + (c^2*d^2*e + 2*d*e^2)*x^2)*sqrt(c^2*x^2 + 1)), x) + (2*c^2*d - e)*log(e
*x^2 + d)/(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2) - (2*c^4*d^4*log(c) + 2*d^2*e^2
*log(c) - d^2*e^2 - (4*d^3*e*log(c) - d^3*e)*c^2 + (c^2*d^2*e^2 - d*e^3)*x^
2 + (c^4*d^2*e^2*x^4 + 2*c^4*d^3*e*x^2 + c^4*d^4)*log(c^2*x^2 + 1) - 2*((c^
4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*
x^2)*log(x) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2)*log(sqrt(c^2*x^2 + 1) + 1
))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^
2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2))*b - 1/4*a/(e^3
*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{x\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.114 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

Optimal result 883
Rubi [A] (verified) 884
Mathematica [C] (warning: unable to verify) 891
Maple [F] 893
Fricas [F] 893
Sympy [F(-1)] 893
Maxima [F] 893
Giac [F] 894
Mupad [F(-1)] 894

Optimal result

Integrand size = 21, antiderivative size = 657

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx &= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2}\\ &-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3}\\ &-\frac{b(c^2d-2e)\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d-e\right)^{3/2}}\\ &+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}\\ &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}\\ &-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}\end{aligned}$$

```
[Out] 1/4*e^2*(a+b*arccsch(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsch(c*x))/d^3/(e+d/x^
2)+1/2*(a+b*arccsch(c*x))^2/b/d^3-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc
h(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(
1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylo
g(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d
^3-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*
d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2
))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/8*b*(c^2*d-2*e)*arctan((c^2
```

```
*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d-e)^(3/2)+b*
arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d-
e)^(1/2)-1/8*b*c*e*(1+1/c^2/x^2)^(1/2)/d^2/(c^2*d-e)/(e+d/x^2)/x
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6439, 5823, 5821, 390, 385, 211, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = &-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}\\
&-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d^3}\\
&-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^3}\\
&-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d^3}\\
&+\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(\frac{d}{x^2}+e\right)}\\
&+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{d^3\sqrt{c^2d-e}}\\
&-\frac{b\sqrt{e}(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{8d^3\left(c^2d-e\right)^{3/2}}\\
&-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}\\
&-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^3}\\
&-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^3}-\frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{8d^2x\left(c^2d-e\right)\left(\frac{d}{x^2}+e\right)}
\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -1/8*(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(d^2*(c^2*d - e)*(e + d/x^2)*x) + (e^2*(
a + b*ArcCsch[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsch[c*x]))/(d^3*
(e + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2*b*d^3) - (b*(c^2*d - 2*e)*Sqrt[e]*
```

```
ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d
- e)^(3/2)) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*
x^2)]*x)])/(d^3*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d
]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsc
h[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]
)/(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt
[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - (b*PolyLog[
2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^3)
- (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]
)/(2*d^3) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c
^2*d) + e]))])/(2*d^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*d^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^5\left(a + b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e + dx^2\right)^3}\, dx, x, \frac{1}{x}\right)$$

$$
\begin{aligned}
&= -\text{Subst}\left(\int \left(\frac{e^2 x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2ex\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} \right.\right. \\
&\qquad \left.\left. + \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{e+dx^2} dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2} dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{x\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3} dx, x, \frac{1}{x}\right)}{d^2} \\
&= \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} \\
&\quad - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad + \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)} dx, x, \frac{1}{x}\right)}{cd^3} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)^2} dx, x, \frac{1}{x}\right)}{4cd^3} \\
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} \\
&\quad - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{(be)\text{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2} dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{cd^3} \\
&\quad - \frac{\left(b\left(c^2d-2e\right)e\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}\left(e+dx^2\right)} dx, x, \frac{1}{x}\right)}{8cd^3\left(c^2d-e\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} \\
&\quad + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{5/2}} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{5/2}} \\
&\quad - \frac{\left(b\left(c^2d-2e\right)e\right)\operatorname{Subst}\left(\int \frac{1}{e-\left(-d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8cd^3\left(c^2d-e\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3} - \frac{b\left(c^2d-2e\right)\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d-e\right)^{3/2}} \\
&\quad + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}} - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{5/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{5/2}} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{5/2}} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2(-d)^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3} - \frac{b(c^2d-2e)\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d-e\right)^{3/2}} \\
&\quad + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3} - \frac{b(c^2d-2e)\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d-e\right)^{3/2}} \\
&\quad + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^3} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} \\
&\quad + \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3} - \frac{b(c^2d-2e)\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d-e\right)^{3/2}} \\
&\quad + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}} - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3} \\
&\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 2081, normalized size of antiderivative = 3.17

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d +
 e*x^2])/(2*d^3) + b*((Sqrt[e]*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt
[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*S
qrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Lo
g[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]*
Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(c^
2*d - e)^(3/2))))/(16*d^2) + (Sqrt[e]*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)
]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*
(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e
)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2
```

```
*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))]
)/(d*(c^2*d - e)^(3/2))))/(16*d^2) - (((5*I)/16)*Sqrt[e]*(-(ArcCsch[c*x]/(I
*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sq
rt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)
])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/S
qrt[d]))/d^(5/2) + (((5*I)/16)*Sqrt[e]*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e
] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e]
 + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c
^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/d^(5/2)
 - (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sq
rt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*A
rcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch
[c*x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]
/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x
])] + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sq
rt[d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])])/(16*d^3) - (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2
- 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqr
t[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x
]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1
 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] +
 Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 -
Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyL
og[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(16*d^3) + (ArcCsch[c*x]^2/2 - ArcCsc
h[c*x]*Log[1 - E^(2*ArcCsch[c*x])] - PolyLog[2, E^(2*ArcCsch[c*x])]/2)/d^3)
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x\left(e\,x^2 + d\right)^3}\,dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^
3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx) + a}{(ex^2 + d)^3 x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{a + b\operatorname{asinh}\left(\frac{1}{cx}\right)}{x\left(e\,x^2 + d\right)^3}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.115 $\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result 896
Rubi [A] (verified) 897
Mathematica [C] (warning: unable to verify) 906
Maple [F] 908
Fricas [F] 908
Sympy [F(-1)] 908
Maxima [F(-2)] 908
Giac [F] 909
Mupad [F(-1)] 909

Optimal result

Integrand size = 21, antiderivative size = 1106

$$\begin{aligned}
\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx &= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&\quad -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} \\
&\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad +\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} \\
&\quad -\frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad +\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} \\
&\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad -\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad +\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad -\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad +\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}$$

```
[Out] 3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccsch(c*x))*ln(1+c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-
d)^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccsch(c*x))*
ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+
(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1
/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c
^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+1/16*b*arc
tanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(
1/2))/(c^2*d-e)^(3/2)/e/d^(1/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)
/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e/d^(1/2)-3
/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c
^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d-e)^(1/2)-3/16*b*arctanh((c^2*d+(-d)^(1/2)
*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2
*d-e)^(1/2)+1/16*(a+b*arccsch(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(
1/2))^2+3/16*(a+b*arccsch(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arc
csch(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arccsch(
c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1+1/c^2/x^2)^(1/2)/
(c^2*d-e)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1+1/c^2/x^
2)^(1/2)/(c^2*d-e)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules

used = {6439, 5793, 5828, 745, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$-\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16\left(c^2d-e\right)e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$-\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$
$$-\frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e}$$
$$-\frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2}$$
$$+\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e}$$
$$-\frac{3b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2}$$
$$+\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$-\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{16\sqrt{-d}e^{5/2}}$$
$$+\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$-\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{16\sqrt{-d}e^{5/2}}$$
$$-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/((c^2*d - e)*e^(3/2)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(3/
2)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*
(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sq
rt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[
e] + d/x)^2) - (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) -
 (3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqr
t[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2*d - e]*e^2) + (b*ArcTanh[(c^2*d
 - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])
/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c
^2*d - e]*e^2) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) + (3*(
a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2
)) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[
1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-
d]*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt
[-(c^2*d) + e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^Ar
cCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*P
olyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(
16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
```

```
0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\right.\right. \\
&\qquad \left.\left. - \frac{d^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\right.\right. \\
&\qquad \left.\left. - \frac{3d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8e^2\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= \frac{(3d)\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} \\
&\quad + \frac{(3d)\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e^2} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}} \\
&\quad - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^2} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^2} \\
&\quad + \frac{(3d)\operatorname{Subst}\left(\int \left(-\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}-\sqrt{-d}x)} - \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)\,dx, x, \frac{1}{x}\right)}{8e^2} \\
&\quad - \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^{3/2}} \\
&\quad + \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^{3/2}} \\
&= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16e^{5/2}} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16e^{5/2}} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16ce^2} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16ce^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16c\left(c^2d-e\right)e} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16c\left(c^2d-e\right)e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} - \frac{3\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16e^{5/2}} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16e^{5/2}} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16c\left(c^2d-e\right)e} + \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16c\left(c^2d-e\right)e} \\
&= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} - \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16e^{5/2}} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16e^{5/2}} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16e^{5/2}} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)dx, x, \operatorname{csch}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{csch}^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} \\
&\quad + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\left(c^2d-e\right)^{3/2}e} + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad + \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 2045, normalized size of antiderivative = 1.85

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*((I*c*Sqrt[e]
*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x))
```

```
- ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d
*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*
c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(S
qrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2)))/e^2 - ((I/16)*Sqrt[d]*(((-
I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqr
t[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*
x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*
Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2
*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2)))/e^2 + (5*(-(ArcC
sch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2
*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 +
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*
d) + e]))/Sqrt[d]))/(16*e^2) + (5*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e
*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*
(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*e^2) + (
((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-
2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*Ar
cCsch[c*x])] + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])]))/(Sqrt[d]*e^(5/2)) - (((3*I)/128)*(Pi^2 - (4*I)*Pi*
ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*L
og[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*
ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*
Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log
[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*Arc
Csch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt
[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (
(-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyL
og[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(Sqr
```

```
t[d]*e^(5/2)))
```

Maple [F]

$$\int \frac{x^4(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^3}dx$$

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^4}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^4}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{x^4\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.116 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result 911
Rubi [A] (verified) 912
Mathematica [C] (warning: unable to verify) 921
Maple [F] 922
Fricas [F] 922
Sympy [F(-1)] 922
Maxima [F(-2)] 923
Giac [F] 923
Mupad [F(-1)] 923

Optimal result

Integrand size = 21, antiderivative size = 1106

$$\begin{aligned}
\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx &= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} \\
&- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}
\end{aligned}$$

```
[Out] -1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1
/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(3/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)*e
^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^
(3/2)-1/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsch(c*x))*ln(
1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)
^(3/2)/e^(3/2)-1/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsch(
c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/
2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(1+1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16
*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/
x^2)^(1/2))/d^(3/2)/e/(c^2*d-e)^(1/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1
/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d-e)^(
1/2)+1/16*(a+b*arccsch(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2
+1/16*(a+b*arccsch(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arccsch(c
*x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arccsch(c*x))
/d/e/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1+1/c^2/x^2)^(1/2)/(c^2*d-e)/(-d)^(
1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1+1/c^2/x^2)^(1/2)/(c^2*d-
e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules

used = {6439, 5823, 5793, 5828, 745, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$-\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$
$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e}$$
$$-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}}$$
$$-\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e}$$
$$-\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}}$$
$$-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$
$$+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{16(-d)^{3/2}e^{3/2}}$$
$$-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$
$$+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{16(-d)^{3/2}e^{3/2}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$
$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[1 + 1/(c^2*x^2)])/(Sqrt[-d]*(c^2*d - e)*Sqrt[e]*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqrt[
e]*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(S
qrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e]
 - d/x)) - (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x
)^2) - (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[
(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x
^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt
[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*Sqr
t[c^2*d - e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt
[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*Ar
cTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/
(c^2*x^2)])])/(16*d^(3/2)*Sqrt[c^2*d - e]*e) - ((a + b*ArcCsch[c*x])*Log[1
 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/
2)*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sq
rt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsch[c*x
])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16
*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog
[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(16*(-d
)^(3/2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqr
t[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^
ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (
b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(
16*(-d)^(3/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d}$$

$$+ \frac{e\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d\left(a+b\text{arcsinh}\right.\right.}{16e^2\left(\sqrt{-d}\sqrt{e}+\right.}}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} - \frac{3\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}$$

$$+ \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}$$

$$- \frac{3\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e} + \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e}$$

$$- \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{e}} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{e}}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\text{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$- \frac{a+b\text{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3\text{Subst}\left(\int \left(-\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{8e}$$
$$+ \frac{\text{Subst}\left(\int \left(-\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{2e}$$
$$+ \frac{(3b)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{16cde}$$
$$- \frac{(3b)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{16cde}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{4cde} + \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{4cde}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2\sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd\left(c^2d-e\right)} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd\left(c^2d-e\right)} + \frac{3\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16de^{3/2}} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16de^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4de^{3/2}} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16cde} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16cde} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cde} - \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4cde}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\,(c^2d-e)\sqrt{e}\,\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\,(c^2d-e)\sqrt{e}\,\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\,\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\,\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16cd\,(c^2d-e)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16cd\,(c^2d-e)} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16de^{3/2}} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4de^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}} \\
&\quad - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} + \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16de^{3/2}} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16de^{3/2}} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16de^{3/2}} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{16de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4de^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{4de^{3/2}} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 2053, normalized size of antiderivative = 1.86

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-1/16*I)*((I*c*Sqrt[e]*Sqrt[1 +
1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[
c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e])
+ (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[
d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I
*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*c*Sqrt[
e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) -
 ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] +
c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(S
qrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) - (-(ArcCsch[c
*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt
[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^
2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) +
e]))/Sqrt[d])/(16*d*e) - (-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I
*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt
[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sq
rt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d])/(16*d*e) + ((I/128)*(P
i^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsc
h[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x]
)] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt
[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])
] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]
]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
(4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] +
 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])]))/(d^(3/2)*e^(3/2)) - ((I/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*A
rcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c
```

```
*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] -
 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1
 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 -
 (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*Ar
cSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d]
)/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(d^(3/2)*e^(3/2)))
```

Maple [F]

$$\int \frac{x^2(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^3}dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^2}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)x^2}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{x^2\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.117 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx$

Optimal result 925
Rubi [A] (verified) 926
Mathematica [C] (warning: unable to verify) 934
Maple [F] 935
Fricas [F] 935
Sympy [F(-1)] 935
Maxima [F(-2)] 936
Giac [F] 936
Mupad [F(-1)] 936

Optimal result

Integrand size = 18, antiderivative size = 1096

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx &= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&\quad -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad +\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&\quad -\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}+\frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad +\frac{5b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}}+\frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d-e\right)^{3/2}} \\
&\quad +\frac{5b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}}+\frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d-e\right)^{3/2}} \\
&\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&\quad -\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&\quad -\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}
\end{aligned}$$

```
[Out] 1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+
1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(3/2)+1/16*b*e*arctanh((c^2*d+(-d)^(1/2
)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-
e)^(3/2)+5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1
/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)+5/16*b*arctanh((c^2*d+(-d)
```

```
^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c
^2*d-e)^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccsch(c
*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2
)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*
arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*
d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polyl
og(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(
-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+
(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1
/2)+1/16*(a+b*arccsch(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2-
5/16*(a+b*arccsch(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccsch(c*x
))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arccsch(c*x))/d^
2/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1+1/c^2/x^2)^(1/2)/(-d)^(3/2)/
(c^2*d-e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1+1/c^2/x^2)^(1/2)/(-
d)^(3/2)/(c^2*d-e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {6429, 5823, 5793, 5828, 745, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx &= -\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}\\ &\quad -\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(c^2d-e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}\\ &\quad -\frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}\\ &\quad +\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}\\ &\quad +\frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d-e\right)^{3/2}}+\frac{5b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}}\\ &\quad +\frac{be\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d-e\right)^{3/2}}+\frac{5b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}}\\ &\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}\\ &\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{16(-d)^{5/2}\sqrt{e}}\\ &\quad +\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}\\ &\quad -\frac{3\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{16(-d)^{5/2}\sqrt{e}}\\ &\quad -\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}\\ &\quad -\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/((-d)^(3/2)*(c^2*d - e)*(Sqrt[-d]
*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*
d - e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^
(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[
-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^(3/2)*(Sqrt[-
```

```
d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] +
 d/x)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
 - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d - e]) + (b*e*ArcTanh[(
c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^
2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqr
t[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sq
rt[c^2*d - e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqr
t[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (3*(
a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sq
rt[e]) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[
e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x]
)*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*
(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] - Sqrt[-(c^2*d) + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[
e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
```

```
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6429

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d^2} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad + \frac{(2e)\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2} \\
&\quad - \frac{e^2\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\text{arcsinh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d\left(a+b\text{arcsi}\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}\right.}{d^2}
\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d}$$
$$+ \frac{3\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8d} - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{2d}$$
$$- \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{d}$$
$$- \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}}$$
$$- \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{-d}} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arcsinh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{-d}}$$

$$
\begin{aligned}
&= \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad + \frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16cd^2} \\
&\quad + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16cd^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2cd^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2cd^2} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \left(-\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)}{8d} \\
&\quad - \frac{\operatorname{Subst}\left(\int \left(-\frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)}{d} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sinh(x)}\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&\quad - \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16c(-d)^{3/2}} \\
&\quad + \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16c(-d)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&\quad + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&\quad + \frac{5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16cd^2} \\
&\quad - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16cd^2} \\
&\quad - \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2cd^2} + \frac{b\operatorname{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{d-\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2cd^2} \\
&\quad - \frac{3\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16d^2\sqrt{e}} - \frac{3\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16d^2\sqrt{e}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} + \frac{\operatorname{Subst}\left(\int \frac{a+b\operatorname{arcsinh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} \\
&\quad + \frac{(be)\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2\left(c^2d-e\right)} \\
&\quad - \frac{(be)\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2\left(c^2d-e\right)}
\end{aligned}
$$

$$= \text{Too large to display}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 2038, normalized size of antiderivative = 1.86

$$\int \frac{a + b\text{csch}^{-1}(cx)}{\left(d + ex^2\right)^3}\, dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*((I*c*Sqrt[e]*Sqrt[1 + 1/
(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*
x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) +
(I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d]
 - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*S
qrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*Sqrt[e]*Sqr
t[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCs
ch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e])
+ (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*S
qrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d]
 - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - (3*(-(ArcCsch[c*x]/(I*
Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqr
t[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)]
)*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sq
rt[d]))/(16*d^2) - (3*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(A
rcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[
d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*d^2) + (((3*I)/128)*
(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcC
sch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*
x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
- (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])]
 + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])]))/(d^(5/2)*Sqrt[e]) - (((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x]
 - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcT
```

```
an[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) +
 e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]
*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
(16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] +
 Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16
*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*S
qrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(d^(5/2)*Sqrt[e]
))
```

Maple [F]

$$\int \frac{a + b\ \mathrm{arccsch}\,(cx)}{(e\,x^2 + d)^3}\,dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Fricas [F]

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{(d + ex^2)^3}\,dx = \int \frac{b\,\mathrm{arcsch}\,(cx) + a}{(ex^2 + d)^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\mathrm{csch}^{-1}(cx)}{(d + ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex^2+d)^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\operatorname{asinh}\left(\frac{1}{cx}\right)}{(e\,x^2+d)^3}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^3, x)
```

3.118 $\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 937
Rubi [A] (verified) . . . 938
Mathematica [C] (verified) . . . 943
Maple [F] . . . 944
Fricas [A] (verification not implemented) . . . 944
Sympy [F] . . . 945
Maxima [F(-2)] . . . 945
Giac [F] . . . 946
Mupad [F(-1)] . . . 946

Optimal result

Integrand size = 23, antiderivative size = 413

$$
\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} \\
&\quad +\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce^2\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad -\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad +\frac{b(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{-c^2x^2}} \\
&\quad +\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{105e^3\sqrt{-c^2x^2}}
\end{aligned}
$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*a
rccsch(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsch(c*x))/e^3+1/1680*b*(105*c
^6*d^3+35*c^4*d^2*e+63*c^2*d*e^2-75*e^3)*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2
)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(-c^2*x^2)^(1/2)+8/105*b*c*d^(7/2)*x*arcta
n((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x^2)^(1/2)-1/840*b*
(29*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c^3/e^2/(-c^2*x^2)^(1/
2)+1/42*b*x*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/c/e^2/(-c^2*x^2)^(1/2)-1/168
0*b*(23*c^4*d^2-12*c^2*d*e-75*e^2)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5
/e^2/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 1629, 159, 163, 65, 223, 209, 95, 210}

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3} + \frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{105e^3\sqrt{-c^2x^2}} \\
&\quad + \frac{bx(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} - \frac{bx\sqrt{-c^2x^2-1}(29c^2d+25e)(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bx\sqrt{-c^2x^2-1}(23c^4d^2-12c^2de-75e^2)\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}
\end{aligned}$$

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/1680*(b*(23*c^4*d^2 - 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d +
 e*x^2])/(c^5*e^2*Sqrt[-(c^2*x^2)]) - (b*(29*c^2*d + 25*e)*x*Sqrt[-1 - c^2*
x^2]*(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2
*x^2]*(d + e*x^2)^(5/2))/(42*c*e^2*Sqrt[-(c^2*x^2)]) + (d^2*(d + e*x^2)^(3/
2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsch[c*
x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^3) + (b*(105*c
^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*x*ArcTan[(Sqrt[e]*Sqrt[-1 -
c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*
c*d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(105*e^3*
Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\ &\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{105e^3x\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\ &= \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\ &\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{x\sqrt{-1-c^2x^2}}\,dx}{105e^3\sqrt{-c^2x^2}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}(8d^2-12dex+15e^2x^2)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{210e^3\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}\left(-24c^2d^2e+\frac{3}{2}e^2(29c^2d+25e)x\right)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{630ce^4\sqrt{-c^2x^2}} \\
&= -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad - \frac{(bx)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}\left(48c^4d^3e-\frac{3}{4}e^2(23c^4d^2-12c^2de-75e^2)x\right)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{1260c^3e^4\sqrt{-c^2x^2}} \\
&= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{-48c^6d^4e-\frac{3}{8}e^2(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{1260c^5e^4\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} \\
&\quad +\frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad +\frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3}-\frac{(4bcd^4x)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{105e^3\sqrt{-c^2x^2}} \\
&\quad -\frac{(b(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\,x)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3360c^5e^2\sqrt{-c^2x^2}} \\
&= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} \\
&\quad +\frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad +\frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3}-\frac{(8bcd^4x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{105e^3\sqrt{-c^2x^2}} \\
&\quad +\frac{(b(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\,x)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1-c^2x^2}\right)}{1680c^7e^2\sqrt{-c^2x^2}} \\
&= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} \\
&\quad +\frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad +\frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3}+\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{105e^3\sqrt{-c^2x^2}} \\
&\quad +\frac{(b(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\,x)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{1680c^7e^2\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(23c^4d^2 - 12c^2de - 75e^2)\, x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{b(29c^2d + 25e)\, x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad + \frac{b(105c^6d^3 + 35c^4d^2e + 63c^2de^2 - 75e^3)\, x \arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{-c^2x^2}} \\
&\quad + \frac{8bcd^{7/2}x \arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{105e^3\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.20 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.78

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{32a(d+ex^2)^2\left(8d^2-12dex^2+15e^2x^4\right) + \frac{2be\sqrt{1+\frac{1}{c^2x^2}}x\left(d+ex^2\right)\left(75e^2-2c^2e\left(19d+25ex^2\right)+c^4\left(-41d^2+22dex^2+40e^2x^4\right)\right)}{c^5} + \frac{b}{}}{}$$

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]

[Out] (32*a*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4) + (2*b*e*Sqrt[1 + 1/(
c^2*x^2)]*x*(d + e*x^2)*(75*e^2 - 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^2
+ 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (b*(-128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*App
ellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] + (e*(105*c^6*d^3 + 35*
c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^
2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]
))/(c^5*x) + 32*b*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c
*x])/(3360*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^5(a+b\,\mathrm{arccsch}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.49 (sec) , antiderivative size = 1951, normalized size of antiderivative = 4.72

$$\int x^5\sqrt{d+ex^2}\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^
2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e +
 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x
^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e^3*x^6
 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log
((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 48
*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 +
 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 -
 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3)
, 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^
2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e +
 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*
x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c
^2*d*e + e^2)*x^2 + d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*
c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7
*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c
^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2
*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^7*sqrt(-d
)*d^3*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (10
5*b*c^6*d^3 + 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
```

```
 + e^2) + 64*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*
b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a
*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*
b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2
)))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*(128*b*c^7*sqrt(-d)*d^3*arctan(1/2*(
(c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c
^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (105*b*c^6*d^3 + 35*b
*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^
2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^
4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*
e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*
x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3)]
```

Sympy [F]

$$\int x^5 \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^5 (a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**5*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^5\,\sqrt{e\,x^2+d}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.119 $\int x^3\sqrt{d+ex^2}\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 947
Rubi [A] (verified) . . . 948
Mathematica [C] (verified) . . . 952
Maple [F] . . . 953
Fricas [A] (verification not implemented) . . . 953
Sympy [F] . . . 954
Maxima [F(-2)] . . . 954
Giac [F] . . . 954
Mupad [F(-1)] . . . 955

Optimal result

Integrand size = 23, antiderivative size = 302

$$\begin{aligned}\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = {}& \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} \\ &+ \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}} \\ &- \frac{d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\ &+ \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\ &- \frac{b(15c^4d^2+10c^2de-9e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{-c^2x^2}} \\ &- \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^2\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcc
sch(c*x))/e^2-1/120*b*(15*c^4*d^2+10*c^2*d*e-9*e^2)*x*arctan(e^(1/2)*(-c^2*
x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(-c^2*x^2)^(1/2)-2/15*b*c*d^(5/
2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/(-c^2*x^2)^(1/2
)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/e/(-c^2*x^2)^(1/2)+1/120*b*
(c^2*d-9*e)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 587, 159, 163, 65, 223, 209, 95, 210}

$$\begin{aligned}\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx &= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}\\ &+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}\\ &-\frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^2\sqrt{-c^2x^2}}\\ &-\frac{bx(15c^4d^2+10c^2de-9e^2)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{-c^2x^2}}\\ &+\frac{bx\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}\\ &+\frac{bx\sqrt{-c^2x^2-1}(c^2d-9e)\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}\end{aligned}$$

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(c^2*d - 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[-(c^
2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[-(c^2*x^
2)]) - (d*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + ((d + e*x^2)^(5
/2)*(a + b*ArcCsch[c*x]))/(5*e^2) - (b*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*x*
ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(3/2)*
Sqrt[-(c^2*x^2)]) - (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-
1 - c^2*x^2])])/(15*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{15e^2x\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{x\sqrt{-1-c^2x^2}}\,dx}{15e^2\sqrt{-c^2x^2}} \\
&= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}(-2d+3ex)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{30e^2\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{(bx)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}\left(4c^2d^2-\frac{1}{2}\left(c^2d-9e\right)ex\right)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{60ce^2\sqrt{-c^2x^2}} \\
&= \frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad - \frac{(bx)\operatorname{Subst}\left(\int \frac{-4c^4d^3-\frac{1}{4}e\left(15c^4d^2+10c^2de-9e^2\right)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{60c^3e^2\sqrt{-c^2x^2}} \\
&= \frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{\left(bcd^3x\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{\left(b\left(15c^4d^2+10c^2de-9e^2\right)x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{240c^3e\sqrt{-c^2x^2}} \\
&= \frac{b\left(c^2d-9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{\left(2bcd^3x\right)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{15e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(b\left(15c^4d^2+10c^2de-9e^2\right)x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{120c^5e\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} - \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(b(15c^4d^2+10c^2de-9e^2)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^5e\sqrt{-c^2x^2}} \\
&= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad - \frac{b(15c^4d^2+10c^2de-9e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{-c^2x^2}} \\
&\quad - \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^2\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.54 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.87

$$
\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx
$$

$$
= \frac{16a(d+ex^2)^2(-2d+3ex^2) + \frac{2be\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)(-9e+c^2(7d+6ex^2))}{c^3} - \frac{b\left(-16c^2d^3\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right.\right.}{240e^2\sqrt{d+}}
$$

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]

[Out] (16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) + (2*b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
e*x^2)*(-9*e + c^2*(7*d + 6*e*x^2)))/c^3 - (b*(-16*c^2*d^3*Sqrt[1 + d/(e*x^
2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] + (e*(15*c^4*d^2
 + 10*c^2*d*e - 9*e^2)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*Appell
F1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(c^3*x) +
 16*b*(d + e*x^2)^2*(-2*d + 3*e*x^2)*ArcCsch[c*x])/(240*e^2*Sqrt[d + e*x^2]
)
```

Maple [F]

$$\int x^3(a+b\,\mathrm{arccsch}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 1625, normalized size of antiderivative = 5.38

$$\int x^3\sqrt{d+ex^2}\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
+ d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e - 9*b*
e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2
)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c
^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
+ 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x^3 +
 (7*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 +
 d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4
 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*
sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*
c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e
*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e +
e^2)*x^2 + d*e)) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(
e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5
*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e
 - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2
), -1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d
^2 + d*e)*x^2 + d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log
(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4
*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) + e^2) - 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(24*a*c^5*e^
2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e -
9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2),
```

```
-1/240*(16*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt
(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2
+ d*e)*x^2 + d^2)) - (15*b*c^4*d^2 + 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arcta
n(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2
+ 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 16*(3*b*c^5*e^
2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a
*c^5*d^2 + (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2
+ 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^3(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^3\,\sqrt{e\,x^2+d}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.120 $\int x\sqrt{d+ex^2}\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 956
Rubi [A] (verified) . . . 956
Mathematica [C] (verified) . . . 960
Maple [F] . . . 960
Fricas [A] (verification not implemented) . . . 960
Sympy [F] . . . 961
Maxima [F] . . . 961
Giac [F] . . . 962
Mupad [F(-1)] . . . 962

Optimal result

Integrand size = 21, antiderivative size = 203

$$\begin{aligned}\int x\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx &= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} \\ &+ \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\ &+ \frac{b(3c^2d-e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}} \\ &+ \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e+1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/6*b*(3*c^2*d-e)*x*a
rctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(-c^2*x^2)^
(1/2)+1/6*b*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{6435, 457, 104, 163, 65, 223, 209, 95, 210\}$

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} + \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e\sqrt{-c^2x^2}} + \frac{bx(3c^2d-e)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}$$

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + ((d + e*x
^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (b*(3*c^2*d - e)*x*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[-(c^2*x^2)])
 + (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e
*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}}{x\sqrt{-1-c^2x^2}}\,dx}{3e\sqrt{-c^2x^2}}$$

$$= \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{6e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$
$$+ \frac{(bx)\operatorname{Subst}\left(\int \frac{-c^2d^2-\frac{1}{2}(3c^2d-e)ex}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$
$$- \frac{\left(b(3c^2d-e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12c\sqrt{-c^2x^2}}$$
$$- \frac{(bcd^2x)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$
$$+ \frac{\left(b(3c^2d-e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{6c^3\sqrt{-c^2x^2}}$$
$$- \frac{(bcd^2x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$
$$+ \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e\sqrt{-c^2x^2}} + \frac{\left(b(3c^2d-e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$
$$+ \frac{b(3c^2d-e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}} + \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{-2bd^2\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(3c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x^4\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF}}{12cex\sqrt{1+}}$$

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (-2*b*d^2*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -(
1/(c^2*x^2)), -(d/(e*x^2))] + b*(3*c^2*d - e)*e*Sqrt[1 + 1/(c^2*x^2)]*x^4*S
qrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] + 2*x
*Sqrt[1 + c^2*x^2]*(d + e*x^2)*(b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a*c*(d + e*
x^2) + 2*b*c*(d + e*x^2)*ArcCsch[c*x]))/(12*c*e*x*Sqrt[1 + c^2*x^2]*Sqrt[d
+ e*x^2])
```

Maple [F]

$$\int x(a+b\,\text{arccsch}\,(cx))\sqrt{e\,x^2+d}dx$$

```
[In] int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 1342, normalized size of antiderivative = 6.61

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e
^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 +
 (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)
```

```
/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*d^(3/2)*log(
((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)
*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d
^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)
*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (
c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log(
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 + b*c^2*e
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/2
4*(4*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 +
 d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x
^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*
d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(
e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 8*(b*c^3*e*x^2 +
b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*
sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e
)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^
2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan
(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 +
 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2
+ b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + 2*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d
)*sqrt(e*x^2 + d))/(c^3*e)]
```

Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/3*((e*x^2 + d)^(3/2)*log(sqrt(c^2*x^2 + 1) + 1)/e + 3*integrate(1/3*(c^2*
e*x^3 + c^2*d*x)*sqrt(e*x^2 + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^2
+ 1) + e), x) - 3*integrate(1/3*((3*e*log(c) + e)*c^2*x^3 + (c^2*d + 3*e*lo
g(c))*x + 3*(c^2*e*x^3 + e*x)*log(x))*sqrt(e*x^2 + d)/(c^2*e*x^2 + e), x))*
b + 1/3*(e*x^2 + d)^(3/2)*a/e
```

Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.121 $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal result . . . 963
Rubi [N/A] . . . 963
Mathematica [N/A] . . . 964
Maple [N/A] (verified) . . . 964
Fricas [N/A] . . . 964
Sympy [N/A] . . . 964
Maxima [F(-2)] . . . 965
Giac [N/A] . . . 965
Mupad [N/A] . . . 965

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \operatorname{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arccsch}\left(cx\right))\sqrt{e\,x^2+d}}{x}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\left(cx\right)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 9.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\sqrt{d+ex^2}}{x}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x,x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result . . . 966
Rubi [N/A] . . . 966
Mathematica [N/A] . . . 967
Maple [N/A] (verified) . . . 967
Fricas [N/A] . . . 967
Sympy [N/A] . . . 967
Maxima [F(-2)] . . . 968
Giac [N/A] . . . 968
Mupad [N/A] . . . 968

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \operatorname{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arccsch}\left(cx\right))\sqrt{e\,x^2+d}}{x^3}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\left(cx\right)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 14.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\sqrt{d+ex^2}}{x^3}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**3,x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 5.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{cx}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.123 $\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 969
Rubi [N/A] . . . 969
Mathematica [N/A] . . . 970
Maple [N/A] (verified) . . . 970
Fricas [N/A] . . . 970
Sympy [N/A] . . . 970
Maxima [F(-2)] . . . 971
Giac [N/A] . . . 971
Mupad [N/A] . . . 971

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\,\text{arccsch}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 25.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^2(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^2\,\sqrt{e\,x^2+d}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.124 $\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 972
Rubi [N/A] . . . 972
Mathematica [N/A] . . . 973
Maple [N/A] (verified) . . . 973
Fricas [N/A] . . . 973
Sympy [N/A] . . . 973
Maxima [F(-2)] . . . 974
Giac [N/A] . . . 974
Mupad [N/A] . . . 974

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Int}\left(\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\,\text{arccsch}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 6.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 5.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{e\,x^2+d}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result . . . 975
Rubi [N/A] . . . 975
Mathematica [N/A] . . . 976
Maple [N/A] (verified) . . . 976
Fricas [N/A] . . . 976
Sympy [N/A] . . . 976
Maxima [F(-2)] . . . 977
Giac [N/A] . . . 977
Mupad [N/A] . . . 977

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arccsch}\left(cx\right))\sqrt{e\,x^2+d}}{x^2}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\left(cx\right)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\sqrt{d+ex^2}}{x^2}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**2,x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result . . . 978
Rubi [A] (verified) . . . 979
Mathematica [C] (verified) . . . 982
Maple [F] . . . 982
Fricas [A] (verification not implemented) . . . 983
Sympy [F] . . . 983
Maxima [F(-2)] . . . 983
Giac [F] . . . 984
Mupad [F(-1)] . . . 984

Optimal result

Integrand size = 23, antiderivative size = 389

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx \\
&= -\frac{2bc^3(c^2d-2e)\,x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad + \frac{2bc^2(c^2d-2e)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{b(c^2d-3e)\,ex\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/d/x^3-2/9*b*c^3*(c^2*d-2*e)*x^2*(e*
x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-2/9*b*c*(c^2*d-2*e)*(-c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)+1/9*b*c*(-c^2*x^2-1)^(1/2
)*(e*x^2+d)^(1/2)/x^2/(-c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d-2*e)*x*(1/(c^2*x^2+
1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/
2))*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2
*x^2+1))^(1/2)-1/9*b*(c^2*d-3*e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2
)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-
c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {270, 6437, 12, 485, 597, 545, 429, 506, 422}

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx \\
&= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad - \frac{bex(c^2d-3e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\
&\quad + \frac{2bc^2x(c^2d-2e)\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\
&\quad - \frac{2bc\sqrt{-c^2x^2-1}(c^2d-2e)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{2bc^3x^2(c^2d-2e)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}
\end{aligned}
$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-2*b*c^3*(c^2*d - 2*e)*x^2*Sqrt[d + e*x^2])/(9*d*Sqrt[-(c^2*x^2)]*Sqrt[-1
- c^2*x^2]) - (2*b*c*(c^2*d - 2*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d
*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[-
(c^2*x^2)]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2*b*c^2
*(c^2*d - 2*e)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*
d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
- (b*(c^2*d - 3*e)*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)
])/(9*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*
x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{3/2}}{3dx^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} + \frac{(bcx)\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\sqrt{-1-c^2x^2}}\,dx}{3d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int \frac{2d(c^2d-2e)+(c^2d-3e)ex^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{-c^2x^2}} \\
&= -\frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int \frac{d(c^2d-3e)e+2c^2d(c^2d-2e)ex^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{-c^2x^2}} \\
&= -\frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bc(c^2d-3e)ex)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{-c^2x^2}} \\
&\quad - \frac{(2bc^3(c^2d-2e)ex)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{-c^2x^2}} \\
&= -\frac{2bc^3(c^2d-2e)x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad - \frac{b(c^2d-3e)ex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{(2bc^3(c^2d-2e)x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{9d\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2bc^3(c^2d-2e)\,x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad + \frac{2bc^2(c^2d-2e)\,x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{b(c^2d-3e)\,ex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.03 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.61

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \\
&\quad -\frac{\sqrt{d+ex^2}\left(bc\sqrt{1+\frac{1}{c^2x^2}}x(-d+2c^2dx^2-4ex^2)+3a(d+ex^2)+3b(d+ex^2)\operatorname{csch}^{-1}(cx)\right)}{9dx^3} \\
&\quad -\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d-2e)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)\,|\,\frac{e}{c^2d}\right)+(-2c^4d^2+5c^2de-3e^2)\,\text{EllipticF}\left(i\right.\right.}{9\sqrt{c^2}d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d + 2*c^2*d*x^2 - 4*e*
x^2) + 3*a*(d + e*x^2) + 3*b*(d + e*x^2)*ArcCsch[c*x]))/(d*x^3) - ((I/9)*b*
c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 2*e)*Ellipt
icE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-2*c^4*d^2 + 5*c^2*d*e - 3*e^2)*E
llipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sqrt[1 + c^2*x^2
]*Sqrt[d + e*x^2])
```

Maple [F]

$$
\int \frac{(a+b\ \text{arccsch}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx
$$

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

$$= \frac{2\left(bc^6d^2-2\,bc^4de\right)\sqrt{-c^2}\sqrt{d}x^3E\left(\arcsin\left(\sqrt{-c^2}x\right)\mid\frac{e}{c^2d}\right)-\left(2\,bc^6d^2-\left(4\,bc^4-bc^2\right)de-3\,be^2\right)\sqrt{-c^2}\sqrt{d}x^3F($$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(2*(b*c^6*d^2 - 2*b*c^4*d*e)*sqrt(-c^2)*sqrt(d)*x^3*elliptic_e(arcsin(s
qrt(-c^2)*x), e/(c^2*d)) - (2*b*c^6*d^2 - (4*b*c^4 - b*c^2)*d*e - 3*b*e^2)*
sqrt(-c^2)*sqrt(d)*x^3*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - 3*(b*c
^2*d*e*x^2 + b*c^2*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 1)/(c*x)) - (3*a*c^2*d*e*x^2 + 3*a*c^2*d^2 - (b*c^3*d^2*x - 2*(b*c^5*
d^2 - 2*b*c^3*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^
2*d^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(a+b\operatorname{acsch}(cx)\right)\sqrt{d+ex^2}}{x^4}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result . . . 985
Rubi [A] (verified) . . . 986
Mathematica [C] (verified) . . . 991
Maple [F] . . . 991
Fricas [A] (verification not implemented) . . . 991
Sympy [F] . . . 992
Maxima [F(-2)] . . . 992
Giac [F] . . . 992
Mupad [F(-1)] . . . 993

Optimal result

Integrand size = 23, antiderivative size = 527

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx \\
&= \frac{bc^3(24c^4d^2-19c^2de-31e^2)\,x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad + \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad - \frac{bc^2(24c^4d^2-19c^2de-31e^2)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{2be(6c^4d^2-4c^2de-15e^2)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rccsch(c*x))/d^2/x^3-1/45*b*c^3*(2*c^2*d-e)*e*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2
*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-2/15*b*c^3*e^2*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2
*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+1/75*b*c^3*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x^2*(
e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/45*b*c*(2*c^2*d-e)
*e*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)-2/15*b*c*e^2*(-c
^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/75*b*c*(8*c^4*d^2-3*
c^2*d*e-2*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/25
```

```
*b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4/(-c^2*x^2)^(1/2)-1/75*b*c*(4*c^
2*d-e)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/45*b*c*e
*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/45*b*c^2*(2*c^
2*d-e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1
)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1
)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+2/15*b*c^2*e^2*x*(1/(c^2*x^2+1))^(1
/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e
*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2
+1))^(1/2)-1/75*b*c^2*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(
c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+
d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^
(1/2)+1/75*b*c^2*(4*c^2*d-e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*El
lipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*
x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)-1/45*b*c^2*e^
2*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2)
,(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)-2/15*b*e^3*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^
2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/
2)/d^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 270, 6437, 12, 594, 597, 545, 429, 506, 422}

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx \\
&= \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad - \frac{bc^2x(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{225d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\
&\quad + \frac{2bex(6c^4d^2-4c^2de-15e^2)\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\
&\quad - \frac{bc\sqrt{-c^2x^2-1}(12c^2d+e)\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-c^2x^2-1}(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc^3x^2(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}
\end{aligned}
$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c^3*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*x^2*Sqrt[d + e*x^2])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)
*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*Sqrt[-(c^2*x^2)]) - (b*c*(12*
c^2*d + e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d*x^2*Sqrt[-(c^2*x^2)])
 + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*d*x^4*Sqrt[-(c^2*x^2)]) -
 ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) + (2*e*(d + e*x^2)^(3/2
)*(a + b*ArcCsch[c*x]))/(15*d^2*x^3) - (b*c^2*(24*c^4*d^2 - 19*c^2*d*e - 31
*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (2*
b*e*(6*c^4*d^2 - 4*c^2*d*e - 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x
], 1 - e/(c^2*d)])/(225*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e
*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{15d^2x^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{x^6\sqrt{-1-c^2x^2}}\,dx}{15d^2\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(-d\left(12c^2d+e\right)-e\left(3c^2d+10e\right)x^2\right)}{x^4\sqrt{-1-c^2x^2}}\,dx}{75d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad - \frac{(bcx)\int \frac{-d\left(24c^4d^2-19c^2de-31e^2\right)-2e\left(6c^4d^2-4c^2de-15e^2\right)x^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad - \frac{(bcx)\int \frac{-2de\left(6c^4d^2-4c^2de-15e^2\right)-c^2de\left(24c^4d^2-19c^2de-31e^2\right)x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^3\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad + \frac{(bc^3e(24c^4d^2-19c^2de-31e^2)\,x)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{(2bce(6c^4d^2-4c^2de-15e^2)\,x)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc^3(24c^4d^2-19c^2de-31e^2)\,x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad + \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad + \frac{2be(6c^4d^2-4c^2de-15e^2)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{(bc^3(24c^4d^2-19c^2de-31e^2)\,x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc^3(24c^4d^2-19c^2de-31e^2)\,x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad + \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad - \frac{bc^2(24c^4d^2-19c^2de-31e^2)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{2be(6c^4d^2-4c^2de-15e^2)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.99 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$$

$$= \frac{\sqrt{d+ex^2}\left(-15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}x(-31e^2x^4+dex^2(8-19c^2x^2)+3d^2(3-4c^2x^2+8\right.}{225d^2x^5}$$

$$+\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2-19c^2de-31e^2)\,E\left(i\text{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+(-24c^6d^3+31c^4d^2e\right.}{225\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2
*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 - 19*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8
*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsch[c*x]))/(225*d^2*x^5
) + ((I/225)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^4
*d^2 - 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] +
(-24*c^6*d^3 + 31*c^4*d^2*e + 23*c^2*d*e^2 - 30*e^3)*EllipticF[I*ArcSinh[Sq
rt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\text{arccsch}\,(cx))\sqrt{e\,x^2+d}}{x^6}dx$$

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx =$$

$$-\frac{(24\,bc^8d^3-19\,bc^6d^2e-31\,bc^4de^2)\sqrt{-c^2}\sqrt{d}x^5E(\arcsin\left(\sqrt{-c^2}x\right)\,|\,\frac{e}{c^2d})-(24\,bc^8d^3-(19\,bc^6-12\,bc^4)d^2}{}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*((24*b*c^8*d^3 - 19*b*c^6*d^2*e - 31*b*c^4*d*e^2)*sqrt(-c^2)*sqrt(d)
*x^5*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - (24*b*c^8*d^3 - (19*b*c^
6 - 12*b*c^4)*d^2*e - (31*b*c^4 + 8*b*c^2)*d*e^2 - 30*b*e^3)*sqrt(-c^2)*sqr
t(d)*x^5*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - 15*(2*b*c^2*d*e^2*x^
4 - b*c^2*d^2*e*x^2 - 3*b*c^2*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) - (30*a*c^2*d*e^2*x^4 - 15*a*c^2*d^2*e*x^2 - 45*
a*c^2*d^3 + (9*b*c^3*d^3*x + (24*b*c^7*d^3 - 19*b*c^5*d^2*e - 31*b*c^3*d*e^
2)*x^5 - 4*(3*b*c^5*d^3 - 2*b*c^3*d^2*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
)*sqrt(e*x^2 + d))/(c^2*d^3*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\left(a+b\,\operatorname{acsch}(cx)\right)\sqrt{d+ex^2}}{x^6}\,dx$$

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}\left(b\,\operatorname{arcsch}(cx)+a\right)}{x^6}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.128 $\int x^3 (d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx$

Optimal result . . . 994
Rubi [A] (verified) . . . 995
Mathematica [C] (verified) . . . 1000
Maple [F] . . . 1000
Fricas [A] (verification not implemented) . . . 1000
Sympy [F(-1)] . . . 1002
Maxima [F(-2)] . . . 1002
Giac [F] . . . 1002
Mupad [F(-1)] . . . 1002

Optimal result

Integrand size = 23, antiderivative size = 384

$$\begin{aligned}
&\int x^3 (d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \\
&-\frac{b(3c^4d^2+38c^2de-25e^2)\, x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} \\
&+\frac{b(13c^2d-25e)\, x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} \\
&-\frac{d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2} \\
&-\frac{b(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\, x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{-c^2x^2}} \\
&-\frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{35e^2\sqrt{-c^2x^2}}
\end{aligned}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcc
sch(c*x))/e^2-1/560*b*(35*c^6*d^3+35*c^4*d^2*e-63*c^2*d*e^2+25*e^3)*x*arcta
n(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(-c^2*x^2)^(1/2
)-2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2
/(-c^2*x^2)^(1/2)+1/840*b*(13*c^2*d-25*e)*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1
/2)/c^3/e/(-c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/c/e/
(-c^2*x^2)^(1/2)-1/560*b*(3*c^4*d^2+38*c^2*d*e-25*e^2)*x*(-c^2*x^2-1)^(1/2)
*(e*x^2+d)^(1/2)/c^5/e/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 587, 159, 163, 65, 223, 209, 95, 210}

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = -\frac{d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$
$$+\frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2} - \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{35e^2\sqrt{-c^2x^2}}$$
$$-\frac{bx(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{-c^2x^2}}$$
$$+\frac{bx\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}(13c^2d-25e)\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}}$$
$$-\frac{bx\sqrt{-c^2x^2-1}(3c^4d^2+38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/560*(b*(3*c^4*d^2 + 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e
*x^2])/(c^5*e*Sqrt[-(c^2*x^2)]) + (b*(13*c^2*d - 25*e)*x*Sqrt[-1 - c^2*x^2]
*(d + e*x^2)^(3/2))/(840*c^3*e*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*
(d + e*x^2)^(5/2))/(42*c*e*Sqrt[-(c^2*x^2)]) - (d*(d + e*x^2)^(5/2)*(a + b*
ArcCsch[c*x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^2) -
 (b*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*x*ArcTan[(Sqrt[e]*S
qrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[-(c^2*x^2)])
 - (2*b*c*d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(
35*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{35e^2x\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{x\sqrt{-1-c^2x^2}}\,dx}{35e^2\sqrt{-c^2x^2}} \\
&= -\frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{(d+ex)^{5/2}(-2d+5ex)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{70e^2\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{(d+ex)^{3/2}\left(6c^2d^2-\frac{1}{2}\left(13c^2d-25e\right)ex\right)}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{210ce^2\sqrt{-c^2x^2}} \\
&= \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} \\
&\quad - \frac{(bx)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}\left(-12c^4d^3-\frac{3}{4}e\left(3c^4d^2+38c^2de-25e^2\right)x\right)}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{420c^3e^2\sqrt{-c^2x^2}} \\
&= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} \\
&\quad + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{12c^6d^4+\frac{3}{8}e\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{420c^5e^2\sqrt{-c^2x^2}} \\
&= -\frac{b\left(3c^4d^2+38c^2de-25e^2\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} \\
&\quad + \frac{b\left(13c^2d-25e\right)x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} \\
&\quad + \frac{\left(bcd^4x\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{35e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{\left(b\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}x\sqrt{d+ex}}\,dx,x,x^2\right)}{1120c^5e\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} \\
&\quad + \frac{b(13c^2d-25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} + \frac{(2bcd^4x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{35e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(b(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{560c^7e\sqrt{-c^2x^2}} \\
&= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} \\
&\quad + \frac{b(13c^2d-25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} - \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{35e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(b(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{560c^7e\sqrt{-c^2x^2}} \\
&= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}} \\
&\quad + \frac{b(13c^2d-25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} \\
&\quad - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2} \\
&\quad - \frac{b(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{-c^2x^2}} \\
&\quad - \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{35e^2\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.54 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.79

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a + b \operatorname{csch}^{-1}(cx)\right)\, dx = \frac{96a(d+ex^2)^3(-2d+5ex^2) + \frac{2be\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)(75e^2-2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4)}{c^5}}{}$$

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) + (2*b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
e*x^2)*(75*e^2 - 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40
*e^2*x^4)))/c^5 - (3*b*(-32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/
2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] + (e*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^
2*d*e^2 + 25*e^3)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1,
 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(c^5*x) + 96*b
*(d + e*x^2)^3*(-2*d + 5*e*x^2)*ArcCsch[c*x])/(3360*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^3 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a + b\,\operatorname{arccsch}\left(cx\right)\right) dx$$

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.37 (sec) , antiderivative size = 1943, normalized size of antiderivative = 5.06

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a + b \operatorname{csch}^{-1}(cx)\right)\, dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e -
 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
```

```
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x
^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e^3*x^6
 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 384*
a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2
*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 +
75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2),
 1/3360*(48*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e -
 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*
x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c
^2*d*e + e^2)*x^2 + d*e)) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7
*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d
^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*
e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt((c^2*x
^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2), -1/6720*(192*b*c^7*sqrt(-d)
*d^3*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - 3*(3
5*b*c^6*d^3 + 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + e^2) - 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*
c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 - 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c
^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*
c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^7*e^2), -1/3360*(96*b*c^7*sqrt(-d)*d^3*arctan(1/2*((
c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^
2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - 3*(35*b*c^6*d^3 + 35*b
*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2
*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4
+ 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2
 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \operatorname{arcsch}(cx)+a) x^3\, dx$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int x^3 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.129 $\int x(d+ex^2)^{3/2}\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 1003
Rubi [A] (verified) . . . 1003
Mathematica [C] (verified) . . . 1007
Maple [F] . . . 1008
Fricas [A] (verification not implemented) . . . 1008
Sympy [F] . . . 1009
Maxima [F] . . . 1009
Giac [F] . . . 1009
Mupad [F(-1)] . . . 1010

Optimal result

Integrand size = 21, antiderivative size = 270

$$\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{b(15c^4d^2-10c^2de+3e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{5e\sqrt{-c^2x^2}}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/e+1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2-10
*c^2*d*e+3*e^2)*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/
e^(1/2)/(-c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/(-c^
2*x^2)^(1/2)+1/40*b*(7*c^2*d-3*e)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/
(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {6435, 457, 104, 159, 163, 65, 223, 209, 95, 210}

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$
$$+\frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{5e\sqrt{-c^2x^2}}+\frac{bx(15c^4d^2-10c^2de+3e^2)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}}$$
$$+\frac{bx\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}(7c^2d-3e)\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(7*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[-(c^2
*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[-(c^2*x^2)]
) + ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (b*(15*c^4*d^2 - 10*c^
2*d*e + 3*e^2)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/
(40*c^4*Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(5*e*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_)^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
```

]

Rubi steps

$$\text{integral} = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bcx)\int \frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{-1-c^2x^2}}\,dx}{5e\sqrt{-c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{10e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(-2c^2d^2-\frac{1}{2}\left(7c^2d-3e\right)ex\right)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{20ce\sqrt{-c^2x^2}}$$

$$= \frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bx)\text{Subst}\left(\int \frac{2c^4d^3+\frac{1}{4}e\left(15c^4d^2-10c^2de+3e^2\right)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{20c^3e\sqrt{-c^2x^2}}$$

$$= \frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{10e\sqrt{-c^2x^2}} - \frac{\left(b\left(15c^4d^2-10c^2de+3e^2\right)x\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{80c^3\sqrt{-c^2x^2}}$$

$$= \frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} - \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{5e\sqrt{-c^2x^2}} + \frac{\left(b\left(15c^4d^2-10c^2de+3e^2\right)x\right)\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{40c^5\sqrt{-c^2x^2}}$$

$$= \frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}}$$
$$+ \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{5e\sqrt{-c^2x^2}}$$
$$+ \frac{\left(b(15c^4d^2-10c^2de+3e^2)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{-c^2x^2}}$$
$$= \frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}$$
$$+ \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e}$$
$$+ \frac{b(15c^4d^2-10c^2de+3e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{5e\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.50 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int x\left(d+ex^2\right)^{3/2}\left(a + b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{16a\left(d+ex^2\right)^3}{e} + \frac{2b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(d+ex^2\right)\left(-3e+c^2\left(9d+2ex^2\right)\right)}{c^3} + \frac{b\left(-\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e} + \right.}{80\sqrt{d+ex}}$$

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]

[Out] ((16*a*(d + e*x^2)^3)/e + (2*b*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2)*(-3*e +
c^2*(9*d + 2*e*x^2)))/c^3 + (b*((-8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1,
 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))])/e + ((15*c^4*d^2 - 10*c^2*d*e
+ 3*e^2)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2
, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(c^3*x) + (16*b*(d + e*
x^2)^3*ArcCsch[c*x])/e)/(80*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arccsch}\left(cx\right)\right)dx$$

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 1625, normalized size of antiderivative = 6.02

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
 d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e
^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)
*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*
d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4
*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(
3*b*c^4*d*e - b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))
/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c
^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e
 + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 +
d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^
2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d
)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^5*e^2*x^4 +
 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e - b*c^2
*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), 1/160*(16
*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d
)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2
 + d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^
4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4
*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2)
 + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5
*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x)*
sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), 1/80*(8*b*c^5*sqrt
```

```
(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*
sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) -
(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2
 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*
x^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*
x^2 + d))/(c^5*e)]
```

Sympy [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x(a+b\operatorname{acsch}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d
)*log(sqrt(c^2*x^2 + 1) + 1)/e + 5*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*e*x
^3 + c^2*d^2*x)*sqrt(e*x^2 + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^2 +
 1) + e), x) - 5*integrate(1/5*((5*e^2*log(c) + e^2)*c^2*x^5 + ((5*d*e*log(
c) + 2*d*e)*c^2 + 5*e^2*log(c))*x^3 + (c^2*d^2 + 5*d*e*log(c))*x + 5*(c^2*e
^2*x^5 + (c^2*d*e + e^2)*x^3 + d*e*x)*log(x))*sqrt(e*x^2 + d)/(c^2*e*x^2 +
e), x))*b
```

Giac [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)^{3/2}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.130 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal result 1011
Rubi [N/A] 1011
Mathematica [N/A] 1012
Maple [N/A] (verified) 1012
Fricas [N/A] 1012
Sympy [N/A] 1013
Maxima [F(-2)] 1013
Giac [N/A] 1013
Mupad [N/A] 1014

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \operatorname{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsch}\left(cx\right)\right)}{x}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x,
x)
```

Sympy [N/A]

Not integrable

Time = 65.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 5.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.131 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result . . . 1015
Rubi [N/A] . . . 1015
Mathematica [N/A] . . . 1016
Maple [N/A] (verified) . . . 1016
Fricas [N/A] . . . 1016
Sympy [N/A] . . . 1017
Maxima [F(-2)] . . . 1017
Giac [N/A] . . . 1017
Mupad [N/A] . . . 1018

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \operatorname{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsch}\left(cx\right)\right)}{x^3}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

Sympy [N/A]

Not integrable

Time = 62.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**3,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 5.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.132 $\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result 1019
Rubi [N/A] 1019
Mathematica [N/A] 1020
Maple [N/A] (verified) 1020
Fricas [N/A] 1020
Sympy [F(-1)] 1020
Maxima [F(-2)] 1021
Giac [N/A] 1021
Mupad [N/A] 1021

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Int}\left(x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx$$

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\text{arccsch}\left(cx\right)\right) dx$$

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arcsch}\left(cx\right)+a)x^2\, dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arccsch(c*x))*sqrt(e*x^2
+ d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \operatorname{arcsch}(cx)+a) x^2\, dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 5.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int x^2 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.133 $\int \left(d+ex^2\right)^{3/2}\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result 1022
Rubi [N/A] 1022
Mathematica [N/A] 1023
Maple [N/A] (verified) 1023
Fricas [N/A] 1023
Sympy [N/A] 1023
Maxima [F(-2)] 1024
Giac [N/A] 1024
Mupad [N/A] 1024

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Int}\left(\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsch}\left(cx\right)\right)dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 62.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(e\,x^2+d\right)^{3/2}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.134 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result . . . 1025
Rubi [N/A] . . . 1025
Mathematica [N/A] . . . 1026
Maple [N/A] (verified) . . . 1026
Fricas [N/A] . . . 1026
Sympy [N/A] . . . 1027
Maxima [F(-2)] . . . 1027
Giac [N/A] . . . 1027
Mupad [N/A] . . . 1028

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \operatorname{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsch}\left(cx\right)\right)}{x^2}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

Sympy [N/A]

Not integrable

Time = 57.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**2,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 6.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.135 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result . . . 1029
Rubi [N/A] . . . 1029
Mathematica [N/A] . . . 1030
Maple [N/A] (verified) . . . 1030
Fricas [N/A] . . . 1030
Sympy [N/A] . . . 1031
Maxima [F(-2)] . . . 1031
Giac [N/A] . . . 1031
Mupad [N/A] . . . 1032

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \operatorname{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsch}\left(cx\right)\right)}{x^4}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

Sympy [N/A]

Not integrable

Time = 66.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(a+b\,\text{acsch}\left(cx\right)\right)\left(d+ex^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**4,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.136 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result . . . 1033
Rubi [A] (verified) . . . 1034
Mathematica [C] (verified) . . . 1038
Maple [F] . . . 1039
Fricas [A] (verification not implemented) . . . 1039
Sympy [F(-1)] . . . 1039
Maxima [F(-2)] . . . 1040
Giac [F] . . . 1040
Mupad [F(-1)] . . . 1040

Optimal result

Integrand size = 23, antiderivative size = 492

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{bc^3(8c^4d^2-23c^2de+23e^2)\,x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}$$
$$+\frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}}$$
$$-\frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}}$$
$$+\frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}$$
$$-\frac{bc^2(8c^4d^2-23c^2de+23e^2)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$
$$+\frac{be(4c^4d^2-11c^2de+15e^2)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^(3/2)*(-c^
2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)+1/75*b*c^3*(8*c^4*d^2-23*c^2*d*e+23*e^2
)*x^2*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+1/75*b*c*(8*c^4
*d^2-23*c^2*d*e+23*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/
2)-4/75*b*c*(c^2*d-2*e)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(-c^2*x^2)^(
1/2)-1/75*b*c^2*(8*c^4*d^2-23*c^2*d*e+23*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*
x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(
1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+
1/75*b*e*(4*c^4*d^2-11*c^2*d*e+15*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^
2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {270, 6437, 12, 485, 594, 597, 545, 429, 506, 422}

$$\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&+\frac{bex(4c^4d^2-11c^2de+15e^2)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\
&-\frac{bc^2x(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{75d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\
&-\frac{4bc\sqrt{-c^2x^2-1}(c^2d-2e)\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} \\
&+\frac{bc\sqrt{-c^2x^2-1}(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} \\
&+\frac{bc^3x^2(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}
\end{aligned}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c^3*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*x^2*Sqrt[d + e*x^2])/(75*d*Sqrt[-(
c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*Sqrt
[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*Sqrt[-(c^2*x^2)]) - (4*b*c*(c^2*d - 2
*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*x^2*Sqrt[-(c^2*x^2)]) + (b*c*Sq
rt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^4*Sqrt[-(c^2*x^2)]) - ((d + e*x^2
)^(5/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) - (b*c^2*(8*c^4*d^2 - 23*c^2*d*e +
23*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(75*d*Sqrt
[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (b*e
*(4*c^4*d^2 - 11*c^2*d*e + 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x],
 1 - e/(c^2*d)])/(75*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^
2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
```

```
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} - \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{5/2}}{5dx^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int \frac{\left(d+ex^2\right)^{5/2}}{x^6\sqrt{-1-c^2x^2}}\,dx}{5d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(4d\left(c^2d-2e\right)+\left(c^2d-5e\right)ex^2\right)}{x^4\sqrt{-1-c^2x^2}}\,dx}{25d\sqrt{-c^2x^2}} \\
&= -\frac{4bc\left(c^2d-2e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{(bcx)\int \frac{d\left(8c^4d^2-23c^2de+23e^2\right)+e\left(4c^4d^2-11c^2de+15e^2\right)x^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} \\
&\quad - \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{(bcx)\int \frac{de(4c^4d^2-11c^2de+15e^2)+c^2de(8c^4d^2-23c^2de+23e^2)x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^2\sqrt{-c^2x^2}} \\
&= \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} \\
&\quad - \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{(bce(4c^4d^2-11c^2de+15e^2)x)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{-c^2x^2}} \\
&\quad + \frac{(bc^3e(8c^4d^2-23c^2de+23e^2)x)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{-c^2x^2}} \\
&= \frac{bc^3(8c^4d^2-23c^2de+23e^2)x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad + \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} \\
&\quad - \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad + \frac{be(4c^4d^2-11c^2de+15e^2)x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{(bc^3(8c^4d^2-23c^2de+23e^2)x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{75d\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bc^3(8c^4d^2-23c^2de+23e^2)\,x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad + \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} \\
&\quad - \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\
&\quad - \frac{bc^2(8c^4d^2-23c^2de+23e^2)\,x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{be(4c^4d^2-11c^2de+15e^2)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.59

$$
\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d+ex^2}\left(-15a(d+ex^2)^2+bc\sqrt{1+\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11-23c^2\right.}{75dx^5} \\
&+\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2-23c^2de+23e^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+(-8c^6d^3+27c^4d^2e-34c^2\right.}{75\sqrt{c^2}d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]

[Out] (Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(23*e^2
*x^4 + d*e*x^2*(11 - 23*c^2*x^2) + d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*
(d + e*x^2)^2*ArcCsch[c*x]))/(75*d*x^5) + ((I/75)*b*c*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*EllipticE[I
*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-8*c^6*d^3 + 27*c^4*d^2*e - 34*c^2*d*e
^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sq
rt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsch}\,(cx))}{x^6}\,dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)}{x^6}\,dx =$$

$$-\frac{(8\,bc^8d^3-23\,bc^6d^2e+23\,bc^4de^2)\sqrt{-c^2}\sqrt{d}x^5E(\arcsin\left(\sqrt{-c^2}x\right)\mid\frac{e}{c^2d})-(8\,bc^8d^3-(23\,bc^6-4\,bc^4)d^2e+(2}{}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
[Out] -1/75*((8*b*c^8*d^3 - 23*b*c^6*d^2*e + 23*b*c^4*d*e^2)*sqrt(-c^2)*sqrt(d)*x
^5*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - (8*b*c^8*d^3 - (23*b*c^6 -
 4*b*c^4)*d^2*e + (23*b*c^4 - 11*b*c^2)*d*e^2 + 15*b*e^3)*sqrt(-c^2)*sqrt(d
)*x^5*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + 15*(b*c^2*d*e^2*x^4 + 2
*b*c^2*d^2*e*x^2 + b*c^2*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)) + (15*a*c^2*d*e^2*x^4 + 30*a*c^2*d^2*e*x^2 + 15*a*c^2
*d^3 - (3*b*c^3*d^3*x + (8*b*c^7*d^3 - 23*b*c^5*d^2*e + 23*b*c^3*d*e^2)*x^5
 - (4*b*c^5*d^3 - 11*b*c^3*d^2*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(
e*x^2 + d))/(c^2*d^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**6,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcsch}(cx)+a\right)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.137 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal result . . . 1041
Rubi [A] (verified) . . . 1042
Mathematica [C] (verified) . . . 1047
Maple [F] . . . 1048
Fricas [A] (verification not implemented) . . . 1048
Sympy [F(-1)] . . . 1048
Maxima [F(-2)] . . . 1049
Giac [F] . . . 1049
Mupad [F(-1)] . . . 1049

Optimal result

Integrand size = 23, antiderivative size = 643

$$\begin{aligned}
&\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \\
&-\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&-\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&+\frac{bc(120c^4d^2-159c^2de-37e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&-\frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&+\frac{bc^2(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&-\frac{be(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*a
rccsch(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d-11*e)*(e*x^2+d)^(3/2)*(-c^2*x^2-1
)^(1/2)/d/x^4/(-c^2*x^2)^(1/2)+1/49*b*c*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/
d/x^6/(-c^2*x^2)^(1/2)-1/3675*b*c^3*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^
2+247*e^3)*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/36
75*b*c*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*(-c^2*x^2-1)^(1/2)
```

```
*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/3675*b*c*(120*c^4*d^2-159*c^2*d*e-3
7*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/3675*b*c
^2*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*x*(1/(c^2*x^2+1))^(1/2
)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x
^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1
))^(1/2)-1/3675*b*e*(120*c^6*d^3-249*c^4*d^2*e+71*c^2*d*e^2+210*e^3)*x*(1/(
c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^
2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2
+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 270, 6437, 12, 594, 597, 545, 429, 506, 422}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \frac{2e(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}$$
$$-\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$
$$-\frac{bex(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$
$$+\frac{bc^2x(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{d+ex^2}E\left(\arctan(cx)\middle|1-\frac{e}{c^2d}\right)}{3675d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$
$$+\frac{bc\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}}-\frac{bc\sqrt{-c^2x^2-1}(30c^2d-11e)(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}}$$
$$+\frac{bc\sqrt{-c^2x^2-1}(120c^4d^2-159c^2de-37e^2)\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}}$$
$$-\frac{bc\sqrt{-c^2x^2-1}(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}}$$
$$-\frac{bc^3x^2(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] -1/3675*(b*c^3*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x^2*
Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) - (b*c*(240*c^6*
d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*Sqrt[-1 - c^2*x^2]*Sqrt[d +
e*x^2])/(3675*d^2*Sqrt[-(c^2*x^2)]) + (b*c*(120*c^4*d^2 - 159*c^2*d*e - 37*
e^2)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[-(c^2*x^2)]) - (b
*c*(30*c^2*d - 11*e)*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^4*Sqrt
```

```
[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*Sqrt[-
(c^2*x^2)]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(7*d*x^7) + (2*e*(d
 + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 - 5
28*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[
c*x], 1 - e/(c^2*d)])/(3675*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d
 + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(120*c^6*d^3 - 249*c^4*d^2*e + 71*c^2*
d*e^2 + 210*e^3)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(
3675*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x
^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{35d^2x^8\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} \\
&\quad + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} - \frac{(bcx)\int \frac{\left(d+ex^2\right)^{5/2}\left(-5d+2ex^2\right)}{x^8\sqrt{-1-c^2x^2}}\,dx}{35d^2\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} \\
&\quad + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad + \frac{(bcx)\int \frac{\left(d+ex^2\right)^{3/2}\left(-d\left(30c^2d-11e\right)-e\left(5c^2d+14e\right)x^2\right)}{x^6\sqrt{-1-c^2x^2}}\,dx}{245d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc\left(30c^2d-11e\right)\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(-d\left(120c^4d^2-159c^2de-37e^2\right)-2e\left(15c^4d^2-18c^2de-35e^2\right)x^2\right)}{x^4\sqrt{-1-c^2x^2}}\,dx}{1225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc\left(120c^4d^2-159c^2de-37e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc\left(30c^2d-11e\right)\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad + \frac{(bcx)\int \frac{-d\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)-e\left(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3\right)x^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\left(120c^4d^2-159c^2de-37e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc\left(30c^2d-11e\right)\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad + \frac{(bcx)\int \frac{-de\left(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3\right)-c^2de\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^3\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc(120c^4d^2 - 159c^2de - 37e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(30c^2d - 11e)\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad - \frac{(bce(120c^6d^3 - 249c^4d^2e + 71c^2de^2 + 210e^3)\,x)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{(bc^3e(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,x)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc^3(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad - \frac{bc(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc(120c^4d^2 - 159c^2de - 37e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(30c^2d - 11e)\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad - \frac{be(120c^6d^3 - 249c^4d^2e + 71c^2de^2 + 210e^3)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{(bc^3(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{3675d^2\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc^3(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad - \frac{bc(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc(120c^4d^2 - 159c^2de - 37e^2)\,\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(30c^2d - 11e)\,\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&\quad + \frac{bc^2(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{be(120c^6d^3 - 249c^4d^2e + 71c^2de^2 + 210e^3)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.58

$$
\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx =
$$

$$
-\frac{\sqrt{d+ex^2}\left(105a(5d-2ex^2)(d+ex^2)^2 + bc\sqrt{1+\frac{1}{c^2x^2}}x(247e^3x^6 + de^2x^4(-71+193c^2x^2) - 3d^2ex^2(61-8\right.}{3675d^2}
$$

$$
-\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3 - 528c^4d^2e + 193c^2de^2 + 247e^3)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)\,\middle|\,\frac{e}{c^2d}\right) - 2(120c\right.}{3675\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+}
$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]

[Out] -1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
+ 1/(c^2*x^2)]*x*(247*e^3*x^6 + d*e^2*x^4*(-71 + 193*c^2*x^2) - 3*d^2*e*x^2
*(61 - 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*
c^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/(d^2*x^7) -
((I/3675)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d
^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*EllipticE[I*ArcSinh[Sqrt[c^2]
*x], e/(c^2*d)] - 2*(120*c^8*d^4 - 324*c^6*d^3*e + 221*c^4*d^2*e^2 + 88*c^2
*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsch}(cx))}{x^8}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^8}\,dx = \frac{(240\,bc^{10}d^4 - 528\,bc^8d^3e + 193\,bc^6d^2e^2 + 247\,bc^4de^3)\sqrt{-c^2}\sqrt{d}x^7E(\text{arc}}{}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
[Out] 1/3675*((240*b*c^10*d^4 - 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2 + 247*b*c^4*d
*e^3)*sqrt(-c^2)*sqrt(d)*x^7*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) -
(240*b*c^10*d^4 - 24*(22*b*c^8 - 5*b*c^6)*d^3*e + (193*b*c^6 - 249*b*c^4)*d
^2*e^2 + (247*b*c^4 + 71*b*c^2)*d*e^3 + 210*b*e^4)*sqrt(-c^2)*sqrt(d)*x^7*e
lliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + 105*(2*b*c^2*d*e^3*x^6 - b*c^2
*d^2*e^2*x^4 - 8*b*c^2*d^3*e*x^2 - 5*b*c^2*d^4)*sqrt(e*x^2 + d)*log((c*x*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (210*a*c^2*d*e^3*x^6 - 105*a*c^2*
d^2*e^2*x^4 - 840*a*c^2*d^3*e*x^2 - 525*a*c^2*d^4 + (75*b*c^3*d^4*x - (240*
b*c^9*d^4 - 528*b*c^7*d^3*e + 193*b*c^5*d^2*e^2 + 247*b*c^3*d*e^3)*x^7 + (1
20*b*c^7*d^4 - 249*b*c^5*d^3*e + 71*b*c^3*d^2*e^2)*x^5 - 3*(30*b*c^5*d^4 -
61*b*c^3*d^3*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d
^3*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^8}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**8,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.138 $\int \frac{x^5\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1050
Rubi [A] (verified) . . . 1051
Mathematica [C] (verified) . . . 1055
Maple [F] . . . 1056
Fricas [A] (verification not implemented) . . . 1056
Sympy [F] . . . 1057
Maxima [F(-2)] . . . 1057
Giac [F] . . . 1057
Mupad [F(-1)] . . . 1058

Optimal result

Integrand size = 23, antiderivative size = 329

$$\begin{aligned}\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}\\ &+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}\\ &-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}\\ &+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}\\ &+\frac{b(45c^4d^2+10c^2de+9e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{-c^2x^2}}\\ &+\frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^3\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcc
sch(c*x))/e^3+1/120*b*(45*c^4*d^2+10*c^2*d*e+9*e^2)*x*arctan(e^(1/2)*(-c^2*
x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(-c^2*x^2)^(1/2)+8/15*b*c*d^(5/
2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x^2)^(1/2
)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/e^2/(-c^2*x^2)^(1/2)+d^2*(a
+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d+9*e)*x*(-c^2*x^2-1)^
(1/2)*(e*x^2+d)^(1/2)/c^3/e^2/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 1629, 159, 163, 65, 223, 209, 95, 210}

$$\begin{aligned}\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}\\ &\quad - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}\\ &\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}\\ &\quad + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^3\sqrt{-c^2x^2}}\\ &\quad + \frac{bx(45c^4d^2+10c^2de+9e^2)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{-c^2x^2}}\\ &\quad + \frac{bx\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}}\\ &\quad - \frac{bx\sqrt{-c^2x^2-1}(19c^2d+9e)\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}\end{aligned}$$

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/120*(b*(19*c^2*d + 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(c^3*e^2*S
qrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqr
t[-(c^2*x^2)]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^3 - (2*d*(d +
 e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*Arc
Csch[c*x]))/(5*e^3) + (b*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*x*ArcTan[(Sqrt[e
]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqrt[-(c^2*x^2
)]) + (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])]
)/(15*e^3*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\ &\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{15e^3x\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\ &= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\ &\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{x\sqrt{-1-c^2x^2}}\,dx}{15e^3\sqrt{-c^2x^2}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}\left(8d^2-4dex+3e^2x^2\right)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{30e^3\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}\left(-16c^2d^2e+\frac{1}{2}e^2\left(19c^2d+9e\right)x\right)}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{60ce^4\sqrt{-c^2x^2}} \\
&= -\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad - \frac{(bx)\operatorname{Subst}\left(\int \frac{16c^4d^3e+\frac{1}{4}e^2\left(45c^4d^2+10c^2de+9e^2\right)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{60c^3e^4\sqrt{-c^2x^2}} \\
&= -\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} \\
&\quad + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} - \frac{\left(4bcd^3x\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^3\sqrt{-c^2x^2}} \\
&\quad - \frac{\left(b\left(45c^4d^2+10c^2de+9e^2\right)x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{240c^3e^2\sqrt{-c^2x^2}} \\
&= -\frac{b\left(19c^2d+9e\right)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} \\
&\quad + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} - \frac{\left(8bcd^3x\right)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{15e^3\sqrt{-c^2x^2}} \\
&\quad + \frac{\left(b\left(45c^4d^2+10c^2de+9e^2\right)x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{120c^5e^2\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} \\
&\quad + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^3\sqrt{-c^2x^2}} \\
&\quad + \frac{\left(b(45c^4d^2+10c^2de+9e^2)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^5e^2\sqrt{-c^2x^2}} \\
&= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{b(45c^4d^2+10c^2de+9e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{-c^2x^2}} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^3\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 3.03 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.85

$$
\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx
$$

$$
= \frac{16a(d+ex^2)\left(8d^2-4dex^2+3e^2x^4\right) + \frac{2be\sqrt{1+\frac{1}{c^2x^2}}(d+ex^2)(-9ex+c^2(-13dx+6ex^3))}{c^3} + \frac{b\left(-64c^2d^3\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\right.\right.}{}
$$

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + (2*b*e*Sqrt[1 + 1/(c^2*
x^2)]*(d + e*x^2)*(-9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (b*(-64*c^2*d^3
*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))]
 + (e*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 +
(e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2
*x^2]))/(c^3*x) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcCsch[
c*x])/(240*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 1633, normalized size of antiderivative = 4.96

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
+ d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (45*b*c^4*d^2 + 10*b*c^2*d*e + 9*b*
e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2
)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b
*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^
3 - (13*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2
)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2
+ d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4*d^2 +
10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*s
qrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d
*e + e^2)*x^2 + d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2
)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2
4*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^3 - (13*
b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))
/(c^5*e^3), 1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2
*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 + d*e)*x^2 + d^2)) + (45*b*c^4*d^2 + 10*b*c^2*d*e + 9*b*e^2)*sq
rt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 +
 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^
2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(
24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^3 - (13
```

```
*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)
)/(c^5*e^3), 1/240*(64*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2
*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 + d*e)*x^2 + d^2)) - (45*b*c^4*d^2 + 10*b*c^2*d*e + 9*b*e^2)*sq
rt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sq
rt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 16
*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5
*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^4*e^2*x^3 - (13*b*c^4*d*e + 9*b*c^2*e^2)*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^3)]
```

Sympy [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5(a+b\,\operatorname{acsch}(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.139 $\int \frac{x^3\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1059
Rubi [A] (verified) . . . 1060
Mathematica [C] (verified) . . . 1063
Maple [F] . . . 1064
Fricas [A] (verification not implemented) . . . 1064
Sympy [F] . . . 1065
Maxima [F(-2)] . . . 1065
Giac [F] . . . 1066
Mupad [F(-1)] . . . 1066

Optimal result

Integrand size = 23, antiderivative size = 229

$$\begin{aligned}\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}\\ &+ \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}\\ &- \frac{b(3c^2d+e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{-c^2x^2}}\\ &- \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^2\sqrt{-c^2x^2}}\end{aligned}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^2-1/6*b*(3*c^2*d+e)*x*arctan(e^(1/
2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(-c^2*x^2)^(1/2)-2/3*b
*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/(-c^2*x
^2)^(1/2)-d*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(-c^2*x^2-1)^(1/
2)*(e*x^2+d)^(1/2)/c/e/(-c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 587, 159, 163, 65, 223, 209, 95, 210}

$$\begin{aligned}\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\ &\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\ &\quad - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^2\sqrt{-c^2x^2}} \\ &\quad - \frac{bx(3c^2d+e)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{-c^2x^2}} \\ &\quad + \frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}}\end{aligned}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[-(c^2*x^2)]) - (d*Sqrt
[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c
*x]))/(3*e^2) - (b*(3*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*S
qrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[-(c^2*x^2)]) - (2*b*c*d^(3/2)*x*ArcTa
n[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\int \frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{3e^2x\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\int \frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}}\,dx}{3e^2\sqrt{-c^2x^2}} \\
&= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{6e^2\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(bx)\operatorname{Subst}\left(\int \frac{2c^2d^2+\frac{1}{2}e(3c^2d+e)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce^2\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(bcd^2x)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^2\sqrt{-c^2x^2}} + \frac{\left(b(3c^2d+e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12ce\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(2bcd^2x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3e^2\sqrt{-c^2x^2}} - \frac{\left(b(3c^2d+e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{6c^3e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^2\sqrt{-c^2x^2}} - \frac{\left(b(3c^2d+e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3e\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{b(3c^2d+e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{-c^2x^2}} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^2\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.74 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.03

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{4bd^2\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) - be(3c^2d+e)\sqrt{1+\frac{1}{c^2x^2}}x^4\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}$$

$$12ce^2x\sqrt$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (4*b*d^2*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -(1
/(c^2*x^2)), -(d/(e*x^2))] - b*e*(3*c^2*d + e)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sq
rt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] + 2*x*
Sqrt[1 + c^2*x^2]*(d + e*x^2)*(-4*a*c*d + b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a
*c*e*x^2 + 2*b*c*(-2*d + e*x^2)*ArcCsch[c*x]))/(12*c*e^2*x*Sqrt[1 + c^2*x^2
]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a + b \operatorname{arccsch}(cx))}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 1341, normalized size of antiderivative = 5.86

$$\int \frac{x^3\left(a + b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e
^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 +
 (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*d^(3/2
)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d
+ c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*
d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x
^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 +
 d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 +
 b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3
*e^2), -1/24*(8*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d
^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*
```

```
e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) - 8*(b*c
^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) + 1)/(c*x)) - 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
- 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(4*b*c^3*sqrt(-d)*d*arctan(1
/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1
)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d + b*e)
*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)
*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) -
 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 1)/(c*x)) - 2*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2)]
```

Sympy [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3(a+b\,\text{acsch}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.140 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1067
Rubi [A] (verified) . . . 1067
Mathematica [C] (verified) . . . 1070
Maple [F] . . . 1070
Fricas [B] (verification not implemented) . . . 1070
Sympy [F] . . . 1071
Maxima [F] . . . 1071
Giac [F] . . . 1072
Mupad [F(-1)] . . . 1072

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}$$

```
[Out] b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e/(-c^2*x^
2)^(1/2)+b*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(
-c^2*x^2)^(1/2)+(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6435, 457, 132, 65, 223, 209, 12, 95, 210}

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e\sqrt{-c^2x^2}}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c
^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[d]*x*
ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}}\,dx}{e\sqrt{-c^2x^2}} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1-c^2x}}\,dx, x, x^2\right)}{2e\sqrt{-c^2x^2}} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2\sqrt{-c^2x^2}} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{d}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{-c^2x^2}} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{c\sqrt{-c^2x^2}} \\
&\quad - \frac{(bcdx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{-c^2x^2}} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{c\sqrt{-c^2x^2}} \\
&\quad - \frac{(bcdx)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(a - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e\left(1+c^2x^2\right)}{-c^2d+e},1+c^2x^2\right)}{\sqrt{\frac{c^2\left(d+ex^2\right)}{c^2d-e}}} + b\operatorname{csch}^{-1}(cx)\right)}{e}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a - (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1, 3
/2, (e*(1 + c^2*x^2))/(-(c^2*d) + e), 1 + c^2*x^2])/Sqrt[(c^2*(d + e*x^2))/
(c^2*d - e)] + b*ArcCsch[c*x]))/e
```

Maple [F]

$$\int \frac{x(a+b\ \text{arccsch}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(113) = 226.

Time = 0.34 (sec) , antiderivative size = 1064, normalized size of antiderivative = 7.88

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{4\sqrt{ex^2+d}dbc\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + bc\sqrt{d}\log\left(\frac{\left(c^4d^2+6c^2de+e^2\right)x^4+8\left(c^2d^2+de\right)x^2-4\left(\left(c^3d+ce\right)x^3+2cdx\right)\sqrt{ex^2+d}\sqrt{d}\sqrt{\frac{c^2x^2}{c^2x}}\right.}{x^4}\right.$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*
x)) + b*c*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*
x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*log(8*c^
4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^
3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*
(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(
d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - 2*
b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e
)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)))
/(c*e), 1/4*(2*b*c*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e
*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 +
d*e)*x^2 + d^2)) + 4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) + 4*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c
^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2))/(c*e
), 1/2*(b*c*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 +
d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^
2 + d^2)) + 2*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*c - b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c
^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^
2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)))/(c*e)]
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x(a+b\,\operatorname{acsch}(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] b*(sqrt(e*x^2 + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate((c^2*e*x^3 + c^
2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2*e*x^2 + e)
```

```
*sqrt(e*x^2 + d)), x) - integrate(((e*log(c) + e)*c^2*x^3 + (c^2*d + e*log(
c))*x + (c^2*e*x^3 + e*x)*log(x))/((c^2*e*x^2 + e)*sqrt(e*x^2 + d)), x)) +
sqrt(e*x^2 + d)*a/e
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.141 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1073
Rubi [N/A] . . . 1073
Mathematica [N/A] . . . 1074
Maple [N/A] (verified) . . . 1074
Fricas [N/A] . . . 1074
Sympy [N/A] . . . 1074
Maxima [F(-2)] . . . 1075
Giac [N/A] . . . 1075
Mupad [N/A] . . . 1075

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\sqrt{ex^2+d}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.142 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1076
Rubi [N/A] . . . 1076
Mathematica [N/A] . . . 1077
Maple [N/A] (verified) . . . 1077
Fricas [N/A] . . . 1077
Sympy [N/A] . . . 1077
Maxima [F(-2)] . . . 1078
Giac [N/A] . . . 1078
Mupad [N/A] . . . 1078

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x^3\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\sqrt{ex^2+dx^3}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 28.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 5.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.143 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1079
Rubi [N/A] . . . 1079
Mathematica [N/A] . . . 1080
Maple [N/A] (verified) . . . 1080
Fricas [N/A] . . . 1080
Sympy [N/A] . . . 1080
Maxima [F(-2)] . . . 1081
Giac [N/A] . . . 1081
Mupad [N/A] . . . 1081

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \text{arccsch}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 13.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\,\text{acsch}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 5.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.144 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1082
Rubi [N/A] . . . 1082
Mathematica [N/A] . . . 1083
Maple [N/A] (verified) . . . 1083
Fricas [N/A] . . . 1083
Sympy [N/A] . . . 1083
Maxima [F(-2)] . . . 1084
Giac [N/A] . . . 1084
Mupad [N/A] . . . 1084

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex^2}}, x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 5.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.145 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1085
Rubi [A] (verified) . . . 1086
Mathematica [A] (verified) . . . 1088
Maple [F] . . . 1089
Fricas [A] (verification not implemented) . . . 1089
Sympy [F] . . . 1089
Maxima [F(-2)] . . . 1090
Giac [F] . . . 1090
Mupad [F(-1)] . . . 1090

Optimal result

Integrand size = 23, antiderivative size = 294

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx &= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} \\ &\quad - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} \\ &\quad - \frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\ &\quad + \frac{bex\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

```
[Out] -(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/d/x+b*c^3*x^2*(e*x^2+d)^(1/2)/d/(-c^2*x
^2)^(1/2)/(-c^2*x^2-1)^(1/2)+b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2
*x^2)^(1/2)-b*c^2*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(
c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^
2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+b*e*x*(1/(c^2*x^2+1))^(1/2)*
(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2
+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))
^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {270, 6437, 12, 486, 21, 433, 429, 506, 422}

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx &= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} \\ &\quad + \frac{bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\ &\quad - \frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\ &\quad + \frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*
Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]
*(a + b*ArcCsch[c*x]))/(d*x) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)
/(d*(1 + c^2*x^2))]) + (b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/
(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 +
 c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x^2\sqrt{-1-c^2x^2}}\,dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} - \frac{(bcx)\int \frac{-e-c^2ex^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} - \frac{(bcex)\int \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} \\
&\quad + \frac{(bcex)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}} + \frac{(bc^3ex)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} \\
&\quad + \frac{bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{dx} \\
&\quad - \frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx &= \frac{\sqrt{d+ex^2}\left(-a+bc\sqrt{1+\frac{1}{c^2x^2}}x - b\operatorname{csch}^{-1}(cx)\right)}{dx} \\
&\quad - \frac{bce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)\,\middle|\,\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x - b*ArcCsch[c*x]))/(d*x)
 - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt
[-(e/d)]*x], (c^2*d)/e])/(d*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.61

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx =$$
$$-\frac{\sqrt{-c^2}bc^4d^{\frac{3}{2}}xE(\arcsin\left(\sqrt{-c^2}x\right) \mid \frac{e}{c^2d}) + \sqrt{ex^2 + d}bc^2d\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (bc^4d + be)\sqrt{-c^2}\sqrt{d}xF(\arc}{c^2d^2x}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(-c^2)*b*c^4*d^(3/2)*x*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) +
sqrt(e*x^2 + d)*b*c^2*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
- (b*c^4*d + b*e)*sqrt(-c^2)*sqrt(d)*x*elliptic_f(arcsin(sqrt(-c^2)*x), e/(
c^2*d)) - (b*c^3*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - a*c^2*d)*sqrt(e*x^2 +
d))/(c^2*d^2*x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\operatorname{acsch}(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x^2\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.146 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result . . . 1091
Rubi [A] (verified) . . . 1092
Mathematica [C] (verified) . . . 1095
Maple [F] . . . 1096
Fricas [A] (verification not implemented) . . . 1096
Sympy [F] . . . 1096
Maxima [F(-2)] . . . 1097
Giac [F] . . . 1097
Mupad [F(-1)] . . . 1097

Optimal result

Integrand size = 23, antiderivative size = 425

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx &= -\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{bc(2c^2d+5e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} \\ &+ \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\ &+ \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} \\ &+ \frac{bc^2(2c^2d+5e)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\ &- \frac{be(c^2d+6e)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

```
[Out] -1/3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsch(c*x))*(e*x
^2+d)^(1/2)/d^2/x-1/9*b*c^3*(2*c^2*d+5*e)*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2
)^(1/2)/(-c^2*x^2-1)^(1/2)-1/9*b*c*(2*c^2*d+5*e)*(-c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/9*b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/
x^2/(-c^2*x^2)^(1/2)+1/9*b*c^2*(2*c^2*d+5*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x
^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1
/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
-1/9*b*e*(c^2*d+6*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*
x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^2)^(1/2)
/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 270, 6437, 12, 594, 597, 545, 429, 506, 422}

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx &= \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\ &- \frac{bex(c^2d+6e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\ &+ \frac{bc^2x(2c^2d+5e)\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} \\ &- \frac{bc\sqrt{-c^2x^2-1}(2c^2d+5e)\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} \\ &+ \frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{bc^3x^2(2c^2d+5e)\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}\end{aligned}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(b*c^3*(2*c^2*d + 5*e)*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2]) - (b*c*(2*c^2*d + 5*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/
(9*d^2*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^
2*Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2
*e*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d + 5*e)
*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^2*Sqrt[-(c^2
*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(c^2*
d + 6*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^3*Sq
rt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{3d^2x^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{x^4\sqrt{-1-c^2x^2}}\,dx}{3d^2\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} + \frac{(bcx)\int \frac{-d\left(2c^2d+5e\right)-e\left(c^2d+6e\right)x^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc\left(2c^2d+5e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} \\
&\quad + \frac{(bcx)\int \frac{-de\left(c^2d+6e\right)-c^2de\left(2c^2d+5e\right)x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^3\sqrt{-c^2x^2}} \\
&= -\frac{bc\left(2c^2d+5e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} \\
&\quad - \frac{\left(bc^3e\left(2c^2d+5e\right)x\right)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{-c^2x^2}} - \frac{\left(bce\left(c^2d+6e\right)x\right)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{bc(2c^2d+5e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} \\
&\quad - \frac{be(c^2d+6e)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{(bc^3(2c^2d+5e)\,x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{9d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{bc(2c^2d+5e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x} \\
&\quad + \frac{bc^2(2c^2d+5e)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad - \frac{be(c^2d+6e)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.56

$$
\begin{aligned}
&\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \\
&\quad -\frac{\sqrt{d+ex^2}\left(3a(d-2ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}x(-d+2c^2dx^2+5ex^2)+3b(d-2ex^2)\operatorname{csch}^{-1}(cx)\right)}{9d^2x^3} \\
&\quad -\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d+5e)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)\,\middle|\,\frac{e}{c^2d}\right)-2(c^4d^2+2c^2de-3e^2)\operatorname{EllipticF}\left(i\right.\right.}{9\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(3*a*(d - 2*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d
+ 2*c^2*d*x^2 + 5*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsch[c*x]))/(d^2*x^3) - ((I
```

```
/9)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d + 5*e)*
EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] - 2*(c^4*d^2 + 2*c^2*d*e - 3*e
^2)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 +
c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^4 \sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.64

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx$$

$$= \frac{(2\,bc^6d^2 + 5\,bc^4de)\sqrt{-c^2}\sqrt{d}x^3 E(\arcsin\left(\sqrt{-c^2}x\right) \mid \frac{e}{c^2d}) - (2\,bc^6d^2 + (5\,bc^4 + bc^2)de + 6\,be^2)\sqrt{-c^2}\sqrt{d}x^3 F(\text{ar}}{}$$

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*((2*b*c^6*d^2 + 5*b*c^4*d*e)*sqrt(-c^2)*sqrt(d)*x^3*elliptic_e(arcsin(s
qrt(-c^2)*x), e/(c^2*d)) - (2*b*c^6*d^2 + (5*b*c^4 + b*c^2)*d*e + 6*b*e^2)*
sqrt(-c^2)*sqrt(d)*x^3*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + 3*(2*b
*c^2*d*e*x^2 - b*c^2*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 1)/(c*x)) + (6*a*c^2*d*e*x^2 - 3*a*c^2*d^2 + (b*c^3*d^2*x - (2*b*c^
5*d^2 + 5*b*c^3*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(
c^2*d^3*x^3)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{a + b\operatorname{acsch}(cx)}{x^4\sqrt{d + ex^2}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}x^4}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.147 $\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1098
Rubi [A] (verified) . . . 1098
Mathematica [C] (verified) . . . 1102
Maple [F] . . . 1103
Fricas [A] (verification not implemented) . . . 1103
Sympy [F(-1)] . . . 1104
Maxima [F(-2)] . . . 1104
Giac [F] . . . 1105
Mupad [F(-1)] . . . 1105

Optimal result

Integrand size = 23, antiderivative size = 256

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{b(9c^2d+e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{-c^2x^2}} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3-1/6*b*(9*c^2*d+e)*x*arctan(e^(1/
2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(-c^2*x^2)^(1/2)-8/3*b
*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x
^2)^(1/2)-d^2*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsch(c*x))
*(e*x^2+d)^(1/2)/e^3+1/6*b*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(-c^2
*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {272, 45, 6437, 12, 1629, 163, 65, 223, 209, 95, 210}

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^3\sqrt{-c^2x^2}}$$
$$- \frac{bx(9c^2d+e)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}}$$

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[-(c^2*x^2)]) - (d^2*
(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*A
rcCsch[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (b*(
9*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6
*c^2*e^(5/2)*Sqrt[-(c^2*x^2)]) - (8*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(S
qrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{(bcx)\int \frac{-8d^2-4dex^2+e^2x^4}{3e^3x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{(bcx)\int \frac{-8d^2-4dex^2+e^2x^4}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3e^3\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{-8d^2-4dex+e^2x^2}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6e^3\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(bx)\operatorname{Subst}\left(\int \frac{8c^2d^2e+\frac{1}{2}e^2(9c^2d+e)x}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce^4\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(4bcd^2x)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^3\sqrt{-c^2x^2}} \\
&\quad + \frac{(b(9c^2d+e)\,x)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12ce^2\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{(8bcd^2x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$
$$- \frac{\left(b(9c^2d+e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{6c^3e^2\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$
$$+ \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$
$$- \frac{\left(b(9c^2d+e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3e^2\sqrt{-c^2x^2}}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}$$
$$- \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}$$
$$- \frac{b(9c^2d+e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{-c^2x^2}} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.84 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.02

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{16bd^2\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) - be(9c^2d+e)\sqrt{1+}$$

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]

[Out] (16*b*d^2*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -(
1/(c^2*x^2)), -(d/(e*x^2))] - b*e*(9*c^2*d + e)*Sqrt[1 + 1/(c^2*x^2)]*x^4*S
qrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] + 2*x
*Sqrt[1 + c^2*x^2]*(b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*(8*d^2
+ 4*d*e*x^2 - e^2*x^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsch[c*x]))
/(12*c*e^3*x*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 1719, normalized size of antiderivative = 6.71

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] [1/24*((9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e^2)*x^2)*sqrt(e)*log(8*c^4*
e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
+ (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 + d)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(b*c^3*d*e*x^2 + b*
c^3*d^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x
^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a
*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqr
t(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2 + b*d*e + (9*b*
c^2*d*e + b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqr
t(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e
 + e^2)*x^2 + d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqr
t(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(b*c^3*
d*e*x^2 + b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*
d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*
e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^
2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/24*(32*(b*c^3*d*e*x
^2 + b*c^3*d^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^
2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e
)*x^2 + d^2)) - (9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e^2)*x^2)*sqrt(e)*l
og(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c
^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + e^2) - 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*
x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(2*a*c^3*e^
```

```
2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/1
2*(16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 +
2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^
4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e
^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*
sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 +
 d*e)) - 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 + d)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(2*a*c^3*e^2*x^4 - 8
*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^5}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^5\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1106
Rubi [A] (verified) . . . 1106
Mathematica [C] (verified) . . . 1109
Maple [F] . . . 1110
Fricas [B] (verification not implemented) . . . 1110
Sympy [F] . . . 1111
Maxima [F(-2)] . . . 1111
Giac [F] . . . 1111
Mupad [F(-1)] . . . 1112

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}}$$

```
[Out] b*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(-c^2*x^2)
^(1/2)+2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e
^2/(-c^2*x^2)^(1/2)+d*(a+b*arccsch(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsch(c
*x))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {272, 45, 6437, 12, 587, 163, 65, 223, 209, 95, 210}

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e^2\sqrt{-c^2x^2}}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cCsch[c*x]))/e^2 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x
^2])])/(e^(3/2)*Sqrt[-(c^2*x^2)]) + (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcx)\int \frac{2d+ex^2}{e^2x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcx)\int \frac{2d+ex^2}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{2d+ex}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e^2\sqrt{-c^2x^2}}$$

$$
\begin{aligned}
&= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad - \frac{(bcdx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{e^2\sqrt{-c^2x^2}} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{-c^2x^2}} \\
&= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad - \frac{(2bcdx)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{ce\sqrt{-c^2x^2}} \\
&= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}} + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{ce\sqrt{-c^2x^2}} \\
&= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} \\
&\quad + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$
\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{-2bd\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) + cx\left(bce\sqrt{1+\frac{1}{c^2x}}\right.}{2ce^2x}
$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]

[Out] (-2*b*d*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -(1/
(c^2*x^2)), -(d/(e*x^2))] + c*x*(b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x^3*Sqrt[1 + (
e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] + 2*Sqrt[1 + c
^2*x^2]*(2*d + e*x^2)*(a + b*ArcCsch[c*x])))/(2*c*e^2*x*Sqrt[1 + c^2*x^2]*S
qrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\,\operatorname{arccsch}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(136) = 272$.

Time = 0.35 (sec) , antiderivative size = 1274, normalized size of antiderivative = 7.96

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] [1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(
c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d
)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)*sq
rt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(b*c*e
*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d
*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d
))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2
))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 2*(b*c*e*x^2 + 2*b*c*d)*sqr
t(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c*e*x^
2 + b*c*d)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)
*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/
(c*e^3*x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(1/2*((c^3
*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + (b*e*x^2 + b*d)*sqrt(e)*l
og(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c
^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d)
)/(c*e^3*x^2 + c*d*e^2), 1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(1/2*((c
^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2
*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (b*e*x^2 + b*d)*sqrt(-e
)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 2*(b*c*
```

```
e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) + 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2)]
```

Sympy [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^3(a+b\,\operatorname{acsch}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^3\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1113
Rubi [A] (verified) . . . 1113
Mathematica [C] (verified) . . . 1115
Maple [F] . . . 1115
Fricas [B] (verification not implemented) . . . 1115
Sympy [F] . . . 1116
Maxima [F] . . . 1116
Giac [F] . . . 1116
Mupad [F(-1)] . . . 1117

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

```
[Out] -b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/d^(1/2)/(-c^2*x
^2)^(1/2)+(-a-b*arccsch(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6435, 457, 95, 210}

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(Sqrt[d]*e*Sqrt[-(c^2*x^2)])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{-c^2x^2}} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{-c^2x^2}} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{b\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-2cx\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2cex\sqrt{d+ex^2}}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2)
)] - 2*c*x*(a + b*ArcCsch[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x(a+b\,\text{arccsch}\,(cx))}{\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.49

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \left[-\frac{4\sqrt{ex^2+d}bd\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+4\sqrt{ex^2+d}ad-(bex^2+bd)\sqrt{d}\log\left(\frac{(c^4d\right.}{4\left(de^2x^2+d^2\right.}\right.$$

$$\left.-\frac{2\sqrt{ex^2+d}bd\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+2\sqrt{ex^2+d}ad+(bex^2+bd)\sqrt{-d}\arctan\left(\frac{((c^3d+ce)x^3+2\,cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{\frac{c}{2\left(c^2dex^4+(c^2d^2+de)x^2+d^2\right)}}\right.}{2\left(de^2x^2+d^2e\right)}\right]$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 + 6*c^
2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*
sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4))/(d*e^2
*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2
```

```
*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d + (b*e*x^2 + b*d)*sqrt(-d)*arcta
n(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2
+ 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)))/(d*e^2*x^2 + d^
2*e)]
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x(a+b\,\operatorname{acsch}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -(c^2*integrate(x/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2
*e*x^2 + e)*sqrt(e*x^2 + d)), x) + log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x^2 +
 d)*e) + integrate(((e*log(c) - e)*c^2*x^3 - (c^2*d - e*log(c))*x + (c^2*e*
x^3 + e*x)*log(x))/((c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(e*x^2 +
d)), x))*b - a/(sqrt(e*x^2 + d)*e)
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.150 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1118
Rubi [N/A] . . . 1118
Mathematica [N/A] . . . 1119
Maple [N/A] (verified) . . . 1119
Fricas [N/A] . . . 1119
Sympy [N/A] . . . 1120
Maxima [F(-2)] . . . 1120
Giac [N/A] . . . 1120
Mupad [N/A] . . . 1121

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

Sympy [N/A]

Not integrable

Time = 70.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x\left(e\,x^2 + d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1122
Rubi [N/A] . . . 1122
Mathematica [N/A] . . . 1123
Maple [N/A] (verified) . . . 1123
Fricas [N/A] . . . 1123
Sympy [F(-1)] . . . 1124
Maxima [F(-2)] . . . 1124
Giac [N/A] . . . 1124
Mupad [N/A] . . . 1125

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\text{arccsch}\left(cx\right)}{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(3/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 5.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^3 \left(d + ex^2\right)^{3/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^3 \left(e\,x^2 + d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.152 $\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1126
Rubi [N/A] . . . 1126
Mathematica [N/A] . . . 1127
Maple [N/A] (verified) . . . 1127
Fricas [N/A] . . . 1127
Sympy [N/A] . . . 1128
Maxima [F(-2)] . . . 1128
Giac [N/A] . . . 1128
Mupad [N/A] . . . 1129

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\text{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [N/A]

Not integrable

Time = 134.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4(a+b\,\operatorname{acsch}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**4*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 6.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.153 $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1130
Rubi [N/A] . . . 1130
Mathematica [N/A] . . . 1131
Maple [N/A] (verified) . . . 1131
Fricas [N/A] . . . 1131
Sympy [N/A] . . . 1132
Maxima [F(-2)] . . . 1132
Giac [N/A] . . . 1132
Mupad [N/A] . . . 1133

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \text{arccsch}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\left(cx\right)+a)x^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [N/A]

Not integrable

Time = 33.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^2(a+b\,\operatorname{acsch}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^2\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.154 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result . . . 1134
Rubi [A] (verified) . . . 1134
Mathematica [A] (verified) . . . 1135
Maple [F] . . . 1136
Fricas [A] (verification not implemented) . . . 1136
Sympy [F] . . . 1136
Maxima [F] . . . 1136
Giac [F] . . . 1137
Mupad [F(-1)] . . . 1137

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

```
[Out] x*(a+b*arccsch(c*x))/d/(e*x^2+d)^(1/2)-b*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1
)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/
d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {197, 6427, 12, 429}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[d + e*x^2]*Ellipti
cF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sq
rt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{d\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}} \\ &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}} \\ &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} \\ &+ \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-c^2}x\right), \frac{e}{c^2d}\right)}{\sqrt{-c^2}d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x
*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[Sqrt[-c^2]*x], e/(c^2*d)])/(Sqrt[-c^2
]*d*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \frac{\sqrt{ex^2 + d}bc^2dx \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + \sqrt{ex^2 + d}ac^2dx - (bex^2 + bd)\sqrt{-c^2}\sqrt{d}F(\arcsin}{c^2d^2ex^2 + c^2d^3}$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] (sqrt(e*x^2 + d)*b*c^2*d*x*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + sqrt(e*x^2 + d)*a*c^2*d*x - (b*e*x^2 + b*d)*sqrt(-c^2)*sqrt(d)*ellipti
c_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)))/(c^2*d^2*e*x^2 + c^2*d^3)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{a + b\operatorname{acsch}(cx)}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{b\operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x) + a*
x/(sqrt(e*x^2 + d)*d)
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(3/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.155 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result . . . 1138
Rubi [A] (verified) . . . 1139
Mathematica [C] (verified) . . . 1142
Maple [F] . . . 1142
Fricas [A] (verification not implemented) . . . 1142
Sympy [F] . . . 1143
Maxima [F(-2)] . . . 1143
Giac [F] . . . 1143
Mupad [F(-1)] . . . 1144

Optimal result

Integrand size = 23, antiderivative size = 321

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{2bex\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

```
[Out] (-a-b*arccsch(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsch(c*x))/d^2/(e*x^2
+d)^(1/2)+b*c^3*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
+b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)-b*c^2*x*(1/(c^
2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/
d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d
)/d/(c^2*x^2+1))^(1/2)+2*b*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*Elli
pticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^
2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {277, 197, 6437, 12, 597, 545, 429, 506, 422}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}$$
$$-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} + \frac{2bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$
$$-\frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$
$$+\frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*
c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]) - (a + b*ArcCs
ch[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsch[c*x]))/(d^2*Sqrt[d +
 e*x^2]) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/
(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))
]) + (2*b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^3*S
qrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-d-2ex^2}{d^2x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-d-2ex^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} \\
&\quad - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2de-c^2dex^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} \\
&\quad + \frac{(2bcex)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{-c^2x^2}} + \frac{(bc^3ex)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{-c^2x^2}} \\
&= \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} \\
&\quad + \frac{2bex\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{(-1-c^2x^2)^{3/2}}\,dx}{d^2\sqrt{-c^2x^2}} \\
&= \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} \\
&\quad - \frac{2ex\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{2bex\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx), 1-\frac{e}{c^2d}\right)}{d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.63

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\operatorname{csch}^{-1}(cx)}{d^2x\sqrt{d+ex^2}} + \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+(-c^2d+2e)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right),\frac{e}{c^2d}\right)\right)}{\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^2
)*ArcCsch[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
rt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-(
c^2*d) + 2*e)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\operatorname{arccsch}(cx)}{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.85

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx =$$

$$-\frac{(bc^4dex^3+bc^4d^2x)\sqrt{-c^2}\sqrt{d}E(\arcsin\left(\sqrt{-c^2}x\right)|\frac{e}{c^2d})-((bc^4de+2\,be^2)x^3+(bc^4d^2+2\,bde)x)\sqrt{-c^2}\sqrt{d}F(\text{ar}}{}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^4*d*e*x^3 + b*c^4*d^2*x)*sqrt(-c^2)*sqrt(d)*elliptic_e(arcsin(sqrt(-
c^2)*x), e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*x^3 + (b*c^4*d^2 + 2*b*d*e)*x)
*sqrt(-c^2)*sqrt(d)*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + (2*b*c^2*
```

```
d*e*x^2 + b*c^2*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 1)/(c*x)) + (2*a*c^2*d*e*x^2 + a*c^2*d^2 - (b*c^3*d*e*x^3 + b*c^3*d^2*x)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d^3*e*x^3 + c^2*d^4*x
)
```

Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}}\,dx = \int \frac{a + b\,\operatorname{acsch}\left(cx\right)}{x^2 \left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}}\,dx = \int \frac{b\,\operatorname{arcsch}\left(cx\right) + a}{\left(ex^2 + d\right)^{\frac{3}{2}} x^2}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2\left(e\,x^2 + d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.156 $\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result . . . 1145
Rubi [A] (verified) . . . 1145
Mathematica [C] (verified) . . . 1149
Maple [F] . . . 1150
Fricas [B] (verification not implemented) . . . 1150
Sympy [F(-1)] . . . 1151
Maxima [F(-2)] . . . 1151
Giac [F] . . . 1152
Mupad [F(-1)] . . . 1152

Optimal result

Integrand size = 23, antiderivative size = 251

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{-c^2x^2}} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$

```
[Out] -1/3*d^2*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctan(e^(1/2)*(-c^2*x^
2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(-c^2*x^2)^(1/2)+8/3*b*c*x*arctan((e*
x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(-c^2*x^2)^(1/2)+2*d*(
a+b*arccsch(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(-c^2*x^2-1)^(1/2)/(c^2*d
-e)/e^2/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)
/e^3
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {272, 45, 6437, 12, 1628, 163, 65, 223, 209, 95, 210}

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}$$
$$+ \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{-c^2x^2}}$$
$$+ \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^3\sqrt{-c^2x^2}} + \frac{bcdx\sqrt{-c^2x^2-1}}{3e^2\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*x*Sqrt[-1 - c^2*x^2])/(3*(c^2*d - e)*e^2*Sqrt[-(c^2*x^2)]*Sqrt[d + e
*x^2]) - (d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b
*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x
]))/e^3 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(e
^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
```

```
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\int \frac{8d^2+12dex^2+3e^2x^4}{3e^3x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\int \frac{8d^2+12dex^2+3e^2x^4}{x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^3\sqrt{-c^2x^2}} \\
&= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{8d^2+12dex+3e^2x^2}{x\sqrt{-1-c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e^3\sqrt{-c^2x^2}} \\
&= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{4d^2\left(c^2d-e\right)+\frac{3}{2}d\left(c^2d-e\right)ex}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3d\left(c^2d-e\right)e^3\sqrt{-c^2x^2}} \\
&= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{(4bcdx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^3\sqrt{-c^2x^2}} \\
&\quad - \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e^2\sqrt{-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{(8bcdx)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}} \\
&\quad + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1-c^2x^2}\right)}{ce^2\sqrt{-c^2x^2}} \\
&= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} \\
&\quad + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}} + \frac{(bx)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{ce^2\sqrt{-c^2x^2}} \\
&= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} \\
&\quad + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{-c^2x^2}} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96

$$
\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{\frac{2bcde\sqrt{1+\frac{1}{c^2x^2}}x\left(d+ex^2\right)}{c^2d-e} + 2a\left(8d^2+12dex^2+3e^2x^4\right) + bc\left(d+ex^2\right)\left(-\frac{8d\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1\right.}{c^2}\right.}
$$

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((2*b*c*d*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d - e) + 2*a*(8*d^2 +
 12*d*e*x^2 + 3*e^2*x^4) + (b*c*(d + e*x^2)*((-8*d*Sqrt[1 + d/(e*x^2)]*Appe
llF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))])/c^2 + (3*e*Sqrt[1 + 1/(
c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e
*x^2)/d)])/Sqrt[1 + c^2*x^2]))/x + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*Arc
Csch[c*x])/(6*e^3*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(213) = 426$.

Time $= 0.48$ (sec) , antiderivative size $= 2421$, normalized size of antiderivative $= 9.65$

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e
- b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*
e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt
(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3
*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2
 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(b*c^3*d^3 - b
*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*s
qrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c
^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e
^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e
*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 - c*d^2*e^
4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), -1/6*(3*(b*c^
2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2
)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e
)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e))
- 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 1)/(c*x)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 +
2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x
^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d
)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(8*a*c^3*d^3 - 8*
a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^
2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e
*x^2 + d))/(c^3*d^3*e^3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*
e^4 - c*d*e^5)*x^2), 1/12*(16*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e
```

```
^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*
e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c
^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*
e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x
^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^
4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2
) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^
3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 1)/(c*x)) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3
)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 - c*d^2*e^4
+ (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), 1/6*(8*(b*c^3*d
^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*
x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt
(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2
)) - 3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e -
b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2
+ d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*
x^2 + d*e)) + 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4
+ 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2
 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c
^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 -
c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^5}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^5\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^3\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result . . . 1153
Rubi [A] (verified) . . . 1153
Mathematica [C] (verified) . . . 1156
Maple [F] . . . 1156
Fricas [B] (verification not implemented) . . . 1156
Sympy [F(-1)] . . . 1157
Maxima [F] . . . 1157
Giac [F] . . . 1158
Mupad [F(-1)] . . . 1158

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}}$$

```
[Out] 1/3*d*(a+b*arccsch(c*x))/e^2/(e*x^2+d)^(3/2)-2/3*b*c*x*arctan((e*x^2+d)^(1/
2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(-c^2*x^2)^(1/2)+(-a-b*arccsch(c
*x))/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(-c^2*x^2-1)^(1/2)/(c^2*d-e)/e/(-c^2*x^2
)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {272, 45, 6437, 12, 587, 157, 95, 210}

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}} - \frac{bcx\sqrt{-c^2x^2-1}}{3e\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x*Sqrt[-1 - c^2*x^2])/((c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt[d + e*
x^2]) + (d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsch
[c*x])/(e^2*Sqrt[d + e*x^2]) - (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqr
t[-1 - c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{3e^2x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{-c^2x^2}} \\
&= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^2\sqrt{-c^2x^2}} \\
&= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\operatorname{Subst}\left(\int \frac{-2d-3ex}{x\sqrt{-1-c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^2\sqrt{-c^2x^2}} \\
&= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{d(c^2d-e)}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3d\left(c^2d-e\right)e^2\sqrt{-c^2x^2}} \\
&= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^2\sqrt{-c^2x^2}} \\
&= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} \\
&\quad - \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(2bcx)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3e^2\sqrt{-c^2x^2}}
\end{aligned}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$
$$- \frac{a+b\operatorname{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx =$$
$$-\frac{\frac{bce\sqrt{1+\frac{1}{c^2x^2}}x\left(d+ex^2\right)}{c^2d-e} + a(2d+3ex^2) - \frac{b\sqrt{1+\frac{d}{ex^2}}\left(d+ex^2\right)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + b(2d+3ex^2)\operatorname{csch}^{-1}(cx)}{3e^2\left(d+ex^2\right)^{3/2}}$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] -1/3*((b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d - e) + a*(2*d + 3*
e*x^2) - (b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, -(1/(c
^2*x^2)), -(d/(e*x^2))])/(c*x) + b*(2*d + 3*e*x^2)*ArcCsch[c*x])/(e^2*(d +
e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^3(a+b\ \operatorname{arccsch}(cx))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(143) = 286.

Time = 0.37 (sec) , antiderivative size = 786, normalized size of antiderivative = 4.65

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \left[-\frac{2\left(2\,bc^2d^3-2\,bd^2e+3\left(bc^2d^2e-bde^2\right)x^2\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (bc^2\right.$$
$$\left.-\frac{\left(bc^2d^3+\left(bc^2de^2-be^3\right)x^4-bd^2e+2\left(bc^2d^2e-bde^2\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\left(\left(c^3d+ce\right)x^3+2\,cdx\right)\sqrt{ex^2+d}\sqrt{-d}\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\left(c^2dex^4+\left(c^2d^2+de\right)x^2+d^2\right)}\right)+}{3\left(c^2d\right.}\right.$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (
b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(d)
*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d +
 c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 8*d^2)/x^4) + 2*(2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2
+ (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 +
 d))/(c^2*d^4*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3 -
d^2*e^4)*x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(
b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*
x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c
^2*d^2 + d*e)*x^2 + d^2)) + (2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d
*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + (2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c*d*e^2*
x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d^4
*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3 - d^2*e^4)*x^2)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arcsch}\left(cx\right)+a\right)x^3}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*inte
grate(x^3*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^3\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result . . . 1159
Rubi [A] (verified) . . . 1159
Mathematica [C] (verified) . . . 1161
Maple [F] . . . 1161
Fricas [B] (verification not implemented) . . . 1162
Sympy [F(-1)] . . . 1162
Maxima [F] . . . 1163
Giac [F] . . . 1163
Mupad [F(-1)] . . . 1163

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}$$

```
[Out] 1/3*(-a-b*arccsch(c*x))/e/(e*x^2+d)^(3/2)-1/3*b*c*x*arctan((e*x^2+d)^(1/2)/
d^(1/2)/(-c^2*x^2-1)^(1/2))/d^(3/2)/e/(-c^2*x^2)^(1/2)+1/3*b*c*x*(-c^2*x^2-
1)^(1/2)/d/(c^2*d-e)/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6435, 457, 98, 95, 210}

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}} + \frac{bcx\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x^2
]) - (a + b*ArcCsch[c*x])/(3*e*(d + e*x^2)^(3/2)) - (b*c*x*ArcTan[Sqrt[d +
e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*d^(3/2)*e*Sqrt[-(c^2*x^2)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\text{integral} = -\frac{a + b\text{csch}^{-1}(cx)}{3e\left(d + ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6de\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3de\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{-\frac{2a}{e} + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}x\left(d+ex^2\right)}{d\left(c^2d-e\right)} + \frac{b\sqrt{1+\frac{d}{ex^2}}\left(d+ex^2\right)\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex} - \frac{2b\operatorname{csch}^{-1}(c}{e}}{6\left(d+ex^2\right)^{3/2}}$$

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((-2*a)/e + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2))/(d*(c^2*d - e)) + (
b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)),
-(d/(e*x^2))])/(c*d*e*x) - (2*b*ArcCsch[c*x])/e)/(6*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x(a+b\ \text{arccsch}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(120) = 240$.

Time = 0.35 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.85

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \left[-\frac{4\left(bc^2d^3-bd^2e\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - \left(bc^2d^3+\left(bc^2de^2-be^3\right)x^4-b\right.}{}\right.$$

$$\left. -\frac{\left(bc^2d^3+\left(bc^2de^2-be^3\right)x^4-bd^2e+2\left(bc^2d^2e-bde^2\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\left(\left(c^3d+ce\right)x^3+2\,cdx\right)\sqrt{ex^2+d}\sqrt{-d}\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\left(c^2dex^4+\left(c^2d^2+de\right)x^2+d^2\right)}\right)+}{6\left(c^2d^5e-d^4e^2+\left(c^2d^3e^3-d^{2}\right.\right.}\right]$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(b*c^2*d^3 - b*d^2*e)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e
+ 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x
^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d
)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c^2*d^3 - a*d^
2*e - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^
2 - d^3*e^3)*x^2), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e +
 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
+ (c^2*d^2 + d*e)*x^2 + d^2)) + 2*(b*c^2*d^3 - b*d^2*e)*sqrt(e*x^2 + d)*log
((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(a*c^2*d^3 - a*d^2*e -
(b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d
))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^
3*e^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x) -
1/3*a/((e*x^2 + d)^(3/2)*e)
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1164
Rubi [N/A] 1164
Mathematica [N/A] 1165
Maple [N/A] (verified) 1165
Fricas [N/A] 1165
Sympy [F(-1)] 1166
Maxima [F(-2)] 1166
Giac [N/A] 1166
Mupad [N/A] 1167

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x\left(e\,x^2 + d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.160 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1168
Rubi [N/A] 1168
Mathematica [N/A] 1169
Maple [N/A] (verified) 1169
Fricas [N/A] 1169
Sympy [F(-1)] 1170
Maxima [F(-2)] 1170
Giac [N/A] 1170
Mupad [N/A] 1171

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 23.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\text{arccsch}\left(cx\right)}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^3\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{a + b\,\operatorname{asinh}\left(\frac{1}{cx}\right)}{x^3\left(e\,x^2 + d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.161 $\int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result . . . 1172
Rubi [N/A] . . . 1172
Mathematica [N/A] . . . 1173
Maple [N/A] (verified) . . . 1173
Fricas [N/A] . . . 1173
Sympy [F(-1)] . . . 1174
Maxima [F(-2)] . . . 1174
Giac [N/A] . . . 1174
Mupad [N/A] . . . 1175

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 17.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6(a+b\ \text{arccsch}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\left(cx\right)+a)x^6}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^6*arccsch(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)x^6}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^6\left(a+b\,\text{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.162 $\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result . . . 1176
Rubi [N/A] . . . 1176
Mathematica [N/A] . . . 1177
Maple [N/A] (verified) . . . 1177
Fricas [N/A] . . . 1177
Sympy [F(-1)] . . . 1178
Maxima [F(-2)] . . . 1178
Giac [N/A] . . . 1178
Mupad [N/A] . . . 1179

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \operatorname{Int}\left(\frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\ \text{arccsch}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\left(cx\right)+a)x^4}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)x^4}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 5.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^4\left(a+b\,\text{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.163 $\int \frac{x^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result 1180
Rubi [A] (verified) 1181
Mathematica [A] (verified) 1183
Maple [F] 1184
Fricas [A] (verification not implemented) 1184
Sympy [F(-1)] 1184
Maxima [F] 1185
Giac [F] 1185
Mupad [F(-1)] 1185

Optimal result

Integrand size = 23, antiderivative size = 359

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc^2x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{bx\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

```
[Out] 1/3*x^3*(a+b*arccsch(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(-c^2*x^2-1)^(1/2)
/d/(c^2*d-e)/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+1/3*b*c^3*x^2*(e*x^2+d)^(1/2)
/d/(c^2*d-e)/e/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/3*b*c^2*x*(1/(c^2*x^2+
1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/
2))*(e*x^2+d)^(1/2)/d/(c^2*d-e)/e/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x
^2+d)/d/(c^2*x^2+1))^(1/2)+1/3*b*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*
EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(c^2
*d-e)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {270, 6437, 12, 482, 433, 429, 506, 422}

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$
$$+\frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$
$$-\frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\left|1-\frac{e}{c^2d}\right.\right)}{3de\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$
$$+\frac{bcx^2\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3de\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)}$$

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x^2*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x
^2]) + (b*c^3*x^2*Sqrt[d + e*x^2])/(3*d*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2]) + (x^3*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c
^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d*(c^2*d - e
)*e*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]
) + (b*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^2*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{x^3\left(a + b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d + ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{x^2}{3d\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{x^2}{\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d\sqrt{-c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d\left(-c^2d+e\right)\sqrt{-c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d\left(-c^2d+e\right)\sqrt{-c^2x^2}} - \frac{\left(bc^3x\right)\int \frac{x^2}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d\left(-c^2d+e\right)\sqrt{-c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{bx\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} - \frac{\left(bc^3x\right)\int \frac{\sqrt{d+ex^2}}{\left(-1-c^2x^2\right)^{3/2}}\,dx}{3de\left(-c^2d+e\right)\sqrt{-c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{bx\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.53

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x^2\left(a(c^2d-e)\,x+bc\sqrt{1+\frac{1}{c^2x^2}}(d+ex^2)+b(c^2d-e)\,x\operatorname{csch}^{-1}(cx)\right)}{3d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|\frac{c^2d}{e}\right)}{3d\left(c^2d-e\right)\sqrt{-\frac{e}{d}}\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d - e)*x + b*c*Sqrt[1 + 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d -
e)*x*ArcCsch[c*x]))/(3*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 + 1/(
c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], (c^2*d)/e
])/(3*d*(c^2*d - e)*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.03

$$\int \frac{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{(bc^4d^2e-bc^2de^2)\sqrt{ex^2+d}x^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(bc^4de^2x^4+2\,bc^4d^2ex^2+bc^4$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((b*c^4*d^2*e - b*c^2*d*e^2)*sqrt(e*x^2 + d)*x^3*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4
*d^3)*sqrt(-c^2)*sqrt(d)*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + (b*c
^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^4*d^2*e + b*d*e^2)*x^
2)*sqrt(-c^2)*sqrt(d)*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + ((a*c^4
*d^2*e - a*c^2*d*e^2)*x^3 + (b*c^3*d*e^2*x^4 + b*c^3*d^2*e*x^2)*sqrt((c^2*x
^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^4*d^5*e - c^2*d^4*e^2 + (c^4*d^3*e^
3 - c^2*d^2*e^4)*x^4 + 2*(c^4*d^4*e^2 - c^2*d^3*e^3)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{x^2\left(a+b\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.164 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result . . . 1186
Rubi [A] (verified) . . . 1187
Mathematica [C] (verified) . . . 1189
Maple [F] . . . 1189
Fricas [A] (verification not implemented) . . . 1190
Sympy [F(-1)] . . . 1190
Maxima [F] . . . 1190
Giac [F] . . . 1191
Mupad [F(-1)] . . . 1191

Optimal result

Integrand size = 20, antiderivative size = 278

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{bc\sqrt{e}x\sqrt{-1-c^2x^2}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{\frac{d(1+c^2x^2)}{d+ex^2}}\sqrt{d+ex^2}} - \frac{b(3c^2d-2e)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^3\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

```
[Out] 1/3*x*(a+b*arccsch(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsch(c*x))/d^2/(e*
x^2+d)^(1/2)-1/3*b*c*x*(1/(1+e*x^2/d))^(1/2)*(1+e*x^2/d)^(1/2)*EllipticE(x*
e^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2),(1-c^2*d/e)^(1/2))*e^(1/2)*(-c^2*x^2-1)^(
1/2)/d^(3/2)/(c^2*d-e)/(-c^2*x^2)^(1/2)/(d*(c^2*x^2+1)/(e*x^2+d))^(1/2)/(e*
x^2+d)^(1/2)-1/3*b*(3*c^2*d-2*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*
EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(c^2
*d-e)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {198, 197, 6427, 12, 539, 429, 422}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc\sqrt{e}x\sqrt{-c^2x^2-1}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}\sqrt{\frac{d(c^2x^2+1)}{d+ex^2}}} - \frac{bx(3c^2d-2e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsch[c*x]
))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*Sqrt[e]*x*Sqrt[-1 - c^2*x^2]*EllipticE[Ar
cTan[(Sqrt[e]*x)/Sqrt[d]], 1 - (c^2*d)/e])/(3*d^(3/2)*(c^2*d - e)*Sqrt[-(c^
2*x^2)]*Sqrt[(d*(1 + c^2*x^2))/(d + e*x^2)]*Sqrt[d + e*x^2]) - (b*(3*c^2*d
 - 2*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^3*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{3d+2ex^2}{3d^2\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{-c^2x^2}} \\
&= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{3d+2ex^2}{\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d^2\sqrt{-c^2x^2}} \\
&= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} \\
&\quad - \frac{\left(bc(3c^2d-2e)\,x\right)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}} - \frac{(bcex)\int \frac{\sqrt{-1-c^2x^2}}{\left(d+ex^2\right)^{3/2}}\,dx}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}}
\end{aligned}$$

$$= \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}$$
$$- \frac{bc\sqrt{e}x\sqrt{-1-c^2x^2}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{\frac{d(1+c^2x^2)}{d+ex^2}}\sqrt{d+ex^2}}$$
$$- \frac{b(3c^2d-2e)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^3\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.87 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.89

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(-bce\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d-e)\left(3d+2ex^2\right)+b(c^2d-e)\left(3d+2ex^2\right)\csc\right.}{3d^2\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}}$$
$$- \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+2(c^2d-e)\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right),\frac{e}{c^2d}\right)\right)}{3\sqrt{c^2}d^2\left(c^2d-e\right)\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d - e)*(3*d + 2*e
*x^2) + b*(c^2*d - e)*(3*d + 2*e*x^2)*ArcCsch[c*x]))/(3*d^2*(c^2*d - e)*(d
+ e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c
^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + 2*(c^2*d - e)*EllipticF
[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*(c^2*d - e)*Sqrt[1 + c
^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.59

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(bc^4de^2x^4+2\,bc^4d^2ex^2+bc^4d^3)\sqrt{-c^2}\sqrt{d}E(\arcsin\left(\sqrt{-c^2}x\right)\mid\frac{e}{c^2d})-(((bc^4+3\,bc^2)de$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*sqrt(-c^2)*sqrt(d)*e
lliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - (((b*c^4 + 3*b*c^2)*d*e^2 - 2*
b*e^3)*x^4 + (b*c^4 + 3*b*c^2)*d^3 - 2*b*d^2*e + 2*((b*c^4 + 3*b*c^2)*d^2*e
 - 2*b*d*e^2)*x^2)*sqrt(-c^2)*sqrt(d)*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c
^2*d)) + (2*(b*c^4*d^2*e - b*c^2*d*e^2)*x^3 + 3*(b*c^4*d^3 - b*c^2*d^2*e)*x
)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*(
a*c^4*d^2*e - a*c^2*d*e^2)*x^3 + 3*(a*c^4*d^3 - a*c^2*d^2*e)*x - (b*c^3*d*e
^2*x^4 + b*c^3*d^2*e*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(
c^4*d^6 - c^2*d^5*e + (c^4*d^4*e^2 - c^2*d^3*e^3)*x^4 + 2*(c^4*d^5*e - c^2*
d^4*e^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.165 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal result 1192
Rubi [A] (verified) 1193
Mathematica [A] (verified) 1197
Maple [F] 1198
Fricas [F] 1198
Sympy [F(-1)] 1198
Maxima [F] 1199
Giac [F] 1199
Mupad [F(-1)] 1200

Optimal result

Integrand size = 23, antiderivative size = 596

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{be\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)x}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}}$$
$$-\frac{be^2(e(5+m)^2-3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}}$$
$$+\frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(7+m)}$$
$$-\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}-\frac{e(1+m)\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(3+m)(5+m)(7+m)}\right)x(}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{-c^2x^2}\sqrt{-1-}}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsch
(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsch(c*x))/f^5/(5+m)+e^3*(f*x)
^(7+m)*(a+b*arccsch(c*x))/f^7/(7+m)+b*e*(e^2*(m^2+8*m+15)^2-3*c^2*d*e*(3+m)
^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(-
c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(-c^2*x^2)^(
1/2)-b*e^2*(e*(5+m)^2-3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(-c^2*x^2-1)^(1/
2)/c^3/f^3/(4+m)/(6+m)/(m^2+12*m+35)/(-c^2*x^2)^(1/2)+b*e^3*x*(f*x)^(5+m)*(
-c^2*x^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(-c^2*x^2)^(1/2)-b*(c^6*d^3*(2+m)*(4+m)
*(6+m)/(1+m)-e*(1+m)*(e^2*(m^2+8*m+15)^2-3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*
```

```
c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m
)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c^5/f/
(1+m)/(2+m)/(4+m)/(6+m)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 6437, 1823, 1281, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}$$
$$+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$
$$+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}+\frac{be^3x\sqrt{-c^2x^2-1}(fx)^{m+5}}{cf^5(m+6)(m+7)\sqrt{-c^2x^2}}$$
$$-\frac{be^2x\sqrt{-c^2x^2-1}(fx)^{m+3}\left(e(m+5)^2-3c^2d(m^2+13m+42)\right)}{c^3f^3(m+4)(m+5)(m+6)(m+7)\sqrt{-c^2x^2}}$$
$$-\frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{d^3}{(m+1)^2}-\frac{e\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)-3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2\right)}{c^6(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}\right.}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$
$$+\frac{bex\sqrt{-c^2x^2-1}(fx)^{m+1}\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)-3c^2de(m+3)^2\left(m^2+13m+42\right.\right.}{c^5f(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)\sqrt{-c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 - c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^
2)]) - (b*e^2*(e*(5 + m)^2 - 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqr
t[-1 - c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^2)])
 + (b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 - c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[-
(c^2*x^2)]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (3*d^2
*e*(f*x)^(3 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 +
m)*(a + b*ArcCsch[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsch[
c*x]))/(f^7*(7 + m)) - (b*c*(d^3/(1 + m)^2 - (e*(e^2*(15 + 8*m + m^2)^2 - 3
*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 2
2*m^3 + m^4)))/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*
x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -
(c^2*x^2)])/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{3de^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} + \frac{e^3(fx)^{7+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad - \frac{(bcx)\int \frac{(fx)^m\left(\frac{d^3}{1+m}+\frac{3d^2ex^2}{3+m}+\frac{3de^2x^4}{5+m}+\frac{e^3x^6}{7+m}\right)}{\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{3de^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} + \frac{e^3(fx)^{7+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad + \frac{(bx)\int \frac{(fx)^m\left(-\frac{c^2d^3(6+m)}{1+m}-\frac{3c^2d^2e(6+m)x^2}{3+m}+\frac{e^2\left(e(5+m)^2-3c^2d\left(42+13m+m^2\right)\right)x^4}{(5+m)(7+m)}\right)}{\sqrt{-1-c^2x^2}}\,dx}{c(6+m)\sqrt{-c^2x^2}} \\
&= -\frac{be^2\left(e(5+m)^2-3c^2d(42+13m+m^2)\right)x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{3de^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} + \frac{e^3(fx)^{7+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad - \frac{(bx)\int \frac{(fx)^m\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)x^2}{(3+m)(5+m)(7+m)}\right)}{\sqrt{-1-c^2x^2}}\,dx}{c^3(4+m)(6+m)\sqrt{-c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{be\left(e^2(15+8m+m^2)^2 - 3c^2de(3+m)^2\,(42+13m+m^2) + 3c^4d^2(840+638m+179m^2+22m^3+\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad - \frac{be^2(e(5+m)^2 - 3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{3de^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} + \frac{e^3(fx)^{7+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad - \frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m} - \frac{e(1+m)\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right)\right.}{c^3(4+m)(6+m)\sqrt{-c^2x^2}} \\
&= \frac{be\left(e^2(15+8m+m^2)^2 - 3c^2de(3+m)^2\,(42+13m+m^2) + 3c^4d^2(840+638m+179m^2+22m^3+\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad - \frac{be^2(e(5+m)^2 - 3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{3de^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} + \frac{e^3(fx)^{7+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad - \frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m} - \frac{e(1+m)\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right)\right.}{c^3(4+m)(6+m)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{be\left(e^2(15+8m+m^2)^2 - 3c^2de(3+m)^2(42+13m+m^2) + 3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad - \frac{be^2(e(5+m)^2 - 3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{3de^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} + \frac{e^3(fx)^{7+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad - \frac{b\left(\frac{c^6d^3}{(1+m)^2} - \frac{e\left(e^2(15+8m+m^2)^2 - 3c^2de(3+m)^2(42+13m+m^2) + 3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}\right)x(fx)^{1+m}\sqrt{}}{c^5f\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.66

$$
\begin{aligned}
&\int (fx)^m\left(d+ex^2\right)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\
&= x(fx)^m\left(\frac{ad^3}{1+m} + \frac{3ad^2ex^2}{3+m} + \frac{3ade^2x^4}{5+m} + \frac{ae^3x^6}{7+m} + \frac{bd^3\operatorname{csch}^{-1}(cx)}{1+m} + \frac{3bd^2ex^2\operatorname{csch}^{-1}(cx)}{3+m}\right. \\
&\qquad + \frac{3bde^2x^4\operatorname{csch}^{-1}(cx)}{5+m} + \frac{be^3x^6\operatorname{csch}^{-1}(cx)}{7+m} \\
&\qquad + \frac{bcd^3\sqrt{1+\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)^2\sqrt{1+c^2x^2}} \\
&\qquad + \frac{3bcd^2e\sqrt{1+\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(3+m)^2\sqrt{1+c^2x^2}} \\
&\qquad + \frac{3bcde^2\sqrt{1+\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},-c^2x^2\right)}{(5+m)^2\sqrt{1+c^2x^2}} \\
&\qquad \left. + \frac{bce^3\sqrt{1+\frac{1}{c^2x^2}}x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},-c^2x^2\right)}{(7+m)^2\sqrt{1+c^2x^2}}\right)
\end{aligned}
$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5 +
 m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcCsch[c*x])/(1 + m) + (3*b*d^2*e*x^2*A
```

```
rcCsch[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcCsch[c*x])/(5 + m) + (b*e^3*x^6*Ar
cCsch[c*x])/(7 + m) + (b*c*d^3*Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)^2*Sqrt[1 + c^2*x^2]) + (3*b*
c*d^2*e*Sqrt[1 + 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)
/2, -(c^2*x^2)])/((3 + m)^2*Sqrt[1 + c^2*x^2]) + (3*b*c*d*e^2*Sqrt[1 + 1/(c
^2*x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, -(c^2*x^2)])/((5
+ m)^2*Sqrt[1 + c^2*x^2]) + (b*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x^7*Hypergeometr
ic2F1[1/2, (7 + m)/2, (9 + m)/2, -(c^2*x^2)])/((7 + m)^2*Sqrt[1 + c^2*x^2])
)
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 \left(a+b\,\operatorname{arccsch}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\mathrm{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^3 (b\,\operatorname{arcsch}\left(cx\right)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\mathrm{csch}^{-1}(cx)\right)\, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^3 (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x
^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) - (((m^3 + 9*m^2 + 23*m +
15)*b*e^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5 + 3*(m^3 +
 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^
m*x)*x^m*log(x) - ((m^3 + 9*m^2 + 23*m + 15)*b*e^3*f^m*x^7 + 3*(m^3 + 11*m^
2 + 31*m + 21)*b*d*e^2*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x
^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^m*x)*x^m*log(sqrt(c^2*x^2 + 1) + 1
))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + integrate(((m^3 + 9*m^2 + 23*m +
 15)*b*c^2*e^3*f^m*x^8 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^2*d*e^2*f^m*x^6 +
 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*e*f^m*x^4 + (m^3 + 15*m^2 + 71*m +
105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2
+ m^4 + 16*m^3 + 86*m^2 + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 +
m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*x^2 + 1) + 176*m + 105), x) -
 integrate(((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*b*c^2*e^3*f^m*x^8*log(c)
+ (3*(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*d*e^2*f^m*log(c) + (m^4 + 16
*m^3 + 86*m^2 + 176*m + 105)*e^3*f^m*log(c) - (m^3 + 9*m^2 + 23*m + 15)*e^3
*f^m)*b*x^6 + 3*((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*d^2*e*f^m*log(c)
 + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*d*e^2*f^m*log(c) - (m^3 + 11*m^2 +
 31*m + 21)*d*e^2*f^m)*b*x^4 + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*d
^3*f^m*log(c) + 3*(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*d^2*e*f^m*log(c) -
3*(m^3 + 13*m^2 + 47*m + 35)*d^2*e*f^m)*b*x^2 + ((m^4 + 16*m^3 + 86*m^2 + 1
76*m + 105)*d^3*f^m*log(c) - (m^3 + 15*m^2 + 71*m + 105)*d^3*f^m)*b)*x^m/((
m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*
m + 105), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^3 (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (f\,x)^m \left(e\,x^2+d\right)^3 \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*asinh(1/(c*x))), x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result . . . 1201
Rubi [A] (verified) . . . 1202
Mathematica [A] (verified) . . . 1205
Maple [F] . . . 1205
Fricas [F] . . . 1206
Sympy [F] . . . 1206
Maxima [F] . . . 1206
Giac [F] . . . 1207
Mupad [F(-1)] . . . 1207

Optimal result

Integrand size = 23, antiderivative size = 379

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(5+m)} - \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2-2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1+}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{-c^2x^2}\sqrt{-1-}}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsch(c
*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsch(c*x))/f^5/(5+m)-b*e*(e*(3+m)^2-
2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)/(m
^2+5*m+6)/(-c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(-c^2*x^2-1)^(1/2)/c/f^3/(4+
m)/(5+m)/(-c^2*x^2)^(1/2)-b*(c^4*d^2*(2+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(
3+m)^2-2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2
+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c^3/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(
-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 6437, 12, 1281, 470, 372, 371}

$$\begin{aligned}
&\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= \frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)} + \frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)} \\
&\quad + \frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)} + \frac{be^2x\sqrt{-c^2x^2-1}(fx)^{m+3}}{cf^3(m+4)(m+5)\sqrt{-c^2x^2}} \\
&\quad - \frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{e(e(m+3)^2-2c^2d(m^2+9m+20))}{c^4(m+2)(m+3)(m+4)(m+5)} + \frac{d^2}{(m+1)^2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}} \\
&\quad - \frac{bex\sqrt{-c^2x^2-1}(fx)^{m+1}\left(e(m+3)^2-2c^2d(m^2+9m+20)\right)}{c^3f(m+2)(m+3)(m+4)(m+5)\sqrt{-c^2x^2}}
\end{aligned}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 - 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 - c
^2*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)])) + (b*e^2
*x*(f*x)^(3 + m)*Sqrt[-1 - c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)
]) + (d^2*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3
 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCs
ch[c*x]))/(f^5*(5 + m)) - (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 - 2*c^2*d*(
20 + 9*m + m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sq
rt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(
f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} = &\frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} \\ &+ \frac{e^2(fx)^{5+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(5+m)} \\ &- \frac{(bcx)\int \frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)x^2+e^2(3+4m+m^2)x^4\right)}{(1+m)(3+m)(5+m)\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{d^2(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad + \frac{e^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad - \frac{(bcx)\int \frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)x^2+e^2(3+4m+m^2)x^4\right)}{\sqrt{-1-c^2x^2}}\,dx}{(15+23m+9m^2+m^3)\sqrt{-c^2x^2}} \\
&= \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{2de(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad + \frac{(bx)\int \frac{(fx)^m\left(-c^2d^2(3+m)(4+m)(5+m)+e(1+m)\left(e(3+m)^2-2c^2d(20+9m+m^2)\right)x^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{c(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{-c^2x^2}} \\
&= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{2de(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} - \\
&\quad \frac{\left(b(-c^4d^2(2+m)(3+m)(4+m)(5+m)-e(1+m)^2\left(e(3+m)^2-2c^2d(20+9m+m^2)\right)\right)x\right)\int \frac{}{}}{c^3(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{-c^2x^2}} \\
&= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{2de(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} - \\
&\quad \frac{\left(b(-c^4d^2(2+m)(3+m)(4+m)(5+m)-e(1+m)^2\left(e(3+m)^2-2c^2d(20+9m+m^2)\right)\right)x\sqrt{1}\right.}{c^3(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}} \\
&\quad + \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{2de(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad - \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2-2c^2d(20+9m+m^2)\right))\,x(fx)^{1+}}{c^3f(1+m)(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{-c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.76

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$= x(fx)^m \left(\frac{ad^2}{1+m} + \frac{2adex^2}{3+m} + \frac{ae^2x^4}{5+m} + \frac{bd^2\text{csch}^{-1}(cx)}{1+m} + \frac{2bdex^2\text{csch}^{-1}(cx)}{3+m} \right.$$
$$+ \frac{be^2x^4\text{csch}^{-1}(cx)}{5+m} + \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)^2\sqrt{1+c^2x^2}}$$
$$+ \frac{2bcde\sqrt{1+\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(3+m)^2\sqrt{1+c^2x^2}}$$
$$\left. + \frac{bce^2\sqrt{1+\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},-c^2x^2\right)}{(5+m)^2\sqrt{1+c^2x^2}} \right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
(b*d^2*ArcCsch[c*x])/(1 + m) + (2*b*d*e*x^2*ArcCsch[c*x])/(3 + m) + (b*e^2*
x^4*ArcCsch[c*x])/(5 + m) + (b*c*d^2*Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)^2*Sqrt[1 + c^2*x^2]) +
 (2*b*c*d*e*Sqrt[1 + 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5
+ m)/2, -(c^2*x^2)])/((3 + m)^2*Sqrt[1 + c^2*x^2]) + (b*c*e^2*Sqrt[1 + 1/(c
^2*x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, -(c^2*x^2)])/((5
+ m)^2*Sqrt[1 + c^2*x^2]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\,\text{arccsch}\left(cx\right)\right)dx$$

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (fx)^m \left(a+b\,\text{acsch}\,(cx)\right) \left(d+ex^2\right)^2\, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) - (((m^2 + 4*m + 3)*b*e^2*f^m*x^5 + 2*(m^2 + 6*m + 5)*b*d*e*
f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(x) - ((m^2 + 4*m + 3)*b*e^2
*f^m*x^5 + 2*(m^2 + 6*m + 5)*b*d*e*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*
x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2
 + 4*m + 3)*b*c^2*e^2*f^m*x^6 + 2*(m^2 + 6*m + 5)*b*c^2*d*e*f^m*x^4 + (m^2
+ 8*m + 15)*b*c^2*d^2*f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3
 + 9*m^2 + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15)*sq
rt(c^2*x^2 + 1) + 23*m + 15), x) - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c
^2*e^2*f^m*x^6*log(c) + (2*(m^3 + 9*m^2 + 23*m + 15)*c^2*d*e*f^m*log(c) + (
m^3 + 9*m^2 + 23*m + 15)*e^2*f^m*log(c) - (m^2 + 4*m + 3)*e^2*f^m)*b*x^4 +
((m^3 + 9*m^2 + 23*m + 15)*c^2*d^2*f^m*log(c) + 2*(m^3 + 9*m^2 + 23*m + 15)
*d*e*f^m*log(c) - 2*(m^2 + 6*m + 5)*d*e*f^m)*b*x^2 + ((m^3 + 9*m^2 + 23*m +
 15)*d^2*f^m*log(c) - (m^2 + 8*m + 15)*d^2*f^m)*b)*x^m/((m^3 + 9*m^2 + 23*m
 + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (f\,x)^m \left(e\,x^2+d\right)^2 \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.167 $\int (fx)^m \left(d+ex^2\right) \left(a+b\mathbf{csch}^{-1}(cx)\right) \, dx$

Optimal result 1208
Rubi [A] (verified) 1209
Mathematica [A] (verified) 1211
Maple [F] 1211
Fricas [F] 1212
Sympy [F] 1212
Maxima [F] 1212
Giac [F] 1213
Mupad [F(-1)] 1213

Optimal result

Integrand size = 21, antiderivative size = 220

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}$$
$$+ \frac{b(e(1+m)^2-c^2d(2+m)(3+m))\,x(fx)^{1+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{cf(1+m)^2(2+m)(3+m)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsch(c*x))/f
^3/(3+m)+b*e*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c/f/(m^2+5*m+6)/(-c^2*x^2)^(1
/2)+b*(e*(1+m)^2-c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m
],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(-c^2*x^2
)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 6437, 12, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)} + \frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}$$
$$- \frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{d}{(m+1)^2} - \frac{e}{c^2(m+2)(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, -c^2x^2\right)}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$
$$+ \frac{bex\sqrt{-c^2x^2-1}(fx)^{m+1}}{cf\left(m^2+5m+6\right)\sqrt{-c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[-(c^2*x^
2)]) + (d*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m
)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) - (b*c*(d/(1 + m)^2 - e/(c^2*(2 + m)*
(3 + m)))*x*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
 + 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
  + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad - \frac{(bcx)\int \frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{(1+m)(3+m)\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}} \\
&= \frac{d(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad - \frac{(bcx)\int \frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{\sqrt{-1-c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{-c^2x^2}} \\
&= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} - \frac{\left(bc\left(-\frac{e(1+m)^2}{c^2(2+m)}+d(3+m)\right)x\right)\int \frac{(fx)^m}{\sqrt{-1-c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{-c^2x^2}} \\
&= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad + \frac{e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad - \frac{\left(bc\left(-\frac{e(1+m)^2}{c^2(2+m)}+d(3+m)\right)x\sqrt{1+c^2x^2}\right)\int \frac{(fx)^m}{\sqrt{1+c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}
\end{aligned}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{f^3(3+m)} + \frac{bc\left(\frac{e(1+m)^2}{c^2(2+m)} - d(3+m)\right)x(fx)^{1+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{f(1+m)\left(3+4m+m^2\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int (fx)^m\left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)dx$$

$$= x(fx)^m\left(\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)^2\sqrt{1+c^2x^2}} + \frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{1+m} + \frac{bce\sqrt{1+\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{\sqrt{1+c^2x^2}}}{(3+m)^2}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] x*(f*x)^m*((b*c*d*Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, -(c^2*x^2)])/((1 + m)^2*Sqrt[1 + c^2*x^2]) + (((3 + m)*(d*(3 +
m) + e*(1 + m)*x^2)*(a + b*ArcCsch[c*x]))/(1 + m) + (b*c*e*Sqrt[1 + 1/(c^2*
x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/Sqrt[1
+ c^2*x^2])/(3 + m)^2)
```

Maple [F]

$$\int (fx)^m\left(e\,x^2+d\right)\left(a+b\,\operatorname{arccsch}\left(cx\right)\right)dx$$

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b \operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (fx)^m \left(a+b \operatorname{acsch}(cx)\right) \left(d+ex^2\right) \, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral((f*x)**m*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b \operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) - ((b*e*f^m*(m + 1)
*x^3 + b*d*f^m*(m + 3)*x)*x^m*log(x) - (b*e*f^m*(m + 1)*x^3 + b*d*f^m*(m +
3)*x)*x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^2 + 4*m + 3) + integrate((b*c^2*e*
f^m*(m + 1)*x^4 + b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2 + m
^2 + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x^2 + 1) + 4*m + 3)
, x) - integrate(((m^2 + 4*m + 3)*b*c^2*e*f^m*x^4*log(c) + ((m^2 + 4*m + 3)
*c^2*d*f^m*log(c) + (m^2 + 4*m + 3)*e*f^m*log(c) - e*f^m*(m + 1))*b*x^2 + (
(m^2 + 4*m + 3)*d*f^m*log(c) - d*f^m*(m + 3))*b)*x^m/((m^2 + 4*m + 3)*c^2*x
^2 + m^2 + 4*m + 3), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)(b \operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a+b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m\left(a+b\mathbf{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result 1214
Rubi [N/A] 1214
Mathematica [N/A] 1215
Maple [N/A] (verified) 1215
Fricas [N/A] 1215
Sympy [N/A] 1215
Maxima [N/A] 1216
Giac [N/A] 1216
Mupad [N/A] 1216

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\ \text{arccsch}\left(cx\right)\right)}{e\,x^2 + d}dx$$

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(b\,\text{arcsch}\left(cx\right) + a)(fx)^m}{ex^2 + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 41.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(fx)^m \left(a + b\,\text{acsch}\left(cx\right)\right)}{d + ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{ex^2 + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{ex^2 + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{d + ex^2}\,dx = \int \frac{(f\,x)^m \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d}\,dx$$

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.169 $\int \frac{(fx)^m\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result . . . 1217
Rubi [N/A] . . . 1217
Mathematica [N/A] . . . 1218
Maple [N/A] (verified) . . . 1218
Fricas [N/A] . . . 1218
Sympy [F(-1)] . . . 1218
Maxima [N/A] . . . 1219
Giac [N/A] . . . 1219
Mupad [N/A] . . . 1219

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}, x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\, dx = \int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\, dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\ \text{arccsch}\left(cx\right)\right)}{\left(e\, x^2 + d\right)^2} dx$$

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\, dx = \int \frac{(b\,\text{arcsch}\left(cx\right) + a)(fx)^m}{\left(ex^2 + d\right)^2}\, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{\left(ex^2 + d\right)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{\left(ex^2 + d\right)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2}\,dx = \int \frac{(f\,x)^m \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.170 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx$

Optimal result 1220
Rubi [N/A] 1220
Mathematica [N/A] 1221
Maple [N/A] (verified) 1221
Fricas [N/A] 1221
Sympy [F(-1)] 1221
Maxima [N/A] 1222
Giac [N/A] 1222
Mupad [N/A] 1222

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \text{Int}\left(\left(fx\right)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right), x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx$$

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx$$

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\text{arccsch}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arcsch}\left(cx\right)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arcsch}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.171 $\int (fx)^m \sqrt{d+ex^2}\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal result 1223
Rubi [N/A] 1223
Mathematica [N/A] 1224
Maple [N/A] (verified) 1224
Fricas [N/A] 1224
Sympy [N/A] 1224
Maxima [N/A] 1225
Giac [N/A] 1225
Mupad [N/A] 1225

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Int}\left((fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(a+b\,\text{arccsch}\left(cx\right)\right)\sqrt{e\,x^2+d}dx$$

```
[In] int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\left(cx\right)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 53.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (fx)^m \left(a+b\,\text{acsch}\left(cx\right)\right)\sqrt{d+ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 5.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (f\,x)^m\,\sqrt{e\,x^2+d}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.172 $\int \frac{(fx)^m\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result 1226
Rubi [N/A] 1226
Mathematica [N/A] 1227
Maple [N/A] (verified) 1227
Fricas [N/A] 1227
Sympy [N/A] 1227
Maxima [N/A] 1228
Giac [N/A] 1228
Mupad [N/A] 1228

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a + b\ \text{arccsch}\left(cx\right)\right)}{\sqrt{e\, x^2 + d}} dx$$

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(b\,\text{arcsch}\left(cx\right) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 23.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(fx)^m \left(a + b\,\text{acsch}\left(cx\right)\right)}{\sqrt{d + ex^2}}\, dx$$

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 5.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(f\,x)^m \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.173 $\int \frac{(fx)^m\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result 1229
Rubi [N/A] 1229
Mathematica [N/A] 1230
Maple [N/A] (verified) 1230
Fricas [N/A] 1230
Sympy [N/A] 1231
Maxima [N/A] 1231
Giac [N/A] 1231
Mupad [N/A] 1232

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \operatorname{Int}\left(\frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\, dx = \int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\, dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a + b\ \text{arccsch}\left(cx\right)\right)}{\left(e\, x^2 + d\right)^{\frac{3}{2}}} dx$$

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\, dx = \int \frac{(b\,\text{arcsch}\left(cx\right) + a)(fx)^m}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [N/A]

Not integrable

Time = 146.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{(fx)^m \left(a + b\,\text{acsch}\left(cx\right)\right)}{\left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((f*x)**m*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\left(cx\right) + a)(fx)^m}{\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\left(cx\right) + a)(fx)^m}{\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{(f\,x)^m \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{3/2}}\,dx$$

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.174 $\int \frac{x^{11}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal result . . . 1233
Rubi [A] (verified) . . . 1234
Mathematica [A] (verified) . . . 1239
Maple [F] . . . 1239
Fricas [A] (verification not implemented) . . . 1239
Sympy [F(-1)] . . . 1240
Maxima [F] . . . 1240
Giac [F(-2)] . . . 1240
Mupad [F(-1)] . . . 1241

Optimal result

Integrand size = 26, antiderivative size = 395

$$\begin{aligned}\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = &-\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}\\ &-\frac{13b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{3b\left(1-c^2x^2\right)^{7/2}\sqrt{1+c^2x^2}}{70c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}\\ &-\frac{b\left(1-c^2x^2\right)^{9/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}}\\ &+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}\\ &-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}}\\ &+\frac{4b\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}\end{aligned}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsch(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b
*arccsch(c*x))/c^12+7/90*b*(-c^2*x^2+1)^(3/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1
/c^2/x^2)^(1/2)-13/150*b*(-c^2*x^2+1)^(5/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c
^2/x^2)^(1/2)+3/70*b*(-c^2*x^2+1)^(7/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x
^2)^(1/2)-1/90*b*(-c^2*x^2+1)^(9/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^
(1/2)+4/15*b*arctanh((-c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/
x^2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)
^(1/2)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 6445, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\begin{aligned}\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx &= -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}} \\ &+ \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}} \\ &- \frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}} \\ &+ \frac{4b\sqrt{c^2x^2+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{15c^{13}x\sqrt{\frac{1}{c^2x^2}+1}} \\ &- \frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{9/2}}{90c^{13}x\sqrt{\frac{1}{c^2x^2}+1}} + \frac{3b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{7/2}}{70c^{13}x\sqrt{\frac{1}{c^2x^2}+1}} \\ &- \frac{13b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{5/2}}{150c^{13}x\sqrt{\frac{1}{c^2x^2}+1}} \\ &+ \frac{7b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{3/2}}{90c^{13}x\sqrt{\frac{1}{c^2x^2}+1}} - \frac{4b\sqrt{c^2x^2+1}\sqrt{1-c^2x^2}}{15c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}\end{aligned}$$

```
[In] Int[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x
) + (7*b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqrt[1 + 1/(c^2*x^
2)]*x) - (13*b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(150*c^13*Sqrt[1 + 1/
(c^2*x^2)]*x) + (3*b*(1 - c^2*x^2)^(7/2)*Sqrt[1 + c^2*x^2])/(70*c^13*Sqrt[1
 + 1/(c^2*x^2)]*x) - (b*(1 - c^2*x^2)^(9/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqr
t[1 + 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^12) +
 ((1 - c^4*x^4)^(3/2)*(a + b*ArcCsch[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)
*(a + b*ArcCsch[c*x]))/(10*c^12) + (4*b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 -
c^2*x^2]])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} \\
&\quad - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} + \frac{b\int \frac{\sqrt{1-c^4x^4}\left(-8-4c^4x^4-3c^8x^8\right)}{30c^{12}\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} \\
&\quad - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} + \frac{b\int \frac{\sqrt{1-c^4x^4}\left(-8-4c^4x^4-3c^8x^8\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{30c^{13}} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} \\
&\quad - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}\left(-8-4c^4x^4-3c^8x^8\right)}{x\sqrt{1+c^2x^2}}\,dx}{30c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} \\
&\quad - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}\left(8+4c^4x^2+3c^8x^4\right)}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{60c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}$$
$$- \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}}$$
$$- \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \left(\frac{8\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}} + \frac{4c^4x\sqrt{1-c^4x^2}}{\sqrt{1+c^2x}} + \frac{3c^8x^3\sqrt{1-c^4x^2}}{\sqrt{1+c^2x}}\right)\,dx, x, x^2\right)}{60c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}$$
$$- \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{15c^9\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{x^3\sqrt{1-c^4x^2}}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{20c^5\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}}$$
$$- \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^2x}}{x}\,dx, x, x^2\right)}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x\sqrt{1-c^2x}\,dx, x, x^2\right)}{15c^9\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x^3\sqrt{1-c^2x}\,dx, x, x^2\right)}{20c^5\sqrt{1+\frac{1}{c^2x^2}}x}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} \\
&\quad + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} \\
&\quad - \frac{\left(2b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \left(\frac{\sqrt{1-c^2x}}{c^2} - \frac{\left(1-c^2x\right)^{3/2}}{c^2}\right)\,dx, x, x^2\right)}{15c^{9}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \left(\frac{\sqrt{1-c^2x}}{c^6} - \frac{3\left(1-c^2x\right)^{3/2}}{c^6} + \frac{3\left(1-c^2x\right)^{5/2}}{c^6} - \frac{\left(1-c^2x\right)^{7/2}}{c^6}\right)\,dx, x, x^2\right)}{20c^{5}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{13b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{3b\left(1-c^2x^2\right)^{7/2}\sqrt{1+c^2x^2}}{70c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{b\left(1-c^2x^2\right)^{9/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} \\
&\quad + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} \\
&\quad + \frac{\left(4b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{15c^{15}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{13b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad + \frac{3b\left(1-c^2x^2\right)^{7/2}\sqrt{1+c^2x^2}}{70c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{b\left(1-c^2x^2\right)^{9/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^{12}} \\
&\quad - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{10c^{12}} + \frac{4b\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.54

$$\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(768-36c^2x^2+78c^4x^4-5c^6x^6+35c^8x^8)}{1+c^2x^2}+105b\sqrt{1-c^4x^4}(}{3150}$$

```
[In] Integrate[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
+ 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 - 36*c^2*x^2 + 78*c^4*x^4 - 5*c^6*x
^6 + 35*c^8*x^8))/(1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 +
3*c^8*x^8)*ArcCsch[c*x] + 840*b*Log[x + c^2*x^3] - 840*b*Log[1 + c^2*x^2 +
c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^12
```

Maple [F]

$$\int \frac{x^{11}(a+b\,\text{arccsch}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{105\left(3\,bc^{10}x^{10}+3\,bc^8x^8+4\,bc^6x^6+4\,bc^4x^4+8\,bc^2x^2+8\,b\right)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(35\,bc^9x^9}{}$$

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3150*(105*(3*b*c^10*x^10 + 3*b*c^8*x^8 + 4*b*c^6*x^6 + 4*b*c^4*x^4 + 8*b
*c^2*x^2 + 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + (35*b*c^9*x^9 - 5*b*c^7*x^7 + 78*b*c^5*x^5 - 36*b*c^3*x^3 + 76
8*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 420*(b*c^2*x^2
+ b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
```

```
1)/(c^2*x^2 + 1)) + 420*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 105*(3*a*c^10*x^10
+ 3*a*c^8*x^8 + 4*a*c^6*x^6 + 4*a*c^4*x^4 + 8*a*c^2*x^2 + 8*a)*sqrt(-c^4*x^
4 + 1))/(c^14*x^2 + c^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**11*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*lo
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^
12) - 30*integrate((x^11*log(c) + x^11*log(x))*e^(-1/2*log(c^2*x^2 + 1) - 1
/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 30*integrate(1/30*(3*c^10*x^11 -
 3*c^8*x^9 + 4*c^6*x^7 - 4*c^4*x^5 + 8*c^2*x^3 - 8*x)/(sqrt(c^2*x^2 + 1)*sq
rt(c*x + 1)*sqrt(-c*x + 1)*c^10 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^10), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^{11}\left(a+b\,\operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^11*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^11*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.175 $\int \frac{x^7\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal result . . . 1242
Rubi [A] (verified) . . . 1243
Mathematica [A] (verified) . . . 1247
Maple [F] . . . 1247
Fricas [A] (verification not implemented) . . . 1247
Sympy [F(-1)] . . . 1248
Maxima [F] . . . 1248
Giac [F(-2)] . . . 1248
Mupad [F(-1)] . . . 1249

Optimal result

Integrand size = 26, antiderivative size = 264

$$\begin{aligned}\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx &= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}x}\\ &\quad-\frac{b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8}\\ &\quad+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8}\\ &\quad+\frac{b\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x}\end{aligned}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsch(c*x))/c^8+1/18*b*(-c^2*x^2+1)^(3/2)*(c^
2*x^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/30*b*(-c^2*x^2+1)^(5/2)*(c^2*x^2
+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)+1/3*b*arctanh((-c^2*x^2+1)^(1/2))*(c^2*
x^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)
^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/
c^8
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 6445, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{b\sqrt{c^2x^2+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3c^9x\sqrt{\frac{1}{c^2x^2}+1}} - \frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{5/2}}{30c^9x\sqrt{\frac{1}{c^2x^2}+1}} + \frac{b\sqrt{c^2x^2+1}\left(1-c^2x^2\right)^{3/2}}{18c^9x\sqrt{\frac{1}{c^2x^2}+1}} - \frac{b\sqrt{c^2x^2+1}\sqrt{1-c^2x^2}}{3c^9x\sqrt{\frac{1}{c^2x^2}+1}}$$

```
[In] Int[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
+ (b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(18*c^9*Sqrt[1 + 1/(c^2*x^2)]*x
) - (b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(30*c^9*Sqrt[1 + 1/(c^2*x^2)]
*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/
2)*(a + b*ArcCsch[c*x]))/(6*c^8) + (b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 - c^
2*x^2]])/(3*c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} \\
&\quad + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{6c^8\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{6c^9} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} \\
&\quad + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{x\sqrt{1+c^2x^2}}\,dx}{6c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}\left(2+c^4x^2\right)}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{12c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \left(\frac{2\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}} + \frac{c^4x\sqrt{1-c^4x^2}}{\sqrt{1+c^2x}}\right)\,dx, x, x^2\right)}{12c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{6c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1+c^2x}}\,dx, x, x^2\right)}{12c^5\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^2x}}{x}\,dx, x, x^2\right)}{6c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int x\sqrt{1-c^2x}\,dx, x, x^2\right)}{12c^5\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} \\
&\quad + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{6c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \left(\frac{\sqrt{1-c^2x}}{c^2} - \frac{\left(1-c^2x\right)^{3/2}}{c^2}\right)\,dx, x, x^2\right)}{12c^5\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} \\
&\quad + \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{3c^{11}\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}x} \\
&\quad - \frac{b\left(1-c^2x^2\right)^{5/2}\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} \\
&\quad + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3c^9\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.68

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{15a\sqrt{1-c^4x^4}(2+c^4x^4)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28-c^2x^2+3c^4x^4)}{1+c^2x^2}+15b\sqrt{1-c^4x^4}(2+c^4x^4)\,\text{csch}^{-1}(cx)+30b}{90c^8}$$

```
[In] Integrate[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[1 - c^4*x^4]*(28 - c^2*x^2 + 3*c^4*x^4))/(1 + c^2*x^2) + 15*b*Sqrt[1 -
 c^4*x^4]*(2 + c^4*x^4)*ArcCsch[c*x] + 30*b*Log[x + c^2*x^3] - 30*b*Log[1 +
 c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^8
```

Maple [F]

$$\int \frac{x^7(a+b\,\text{arccsch}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.23

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{15\left(bc^6x^6+bc^4x^4+2\,bc^2x^2+2\,b\right)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(3\,bc^5x^5-bc^3x^3+28\,bcx\right)\sqrt{-c^4x^4}}{}$$

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(15*(b*c^6*x^6 + b*c^4*x^4 + 2*b*c^2*x^2 + 2*b)*sqrt(-c^4*x^4 + 1)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (3*b*c^5*x^5 - b*c^3*x^3
 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 15*(b*c^2*x
^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 1)/(c^2*x^2 + 1)) + 15*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1
)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 15*(a*c^6*x^6 + a
*c^4*x^4 + 2*a*c^2*x^2 + 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 + c^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*x
^8 + c^4*x^4 - 2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x +
1)*sqrt(-c*x + 1)*c^8) - 6*integrate((x^7*log(c) + x^7*log(x))*e^(-1/2*log(
c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 6*integrate(1/6*
(c^6*x^7 - c^4*x^5 + 2*c^2*x^3 - 2*x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*c^6 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^6), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^7\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.176 $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal result . . . 1250
Rubi [A] (verified) . . . 1250
Mathematica [A] (verified) . . . 1253
Maple [F] . . . 1253
Fricas [B] (verification not implemented) . . . 1253
Sympy [F] . . . 1254
Maxima [F] . . . 1254
Giac [F] . . . 1254
Mupad [F(-1)] . . . 1255

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} - \frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1-c^2x^2}}\right)}{2c^3\sqrt{-c^2x^2}}$$

```
[Out] -1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(-c^2*x^2-1)^(1/2))/c^3/(-c^2*x^2)^(1/2)
-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/c^4+1/2*b*x*(-c^4*x^4+1)^(1/2)/c
^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {267, 6445, 12, 1586, 1266, 862, 52, 65, 214}

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} + \frac{b\sqrt{c^2x^2+1}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}} - \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}$$

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^5*Sqrt[1 + 1/(c^2*x^2)]*x)
- (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^4) + (b*Sqrt[1 + c^2*x^2]*A
rcTanh[Sqrt[1 - c^2*x^2]])/(2*c^5*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1586

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q]), Int[x^(m + mn*q)*(1 + d*(1/(x^m
n*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && E
qQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} + \frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} - \frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}\,dx}{2c^5} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1+c^2x^2}}\,dx}{2c^5\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{4c^5\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^2x}}{x}\,dx, x, x^2\right)}{4c^5\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} \\
&\quad - \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{4c^5\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}$$
$$+ \frac{\left(b\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{2c^7\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4} + \frac{b\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2c^5\sqrt{1+\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$
$$-\frac{a\sqrt{1-c^4x^4} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}}{1+c^2x^2} + b\sqrt{1-c^4x^4}\operatorname{csch}^{-1}(cx) + b\log\left(x+c^2x^3\right) - b\log\left(1+c^2x^2+c\sqrt{1+}\right.}{2c^4}$$

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(a*Sqrt[1 - c^4*x^4] + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4])
/(1 + c^2*x^2) + b*Sqrt[1 - c^4*x^4]*ArcCsch[c*x] + b*Log[x + c^2*x^3] - b*
Log[1 + c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^4
```

Maple [F]

$$\int \frac{x^3(a+b\ \operatorname{arccsch}(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(110) = 220$.

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.04

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$
$$-\frac{2\sqrt{-c^4x^4+1}bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2\sqrt{-c^4x^4+1}(bc^2x^2+b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (bc^2x^2+b)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4}}{c}\right.}{4\left(c^6x^2+c^4\right.}$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*sqrt(-c^
4*x^4 + 1)*(b*c^2*x^2 + b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) - (b*c^2*x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + (b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c
^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 2*sqrt(
-c^4*x^4 + 1)*(a*c^2*x^2 + a))/(c^6*x^2 + c^4)
```

Sympy [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3(a+b\,\operatorname{acsch}(cx))}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**3*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Maxima [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*((c^4*x^4 - 1)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x
 + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate((x^3*log(c) + x^3*log(x))*e^(-1/2*l
og(c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 2*integrate(1
/2*(c^2*x^3 - x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2 + sqrt
(c*x + 1)*sqrt(-c*x + 1)*c^2), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

Giac [F]

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\operatorname{arcsch}(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.177 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal result 1256
Rubi [N/A] 1256
Mathematica [N/A] 1257
Maple [N/A] (verified) 1257
Fricas [N/A] 1257
Sympy [N/A] 1257
Maxima [N/A] 1258
Giac [N/A] 1258
Mupad [N/A] 1258

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\text{arccsch}\,(cx)}{x\sqrt{-c^4x^4+1}}dx$$

```
[In] int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [N/A]

Not integrable

Time = 7.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x\sqrt{-(cx-1)\,(cx+1)\,(c^2x^2+1)}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(
c*x - 1))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 6.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.178 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal result 1259
Rubi [N/A] 1259
Mathematica [N/A] 1260
Maple [N/A] (verified) 1260
Fricas [N/A] 1260
Sympy [N/A] 1260
Maxima [N/A] 1261
Giac [N/A] 1261
Mupad [N/A] 1261

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\ \text{arccsch}\left(cx\right)}{x^5\sqrt{-c^4x^4+1}}dx$$

```
[In] int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\left(cx\right)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [N/A]

Not integrable

Time = 85.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{acsch}\left(cx\right)}{x^5\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate((a+b*acsch(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x
))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 5.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{cx}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((a + b*asinh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1263

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'^')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])   #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
         #is checked before calling the grading function that is passed.
         #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```